ELECTRONIC
Transformers and Circuits

by

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PREFACE

The purpose of this book is twofold: first, to provide a reference book on the design of transformers for electronic apparatus and, second, to furnish electronic equipment engineers with an understanding of the effects of transformer characteristics on electronic circuits. Familiarity with basic circuit theory and transformer principles is assumed. Conventional transformer design is treated adequately in existing books, so only such phases of it as are pertinent to electronic transformers are included here. The same can be said of circuit theory; only that which is necessary to an understanding of transformer operation is given. It is intended that in this way the book will be encumbered with a minimum of unnecessary material. Mathematical proofs as such are kept to a minimum, but the bases for quantitative results are indicated. The A.I.E.E. "American Standard Definitions of Electrical Terms" gives the meaning of technical words used. Circuit symbols conform to A.S.A. Standards Z32.5—1944 and Z32.10—1944.

Chapter headings, except for the first two, are related to general types of apparatus. This arrangement should make the book more useful. Design data are included which would make tedious reading if grouped together. For instance, the design of an inductor depends on whether it is for power or wave filter work, and the factors peculiar to each are best studied in connection with their respective apparatus.

Parts of the book are based on material already published in the *Proceedings of the Institute of Radio Engineers*, *Electronics*, and *Communications*. Much of it leans heavily upon work done by fellow engineers of the Westinghouse Electric Corporation, the warmth of whose friendship I am privileged to enjoy. To list all their names would be a difficult and inadequate expression of gratitude, but I should be guilty of a gross omission if I did not mention the encouragement given me by Mr. D. G. Little, at whose suggestion this book was written.

R. L.

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I. INTRODUCTION

Transformers are needed in electronic apparatus to provide the different values of plate, filament, and bias voltage required for proper tube operation, and to maintain or modify wave shape and frequency response at different potentials. The very concept of impedance, so characteristic of electronics, almost necessarily presupposes the means of changing from one impedance level to another, and that means is commonly a transformer.

There are three common notions concerning electronic apparatus, all of which are erroneous. The first notion is that if the designer of electronic apparatus uses good circuits the apparatus will perform properly; he may know nothing about the influence that transformers have on the final performance of his apparatus, but he leaves that to the transformer designer and assumes that all will turn out well. The second notion is that the design of such transformers is a somewhat occult art, the secrets of which belong only to the initiated, and are properly outside the orbit of the electronics engineer. There is still another notion that is the reverse of the second one, namely, that since anyone can wrap turns of wire on a core, transformer design is so simple that the electronics engineer need not concern himself with it. With all due recognition of the necessity for division of labor, we will undertake here to show, first, that the electronics engineer cannot dispense with a first-hand knowledge of transformers; second, that transformer design is not foreign to his outlook but is built upon reasonable principles with which he can become readily familiar; and third, that transformer design is not child's play, but must be governed by intelligent application of the laws of physics.

Rectifiers, filters, and amplifiers are discussed in connection with transformers and inductors next, and then basic transformer theory is introduced.

1. Rectifiers. An example will demonstrate the interdependence of circuit and transformer design. Some years ago, a large contract for radio transmitters was let to several manufacturers simultaneously. In one contractor's design, the very low voltage regulation required of the main rectifier was made a subject of special study. The type of

rectifier tube, and the filter that changed the pulsating rectified current into a smooth continuous current, were selected with the idea of conserving material and space as much as possible. In the transformer design, the same idea was followed and every advantage was taken of the circuit used and the materials available to keep the transformer size smaller also. In the design used by another concern for this same apparatus, things were done differently. Although the same tube was used, the filter was more conventional, and the transformer was so restricted by arbitrary specifications that it was unduly large and cumbersome. The total result was a rectifier of considerably greater weight and space than the one previously referred to. The first manufacturer had difficulty convincing the customer's representatives his rectifier was not deficient in some respect. However, every test made upheld the soundness of his design, and so have several years of subsequent field experience.

There are numerous other examples of this principle. Unless the circuit designer has an appreciation of the factors which the transformer contributes, he cannot intelligently appraise these factors, cannot include them in his calculations, and his design suffers. On the other hand, if the transformer designer does not consider the circuit in his calculations, but instead designs by rules of thumb, his design also suffers. The overall result is then very unsatisfactory. This is one reason why some transformers have to be redesigned several times before the various apparatus requirements are met. It is a wasteful procedure, and totally unnecessary if both engineers have an adequate appreciation of the overall problem.

The variety of uses to which electronic transformers are put is a source of bewilderment to the power transformer designer. There is probably no field in which greater variety of uses is made; all are governed by principles of differing significance. The same may be said of size. Volume of space occupied ranges all the way from a unit the size of one's thumb to the giants which power our large broadcast stations, with ratings to correspond. In the ratio of the largest to the smallest, electronic transformer ratings may be said to be infinite, because the lowest rating, in watts, is zero. That is, many transformers are used for voltage step-up only and have no watt rating. In the realm of frequency the range extends from a fraction of one cycle per second up to several million cycles per second. And the end is not yet in sight.

2. Filters. In applying filters and transformers in circuits, engineers are more cognizant of filters, chiefly owing to the widespread use of attenuation charts which have come to us from telephone practice Not so widespread is knowledge of the limitations of filter charts. For ex-

ample, the effects of circuit Q, phase angle, and terminating impedance on attenuation near cut-off frequency are often ignored. Yet performance in the cut-off region is usually important, so that there is a continual gap between calculation and test results in this region. Attention to the qualifying factors closes this gap and makes actual performance predictable. The design of inductors is based solely on the proper values of inductance and Q, and without these filter design becomes merely cut-and-try, regardless of the amount of chart data available. Here again the influence of component design has such repercussion on circuit performance that closest liaison between the circuit and reactor design engineers is necessary. Ideally they should be, and sometimes are, one person; at least they should have full interchange of information and ideas.

3. Amplifiers. The limitations which inhere in transformers often influence the choice of amplifier tubes. For example, if a tube works into a high impedance load over a certain frequency range, it may be a physical impossibility to reduce capacitance effects enough to obtain the required response with tubes which would be desirable from a gain standpoint. This is an instance of the intimate tie-up of transformer design and amplifier performance. Transformer-coupled amplifiers now functioning at frequencies over 500 kc testify to the excellence of coordination between equipment and transformer design.

A corollary of this coordination is the usually hidden fact that the electrical design of much electronic equipment often originates with the transformer designer. Some engineers, recognizing this, relegate a complete amplifier, rectifier, or other part of their work, simply specifying performance required and giving the transformer designer considerable freedom in choice of circuit elements. Others, who cannot make such a separation, prefer to specify all possible operating conditions, but leave only the transformer for the transformer designer. In this latter case, of course, it is part of the designer's function to point out any improvements in performance which would be effected by a better choice of parts.

4. Transformers and Inductors. Like all electronic apparatus, transformers are subject to continual change. This has been especially so in the last few years because of the introduction of new materials and performance requirements. Some of the new materials are

(*a*) Grain-oriented core steel.
(*b*) Solventless impregnating varnish.
(*c*) Inorganic insulating tape.
(*d*) Impervious wire enamel.
(*e*) Low loss, powdered iron cores.

Through the use of these materials, it has been possible to

(*a*). Reduce drastically the size of audio and power transformers and reactors.

(*b*). Extend the upper operating frequency of transformers into the medium r-f range.

(*c*). Make small high voltage units which stand up in service.

(*d*). Make efficient transformers for the non-sinusoidal wave shapes such as are encountered in pulse, video, and sweep amplifiers.

(*e*). Design filters and reactors having sharper cut-off and higher Q than previously was thought possible.

Occasionally someone asks why electronic transformers cannot be designed according to curves or charts showing the relation between volts, turns, wire size, and power rating. Such curves have appeared in magazines and have been used for small control transformers. The idea is that by means of such charts any engineer can design his own transformer. However, this idea has not been found practicable for the following reasons.

(*a*) *Regulation.* This property is rarely negligible in electronic circuits. It often requires care and thought to use the most advantageous winding arrangement in order to obtain the proper IX and IR voltage drops. Sometimes the size is dictated by such considerations.

(*b*) *Frequency Range.* The low frequency end of a transformer operating range in a given circuit is determined by the transformer open-circuit inductance. The high frequency end is governed by the leakage inductance and distributed capacitance. Juggling the various factors, such as core size, number of turns, interleaving, and insulation, in order to obtain the optimum design constitutes a technical problem of the first magnitude.

(*c*) *Voltage.* It would be exceedingly difficult, if not impossible, to reduce to chart form the use of high voltages in the restricted space of a transformer. Circuit considerations are very important here, and the transformer designer must be thoroughly familiar with the functioning of the transformer to insure reliable operation, low cost, and small dimensions.

(*d*) *Size.* Much electronic equipment is cramped for space and, since transformers often constitute the largest items in the equipment, it is imperative that they, too, be of small size. An open-minded attitude toward this condition and the use of good judgment may make it possible to meet the requirements which otherwise might not be fulfilled. The use of new materials, too, can be instrumental in reducing size—in some instances down to a small fraction of former size.

5. Transformer Fundamentals. The simple transformer of Fig. 1 has two windings. The left-hand winding is assumed to be connected to a voltage source and is called the primary winding. The right-hand winding is connected to a load and is called the secondary. The transformer merely delivers to the load a voltage similar to that impressed across its primary, except that it may be smaller or greater in amplitude.

In order for a transformer to perform this function, the voltage across it must vary with respect to time. A d-c voltage such as that of a storage battery produces no voltage in the secondary winding or power in the load. If both varying and d-c voltages are impressed across the primary, only the varying part is delivered to the load. This comes about because the voltage e in the secondary is induced in that winding by the core flux ϕ according to the law

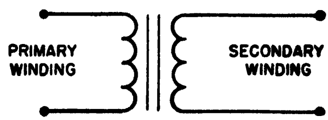


FIG. 1. Simple transformer.

$$e = -\frac{N\,d\phi}{dt} \times 10^{-8} \tag{1}$$

This law may be stated in words as follows: The voltage induced in a coil is proportional to the number of turns and to the time rate of change of magnetic flux in the coil. This rate of change of flux may be large or small. For a given voltage, if the rate of change of flux is small, many turns must be used. Conversely, if a small number of turns is used, a large rate of change of flux is necessary to produce a given voltage. The rate of change of flux can be made large in two ways, namely, by increasing the maximum value of flux and by decreasing the period of time over which the flux change takes place. At low frequencies, the flux changes over a relatively large interval of time, and therefore a large number of turns is required for a given voltage, even though moderately large fluxes are used. As the frequency increases, the time interval between voltage changes is decreased and, for a given flux, fewer turns are needed to produce a given voltage. And so it is that low frequency transformers are characterized by the use of a large number of turns, whereas high frequency transformers have but few turns.

If the flux ϕ did not vary with time, the induced voltage would be zero. Equation 1 is thus the fundamental transformer equation. The voltage variation with time may be of any kind: sinusoidal, exponential, sawtooth, or impulse. The essential condition for inducing a voltage in the secondary is that there be a flux *variation*. Only that part of the flux which links both coils induces a secondary voltage.

If all the flux links both windings, equation 1 shows that equal volts per turn are induced in the primary and secondary, or

$$\frac{e_1}{e_2} = \frac{N_1}{N_2} \tag{2}$$

where e_1 = primary voltage
e_2 = secondary voltage
N_1 = primary turns
N_2 = secondary turns.

6. Sinusoidal Voltage. If the flux variation is sinusoidal,

$$\phi = \Phi_{\max} \sin \omega t$$

where $\Phi_{\max}$ is the peak value of flux, ω is angular frequency, and t is time. Equation 1 becomes

$$e = -N\Phi_{\max}\omega \cos \omega t \times 10^{-8} \tag{3}$$

or the induced voltage also is sinusoidal. This voltage has an effective value

$$E = 0.707 \times 2\pi f N \Phi_{\max} \times 10^{-8}$$
$$= 4.44 f N \Phi_{\max} \times 10^{-8} \tag{4}$$

where f is the frequency of the sine wave. Equation 4 is the relation between voltage and flux for sinusoidal voltage.

Sufficient current is drawn by the primary winding to produce the flux required to maintain the winding voltage. The primary *induced* voltage in an unloaded transformer is just enough lower than the *impressed* voltage to allow this current to flow into the primary winding. If a load is connected across the secondary terminals, the primary induced voltage decreases further, to allow more current to flow into the winding in order that there may be a load current. Thus a loaded transformer primary carries both a magnetizing current and a load current, but only the load part is transformed into secondary load current.

7. Equivalent Circuit and Vector Diagram. With many sine wave electronic transformers, the transformer load is resistive. A tube filament heating load, for example, has 100 per cent power factor. At low frequencies the transformer can be represented by Fig. 2 (*a*), its approximate 1:1 turns ratio equivalent by Fig. 2 (*b*), and its vector diagram for 100 per cent p-f load by Fig. 2 (*c*). Secondary load voltage E_L and load current I_L are in phase. Secondary induced voltage E_S is greater than E_L because it must compensate for the winding resistances

and leakage reactances. The winding resistance and leakage reactance voltage drops are shown in Fig. 2 (*c*) as IR and IX, which are respectively in phase and in quadrature with I_L and E_L. These voltage drops are the sum of secondary and primary winding voltage drops, but the primary values are multiplied by a factor to be derived later. If voltage drops and losses are temporarily forgotten, the same power is

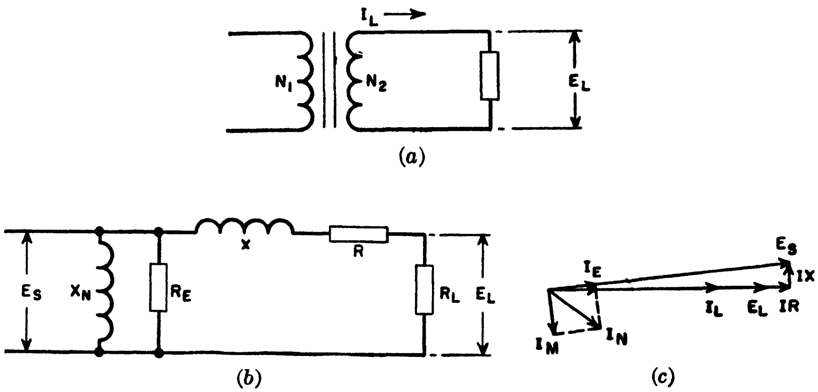


FIG. 2. (*a*) Transformer with resistive load; (*b*) equivalent circuit; (*c*) vector diagram.

delivered to the load as is taken from the line. Let subscripts 1 and 2 denote the respective primary and secondary quantities.

$$E_1 I_1 = E_2 I_2 \tag{5}$$

or

$$\frac{E_1}{E_2} = \frac{I_2}{I_1} \tag{6}$$

so that the voltages are inversely proportional to the currents. Also, from equation 2, they are directly proportional to their respective turns.

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{2a}$$

Now the transformer may be replaced by an impedance Z_1 drawing the same current from the line, so that

$$I_1 = \frac{E_1}{Z_1}$$

Likewise

$$I_2 = \frac{E_2}{Z_2}$$

where Z_2 is the secondary load impedance, in this case R_L. If these expressions for current are substituted in equation 2*a*,

$$\frac{Z_1}{Z_2} = \left(\frac{E_1}{E_2}\right)^2 = \left(\frac{N_1}{N_2}\right)^2 \tag{7}$$

Equation 7 is strictly true only for negligible voltage drops and losses. It is approximately true for voltage drops up to about 10 per cent of the winding voltage or for losses less than 20 per cent of the power delivered; but it is not true when the voltage drops approach in value the winding voltage, or when the losses constitute most of the primary load.

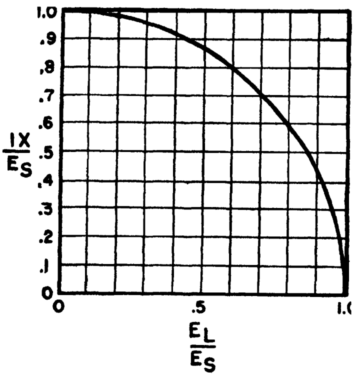


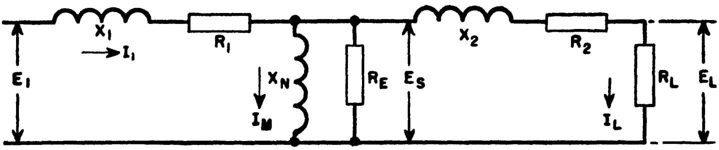
FIG. 3. Relation between reactive voltage drop and load voltage.

Not only does the load impedance bear the relation of equation 7 to the equivalent primary load impedance; the winding reactance and resistance drops IX and IR may also be referred from one winding to the other by the same ratio. This can be seen if the secondary voltage drops are considered part of the load, across which the secondary induced voltage E_S appears. Thus the factor by which the primary voltage drops are multiplied, to refer them to the secondary for addition to the secondary drops, is $(N_2/N_1)^2$.

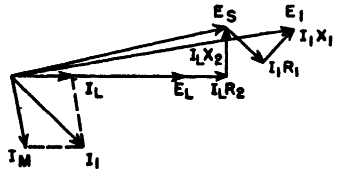
In Fig. 2 (*c*) the IR voltage drop subtracts directly from the terminal voltage across the resistive load, but the IX drop makes virtually no difference. How much the IX drop may be before it becomes appreciable is shown in Fig. 3. If IX drop is 30 per cent of the induced voltage, 4 per cent reduction in load voltage results; 15 per cent IX drop causes but 1 per cent reduction.

8. Magnetizing Current. In addition to the current entering the primary because of the secondary load, there is the core exciting current I_N which flows in the primary whether the secondary load is connected or not. This current is drawn by the primary open-circuit reactance X_N and equivalent core loss resistance R_E and is multiplied by N_1/N_2 when it is referred to the secondary side. It has two components: I_M, the magnetizing component which flows 90 degrees lagging behind induced voltage E_S; and I_E, the core loss current which is in phase with E_S. Ordinarily this current is small and produces neg-

ligible voltage drop in the winding. Practical cases sometimes arise where the magnetizing component becomes of the same order of magnitude as I_L. Because current I_M flows only in the primary, a different equivalent circuit and vector diagram are necessary, as shown in Fig. 4. Note that the leakage reactance voltage drop has a marked effect upon the load voltage, and this effect is larger as I_M increases relative to I_L. Therefore, the statement that IX voltage drop causes



(a)



(b)

FIG. 4. (a) Equivalent circuit and (b) vector diagram for transformer with high magnetizing current.

negligible difference between secondary induced and terminal voltages in transformers with resistive loads is true only for small values of exciting current.

9. Regulation, Efficiency, and Power Factor. Transformer regulation is the difference in the secondary terminal voltage at full load and at no load, expressed as a percentage of full load voltage. For the resistive load of Fig. 2 (a, b, and c),

$$\text{Per cent regulation} = 100\,\frac{(E_S - E_L)}{E_L} \tag{8}$$

Since with low values of leakage reactance $E_S - E_L = IR$,

$$\text{Per cent regulation} = \frac{100IR}{E_L} \tag{9}$$

provided R includes the primary winding resistance multiplied by the factor $(N_2/N_1)^2$ as well as the secondary winding resistance.

Efficiency is the ratio

$$\eta = \frac{\text{Output power}}{\text{Output power plus losses}} \tag{10}$$

where losses include both core and winding losses.

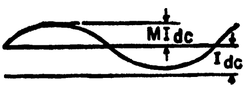
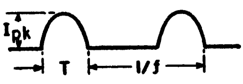
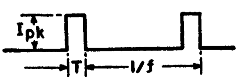
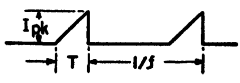
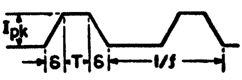
A convenient way of expressing power factor is

$$\text{Power factor} = \frac{\text{Output power plus losses}}{\text{Input volt-amperes}} \tag{11}$$

One of the problems of transformer design is the proper choice of flux to obtain low values of exciting current and high power factor. Low power factor may cause excessive primary winding copper loss, low efficiency, and overheating.

10. Wave Shapes. Although there is a relation between the current and voltage wave shapes in a transformer, the two are frequently not the same. Current waves may be of several shapes, even in 60-cycle plate power transformers. Voltage wave form will be dealt with in subsequent chapters. Common current wave forms are tabulated here for convenience. (See Table I.)

TABLE I. NON-SINUSOIDAL CURRENT WAVE FORMS

Current Wave Shape	Description	I_{rms}	I_{av}
	Direct current with superposed sine wave	$I_{dc}\sqrt{1+\frac{M^2}{2}}$	I_{dc}
	Half-sine loops of T duration and f repetition rate	$I_{pk}\sqrt{\frac{fT}{2}}$	$\frac{2I_{pk}fT}{\pi}$
	Square waves of T duration and f repetition frequency	$I_{pk}\sqrt{fT}$	$I_{pk}fT$
	Sawtooth wave of T duration and f repetition frequency	$I_{pk}\sqrt{\frac{fT}{3}}$	$\frac{I_{pk}fT}{2}$
	Trapezoidal wave of f repetition frequency	$I_{pk}\sqrt{\frac{f(2\delta+3T)}{3}}$	$I_{pk}f(\delta+T)$

Root-mean-square or rms current values are based upon the equation

$$I_{\text{rms}} = \sqrt{f\int_0^T i^2\,dt} \tag{12}$$

where i = current at any instant
f = frequency of repetition of current waves per second
T = duration of current waves in seconds
t = time in seconds.

Average current values are

$$I_{\text{av}} = f\int_0^T i\,dt \tag{13}$$

In the first wave shape, $T = 1/f$.

PROBLEMS

1. A transformer has a 100-turn primary winding, across which are applied 115 volts. Neglecting losses and regulation, determine the number of secondary turns required to develop 460 volts. If a 1.0-amp load is connected to the secondary, what are the primary current, the primary volt-amperes, the load impedance, and the equivalent primary impedance? *Ans.* 400; 4; 460; 460 ohms; 28.8 ohms.

2. In a filament transformer supplying 10 amp at 5 volts, there are 20 secondary turns, the secondary winding resistance is 0.05 ohm, and the total leakage reactance referred to the secondary is 0.1 ohm. If the primary is supplied by a 115-volt line with very low exciting current and the primary winding resistance is 25 ohms, how many primary turns must be used to deliver full output? *Ans.* 362.

3. What are the regulation, efficiency, and power factor of the transformer in Prob. 2 if the core loss is disregarded?
Ans. 21.7, 80.5, and 98.8 per cent respectively.

4. A transformer draws 1-amp magnetizing current from a 115-volt supply line. What is the primary no-load induced voltage if (*a*) the primary winding has zero reactance and 20 ohms resistance, (*b*) the primary winding has 20 ohms reactance and zero resistance. *Ans.* (*a*) 113 volts; (*b*) 95 volts.

5. Derive the rms and average current values in Table I, using equations 12 and 13.

6. If a steady value of direct current is added to any of the last four wave forms, what are the average and rms currents? What are they if the direct current components inherent in these wave forms is subtracted?

II. TRANSFORMER CONSTRUCTION, MATERIALS, AND RATINGS

11. Construction. Most electronic transformers are small, and for small transformers, the shell-type core is usually most suitable because only one coil is required. Figure 5 shows shell-type core and coil assemblies.

The magnetic path is divided, half the flux enclosing one side of the coil, and half the other. The coil opening is called the window. Between the windows is the core tongue, which is twice as wide as the

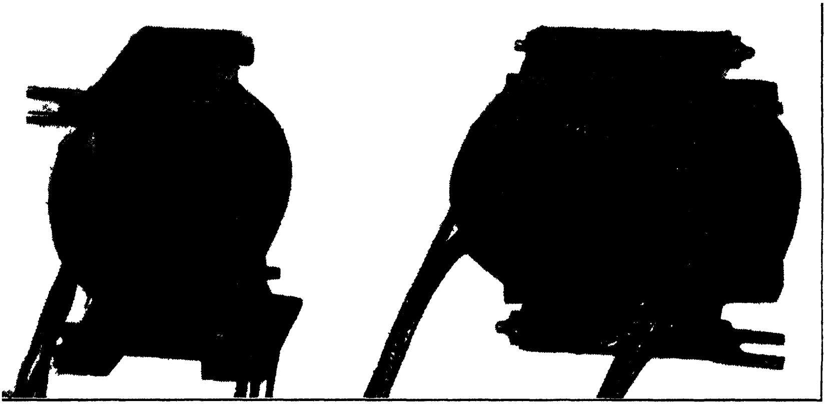


FIG. 5. Transformers with shell-type core.

iron around the rest of the window. The core is built up of thin laminations to reduce eddy current losses; typical shapes are shown in Fig. 6. Alternate stacking of the lamination pairs may be used to reduce magnetic reluctance and keep magnetizing current small. To reduce assembly cost, this alternate stacking is sometimes done in groups of two or more laminations, with some increase in magnetizing current. A wide range of sizes of shell-type laminations is available. At 60 cycles, common thicknesses are 0.014 in., 0.019 in. and 0.025 in.

Shell-type laminations are made with proportions to suit the transformer. In the E-I shape a scrapless lamination is widely used. Two

E's facing each other are first punched, and the punched-out strips are of the right dimensions to form two I's. Then the E's are cut apart.

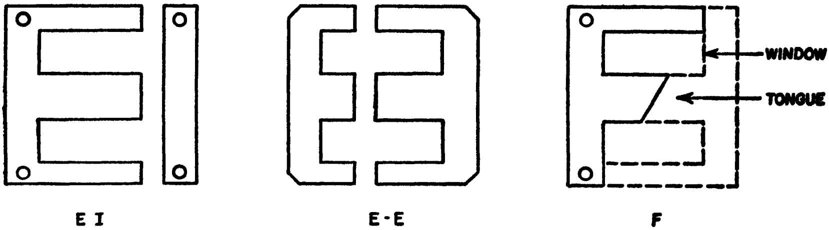


FIG 6. Shell-type laminations.

This economy of material is not justified in transformers in which turns per layer, and hence window width, must be reduced relative to window height.

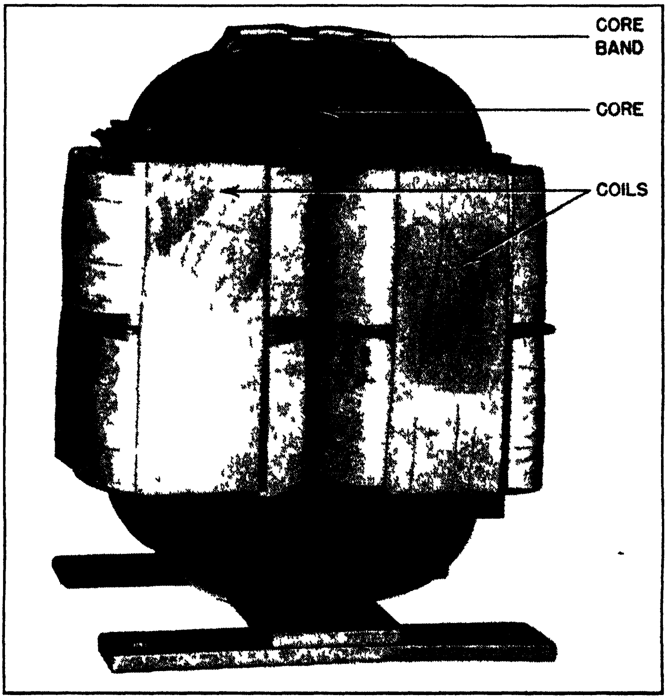


FIG. 7. Core-type transformer.

For some applications, the core-type transformer is preferable. In these there is only one magnetic path, but there are two coils, one on

each leg of the core. A core-type transformer is shown in Fig. 7, and some core-type laminations in Fig. 8.

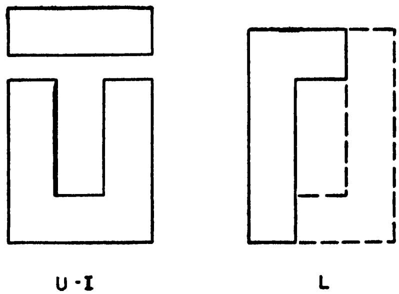


FIG. 8. Core-type laminations

In recent years, cores wound from continuous steel strip have come into use. A widely used shape is illustrated in Fig. 9; it is known as the type C core. Steel strip is first wound to the proper build-up on a

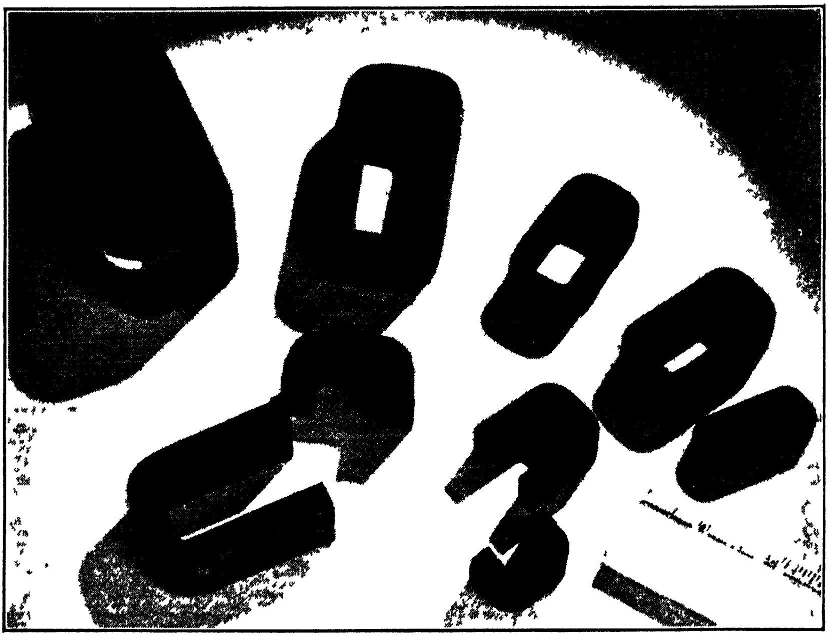


FIG. 9. Type C cores.

mandrel. The wound core is then annealed, impregnated with a bond, and cut in two to permit assembly with the coil. After assembly with the

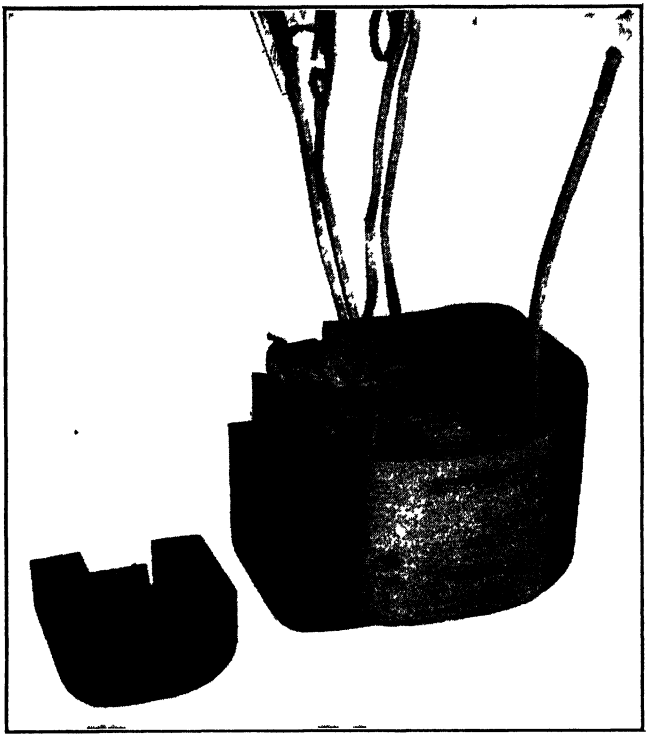


FIG 10. Partly assembled transformer.

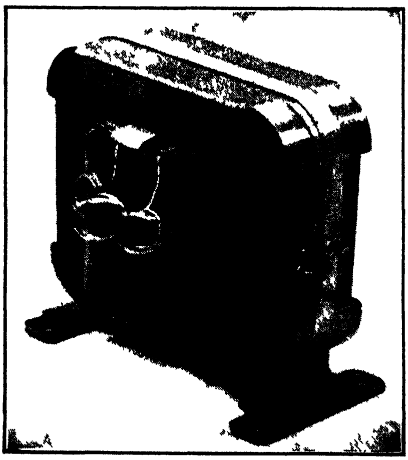


FIG. 11. Assembled type C cores and coil.

coil, the core is held together with a steel band. Several advantages accrue from this construction, which will be discussed in Section 13.

Typical assemblies using two type C cores are shown in Figs. 10 and 11; they correspond to shell-type laminations. Because it is simpler to assemble a single-core loop, a single core is often used, especially in small sizes. See Fig. 12. In 60-cycle service the laminations are usually stacked alternately to produce an overlapping joint. This is approximated in the type C cores with ground gap surfaces which fit

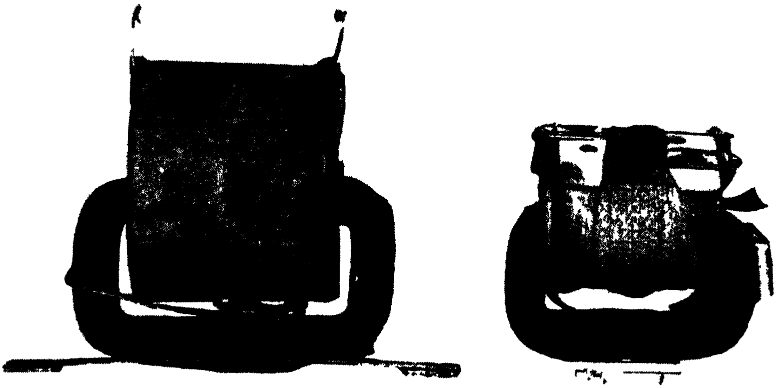


FIG. 12. Single-core single-coil assemblies.

closely together. Either type of core can be used with core gaps: laminations, by butt stacking without overlap, and type C cores, by inserting the desired amount of gap material (paper or micarta) between the loops.

12. Mountings. Both types of cores may be built into neat assemblies with the laminations exposed, and the coils covered by end cases, such as those in the amplifier of Fig. 13. When complete enclosure is desired, assemblies like those in Fig. 14 are used.

The degree of enclosure depends on many conditions, among them the following.

(*a*) *Climate.* In a humid climate, especially in the tropics, copper corrodes readily. Transformers containing fine wire may have open circuits soon after exposure to tropical conditions, and it is preferable to seal them against the entry of moisture.

(*b*) *Temperature Rise.* Transformers handling large amounts of power may become hot because of the electrical losses. To seal them

in containers imposes additional obstacles to the dissipation of this heat. Fortunately the wire size is usually large enough in such units to withstand corrosion without developing open circuits in the windings.

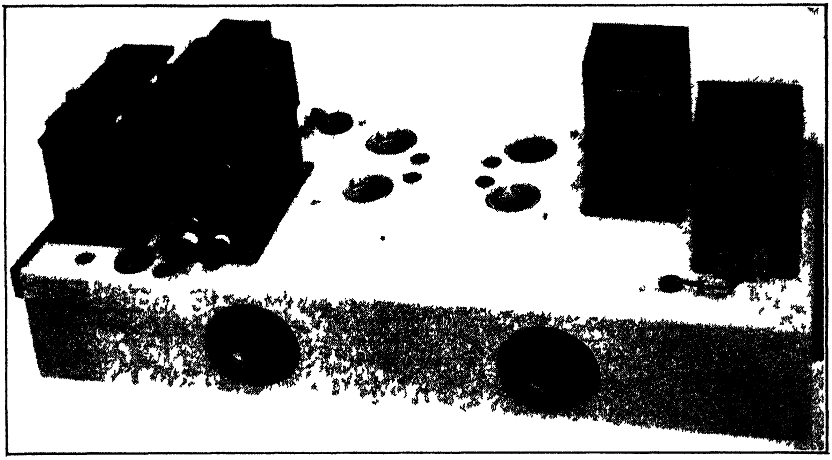


FIG 13 Transformers mounted on amplifier chassis.

(*c*) *Space* Sealing a transformer usually requires more space than mounting the core and coil directly on the chassis or panel. End cases like those in Fig 13 do not require much space but do reduce cooling by convection. When air is used to cool other apparatus, power tubes

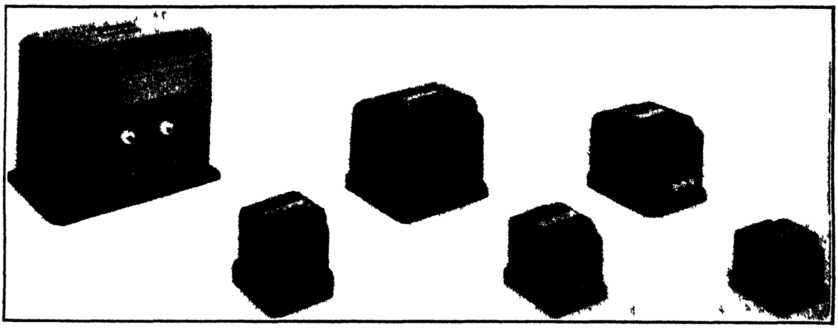


FIG. 14. Fully enclosed transformers.

for instance, it is very often circulated near or through the transformer to prevent the coils from overheating.

(*d*) *Voltage.* In high voltage dry-type transformers, enclosure in a metal case may add to the difficulties of insulating the windings. In

oil-filled transformers, a tank is required for the oil and enclosure is thereby provided.

(*e*) *Appearance.* Generally speaking, enclosed transformers are neater than the open type. This fact is given consideration where space is available, especially in broadcast apparatus.

13. Core Materials. Electronic transformers make use of a large variety of core steels. The principal grades are listed in Table II, together with the maximum permeability, saturation flux density, and chief uses of each grade.

TABLE II. CORE STEEL PROPERTIES

Grade of Steel	Typical Maximum Permeability	Maximum Operating Flux Density (Gauss)	Chief Uses
Silicon steel	8,500	12,000	Small power and voice frequency audio transformers
Hipersil *	30,000	17,000	Larger sizes of power and wide range audio transformers; low frequency r-f transformers
Hipernik †	80,000	10,000	Small, wide range audio transformers; audio filter reactors
Mumetal ‡	200,000	6,000	Small, wide range audio transformers; audio filter reactors
Conpernik §	1,400	‖	Extremely linear and low loss transformers
Powdered iron	80	‖	Low and medium frequency r-f transformers

* Trade name for a high permeability silicon steel. It is a Westinghouse grain-oriented material.

† Trade name for a high permeability nickel alloy. It is a Westinghouse material. Approximate equivalents are General Electric Nicaloi and Allegheny Electric Metal.

‡ Trade name for an Allegheny material. A Western Electric equivalent is Permalloy.

§ Trade name for a constant permeability nickel alloy. It is a Westinghouse material.

‖ These materials are used for low flux density, low loss applications.

As in usual power transformer practice, it is necessary to avoid exceeding saturation flux densities, because high exciting currents produce high winding *IR* drops, high losses, low efficiency, and large size. Curves of induction and core loss are available from manufacturers of laminations. Grades and thickness are designated by numbers such as Armco 58 and Allegheny Transformer A. A wide choice

of silicon steel laminations is available in 0.014-in., 0.019-in., and 0.025-in. thicknesses, with silicon content of approximately 3 to 4 per cent, and with core losses ranging from 0.6 to 1.2 watts per pound at 10,000 gauss 60 cycles (64,500 lines per square inch).

Recent core steel developments affect the size and performance of electronic transformers. Among the new steels is Hipersil, a steel in which the direction of grain orientation is controlled during the manufacturing process. If the flux is made to flow in this preferred or grain-oriented direction, high core inductions may be realized. Type C cores fulfill this requirement, because the strip is wound in the same direction as the flux path.

The material is rolled in three major thicknesses:

No. 29 gage (about 13 to 14 mils thick) for frequencies up to 400 cycles
5 mils thick for frequencies higher than 400 cycles
2 mils thick for frequencies in the low and medium r-f bands

Probably the most remarkable property of this material is its high saturation point. In Fig. 15 the comparison is given in terms of a

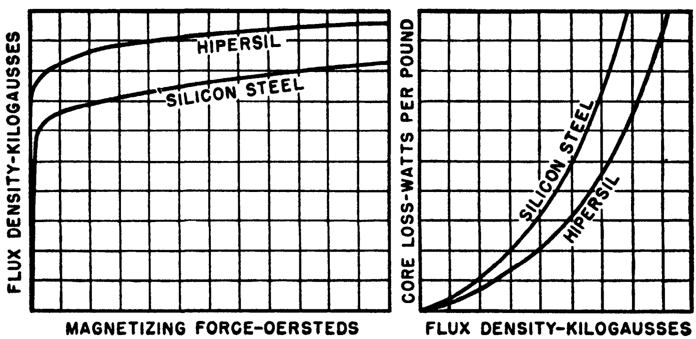


FIG. 15. Saturation and core-loss curves of silicon steel and Hipersil at 60 cycles.

hypothetical 60-cycle working induction using high grade, conventional silicon steel. If this value is assumed to be 100 per cent, the induction obtained with grain-oriented steel is 130 to 150 per cent, with no increase in magnetizing force. Another way of expressing this improvement is shown in Fig. 16 as a comparison of the permeability of the two steels. The permeability of grain-oriented steel is greatly higher at the maximum point, and has the same percentage increase as in Fig. 15 for normal working inductions. Iron loss in Hipersil is less than in silicon steel, as Fig. 15 shows. The decrease in iron loss is chiefly due to a reduction in hysteresis loss; the eddy cur-

rent loss is less affected by grain orientation. Core materials undergo continuous improvement, and future comparisons may widen these differences.

The increase in induction is beneficial in several ways. First, it permits a reduction of core area for the same magnetizing current. Second, it results in a smaller mean length of turn and thus in a reduction in the amount of copper needed. In distribution and power transformers, for maximum benefit the iron and copper losses are reproportioned. In small electronic transformers, the iron loss is usually a small part of the total loss, and the reduction in copper loss is of greater

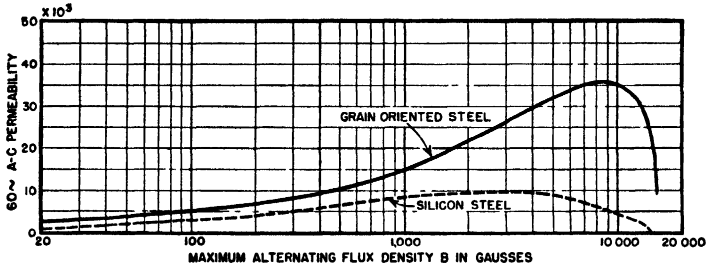


FIG. 16. Permeability of silicon and grain-oriented steel.

significance. Within certain limits, the sum of the two losses determines the size of a transformer, and here the usefulness of grain-oriented steel becomes most apparent.

The foregoing was written with 60-cycle applications particularly in mind. At higher power supply frequencies, such as the 400- and 800-cycle supplies encountered in aircraft and portable equipment, the results are somewhat different. The decrease in iron loss is not so marked, because the eddy current loss forms a larger proportion of the total iron loss. However, it is usual practice to use thin-gage laminations at these frequencies, and much better space factor can be obtained in wound cores than in stacked cores. The increase in permeability is just as effective in these higher frequency applications as at 60 cycles. The net result is a smaller transformer than was formerly possible, though for different reasons and in different proportions.

Reactors which carry direct current are usually smaller when made with grain-oriented than with ordinary silicon steel. At low voltages, where low inductions are involved, grain-oriented steel has greater incremental permeability, and maintains it at high flux densities also. Consequently, a reduction of 50 per cent in weight is often feasible.

Grain-oriented steel does not replace high nickel-iron alloys for audio transformers, when they work at low inductions, and with little

or no direct current. Some nickel-iron alloys have higher permeability at low flux densities; and their use for this purpose continues. But at high inductions, or where considerable amounts of direct current are involved, nickel-iron alloys fade out because of saturation, and grain-oriented steel is used. Lower distortion, extended frequency range, or small size are the result, and sometimes a combination of all three occurs.

Hipersil can be used for transformers in various applications in the low and medium frequency r-f bands, at power levels ranging up to 200 kw. The same is true of video and pulse transformers, which

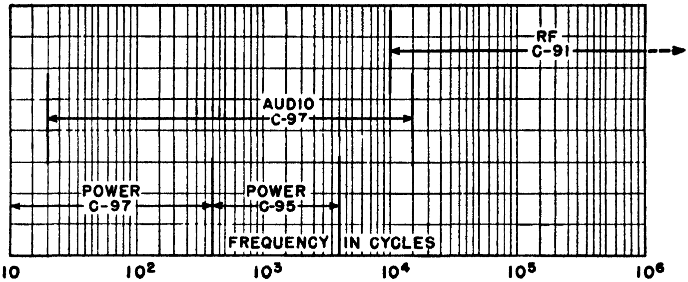


FIG. 17. Use of Hipersil in various frequency zones.

may be regarded as covering an extended frequency range down into the audio range and up into the medium radio frequency range. Such transformers are grouped rather loosely together as r-f transformers in the diagram shown in Fig. 17. In this figure the several classifications, r-f, audio, and power transformers, are shown with respect to their frequency ranges and the approximate gage of the material used for these ranges. The gage is indicated by the symbol number in Table III.

TABLE III. HIPERSIL CORE DATA

Hipersil	Thickness	Typical Hipersil Space Factor *	Typical Space Factor for Silicon Steel *
C-97	0.013 in.	95%	90%
C-95	0.005 in.	90%	80%
C-91	0.002 in.	85%	70%

* Refers to percentage of core volume occupied by metal. The Hipersil figure is for type C cores, and the silicon steel figure is for punched laminations.

14. Windings. Current density in the winding copper is sometimes estimated for design purposes by rules such as 1000 cir mils per amp. These rules are useful in picking out a first choice of wire size for a

given current requirement, but should not be regarded as final. Instead, the temperature rise, regulation, or other performance criterion should govern the final choice of wire size. Regulation is calculated as in Section 9, and temperature rise as in Sections 20 and 21. In Fig. 18 the circular mils per ampere are plotted for small enclosed dry-type transformers with Hipersil cores and a winding temperature rise of

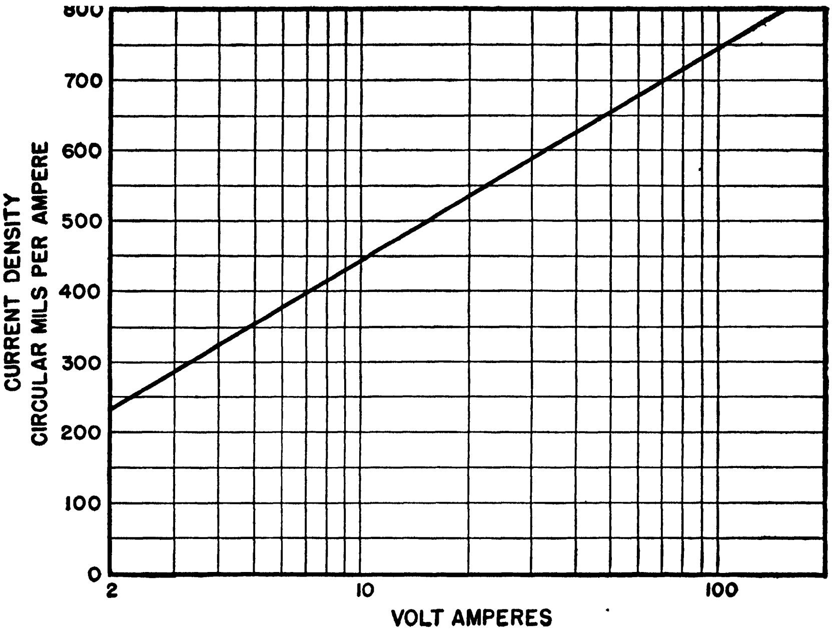


FIG. 18. Current density in windings of small enclosed 60-cycle transformers.

55 centigrade degrees; it can be seen to vary appreciably over this range of sizes.

Space occupied by the wire depends on the wire insulation as well as on the copper section. This is especially noticeable in small wire sizes. Table IV gives the bare and insulation diameters for several commonly used kinds of wire and Table V the turns per square inch of winding space. Space usually can be saved by avoiding the use of cotton or silk wire covering, and instead using enameled wire with paper layer insulation as in Fig. 19. Thickness of layer paper may be governed by layer voltage; it is good practice to use 50 volts per mil of paper. In coils where layer voltage is low, the paper thickness is determined by the mechanical strength necessary to produce even layers and a tightly wound coil. Table VI gives the minimum paper thickness based on this consideration.

TABLE IV. INSULATED WIRE SIZES

B & S Gage	Bare Diameter	Diameter of Insulated Wire								Area in Circular Mils	Ohms per 1000 Feet at 25°C	Feet per Ohm at 25°C	Pounds per 1000 Feet
		Single Enamel	Double Enamel	Single Cotton Enamel	Single Silk Enamel	Single Cotton	Double Cotton	Single Silk	Double Silk				
44	.0020	.0023								4.00	2,700	.3850	.012
43	.0022	.0025								4.84	2,150	.4670	.015
42	.0025	.0029								6.25	1,700	.6050	.019
41	.0028	.0032								7.84	1,350	.7630	.024
40	.0031	.0036	.0039							9.61	1,103	.9550	.030
39	.0035	.0040	.0044							12.25	864	1.204	.038
38	.0040	.0046	.0050							16.00	659	1.519	.048
37	.0045	.0051	.0055							20.30	522	1.915	.060
36	.0050	.0057	.0061	.0095	.0075	.0090	.0130	.0070	.0090	25.00	424	2.414	.076
35	.0056	.0064	.0067	.0102	.0082	.0096	.0136	.0076	.0096	31.40	338	3.045	.096
34	.0063	.0072	.0077	.0109	.0089	.0103	.0143	.0083	.0103	39.70	266	3.839	.120
33	.0071	.0080	.0085	.0117	.0097	.0111	.0151	.0091	.0111	50.40	210	4.841	.152
32	.0080	.0090	.0095	.0127	.0107	.0120	.0160	.0100	.0120	64.00	165	6.105	.19
31	.0089	.0100	.0104	.0137	.0117	.0129	.0169	.0109	.0129	79.20	134	7.698	.24
30	.0100	.0111	.0117	.0148	.0128	.0140	.0180	.0120	.0140	100	106	9.707	.31
29	.0113	.0125	.0130	.0162	.0142	.0153	.0193	.0133	.0153	128	83.1	12.24	.38
28	.0126	.0139	.0145	.0175	.0155	.0166	.0206	.0146	.0166	159	66.4	15.43	.48
27	.0142	.0155	.0161	.0192	.0172	.0182	.0222	.0162	.0182	202	52.5	19.46	.61
26	.0159	.0172	.0178	.0210	.0190	.0199	.0239	.0179	.0199	253	41.7	24.54	.77
25	.0179	.0193	.0200	.0234	.0211	.0222	.0262	.0199	.0219	320	33.0	30.95	.97
24	.0201	.0216	.0222	.0256	.0233	.0244	.0284	.0221	.0241	404	26.2	39.02	1.23
23	.0226	.0242	.0247	.0282	.0259	.0269	0309	.0246	.0266	511	20.7	49.21	1.54
22	.0253	.0271	.0278	.0310	.0287	.0296	.0336	.0273	.0293	645	16.4	62.05	1.95
21	.0285	.0302	.0310	.0344	.0319	.0330	.0370	.0305	.0325	812	13.0	78.25	2.45
20	.0320	.034	.0345	.0385	.0355	.0370	.0410	.0340	.0360	1,020	10.3	98.66	3.09
19	.0359	.038	.0387	.0425	.0395	.0409	.0449	.0379	.0399	1,300	8.14	124.4	3.89
18	.0403	.042	.0431	.0469	.0439	.0453	.0493	.0423	.0443	1,600	6.59	156.9	4.9
17	.0453	.047	.0481	.0521	.0491	.0503	.0543	.0473	.0493	2,030	5.22	197.8	6.2
16	.0508	.053	.0536	.0576	.0546	.0558	.0608	.0528	.0548	2,600	4.07	249.4	7.8
15	.0571	.059	.0605	.0640	.0610	.0621	.0671	.0591	.0611	3,250	3.26	314.5	9.9
14	.0641	.066	.0675	.0711	.0681	.0691	.0741	.0661	.0681	4,100	2.58	396.6	12.4
13	.0719									5,180	2.00	499.3	15.7
12	.0808									6,530	1.59	629.6	19.8
11	.0907									8,235	1.26	794.0	24.9
10	.1019									10,380	1.00	1,001	31.4
9	.1144									13,090	.792	1,262	40.0
8	.1285									16,510	.628	1,592	50.0

TABLE V. TURNS PER SQUARE INCH OF INSULATED WIRE

B & S Gage	Single Enamel Wire	Double Enamel Wire	Single Cotton Enamel Wire	Single Silk Enamel Wire	Single Cotton-Covered Wire	Double Cotton-Covered Wire	Single Silk-Covered Wire	Double Silk-Covered Wire
42	119,000							
41	96,000							
40	77,000	66,200						
39	62,400	51,800						
38	47,300	40,000						
37	38,400	33,100						
36	30,900	26,900	11,100	17,900	12,350	5,920	20,400	12,350
35	24,500	22,300	9,600	14,900	10,900	5,430	17,200	10,900
34	19,300	16,900	8,430	12,700	9,430	4,900	14,500	9,430
33	15,600	13,900	7,280	10,650	8,130	4,380	12,100	8,130
32	12,350	11,100	6,210	8,740	6,940	3,900	10,000	6,940
31	10,000	9,260	5,330	7,300	5,900	3,510	7,780	5,900
30	8,180	7,300	4,580	6,100	5,100	3,090	6,940	5,100
29	6,430	5,920	3,810	4,950	4,270	2,760	5,670	4,270
28	5,200	4,770	3,280	4,170	3,640	2,360	4,690	3,640
27	4,170	3,880	2,720	3,390	3,030	2,080	3,810	3,030
26	3,380	3,160	2,270	2,780	2,520	1,940	3,120	2,520
25	2,690	2,500	1,820	2,240	2,080	1,460	2,530	2,080
24	2,150	2,030	1,530	1,850	1,690	1,230	2,050	1,720
23	1,710	1,650	1,260	1,490	1,380	1,050	1,650	1,420
22	1,370	1,300	1,045	1,220	1,140	883	1,345	1,160
21	1,100	1,045	846	925	915	729	1,075	943
20	860	850	675	793	730	595	862	836
19	693	668	555	640	597	495	700	628
18	568	540	455	518	490	412	563	510
17	455	432	368	417	395	340	450	412
16	357	350	303	338	320	270	360	335
15	288	273	244	270	260	222	287	268
14	230	220	198	216	210	182	229	222
13	179	176						
12	143	141						
11	114	113						
10	90	90						
9	72	72						
8	57	57						

TABLE VI. PAPER-INSULATED COIL DATA

(Courtesy Phelps-Dodge Copper Products Corp.)

B & S Gage	Layer Insulation	Turns per Inch	Space Factor
44	.0005″	369	85%
43	.0005″	340	85%
42	.0005″	304	85%
41	.0007″	265	85%
40	.0007″	239	86%
39	.0007″	215	86%
38	.001″	193	87%
37	.001″	170	87%
36	.001″	155	87%
35	.001″	140	88%
34	.001″	124	88%
33	.0013″	110	88%
32	.0013″	98	88%
31	.0015″	88	88%
30	.0015″	80	89%
29	.0015″	71	89%
28	.0015″	64	89%
27	.0022″	57	89%
26	.0022″	52	89%
25	.0022″	47	90%
24	.0022″	42	90%
23	.005″	37	90%
22	.005″	33	90%
21	.005″	30	90%
20	.005″	26	90%
19	.007″	23	90%
18	.007″	21	90%
17	.007″	19	90%
16	.010″	17	90%
15	.010″	15	90%
14	.010″	13	90%
13	.010″	12	90%
12	.010″	10	90%
11	.010″	9	90%
10	.010″	8	90%

Space factor may refer to linear spacing as across a layer, or to the total coil section area. It is more convenient to use linear space factor in designing layer-wound coils and area space factor in random-wound coils. The values in each case depend largely on the method of winding. For example, it is possible to wind No. 30 enameled wire with 97 per cent linear space factor by hand, but with only 89 per cent on

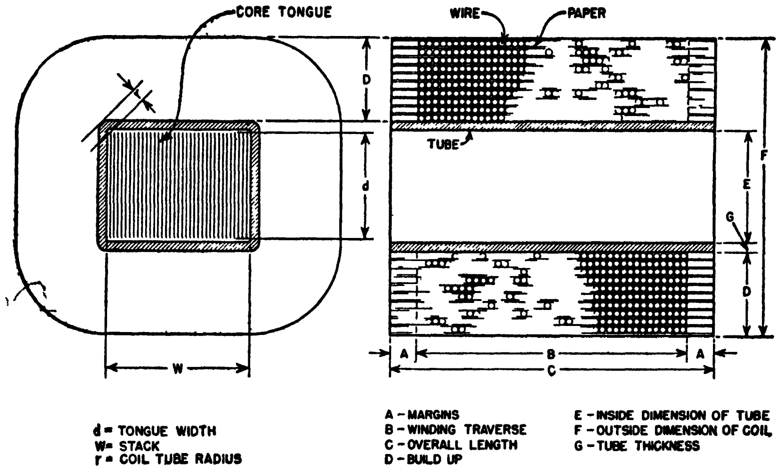


FIG. 19. Paper-insulated coil. (Courtesy of Phelps-Dodge Copper Products Corp.)

an automatic multiple-coil winding machine. (See Fig. 20.) Moreover, values of space factor vary from plant to plant. An average for multiple-coil machines is given in Table VI.

Mean length of turn must be calculated for a coil in order to find its resistance in ohms. This may be found by referring to the side view of Fig. 19. Note that there is a small clearance space between core and coil form or tube. Let d be the core tongue and w the stack. Suppose there are several concentric windings. The length of mean turn of a winding V at distance r from the core and having height D, is

$$MT = 2w + 2d + 2\pi\left(r + \frac{D}{2}\right)$$

$$= 2(w + d) + \pi(2\Sigma D + D) \qquad (14)$$

where ΣD is the sum of all winding heights and insulation thicknesses between winding V and the core.

The mean turn of the winding U just below V ordinarily is calculated before that of winding V. This fact simplifies the calculation of winding V, the mean turn of which is

$$MT_V = MT_U + \pi(D_U + D_V + 2c) \tag{15}$$

where c is the thickness of insulation between U and V.

FIG 20. Winding 20 coils in multiple machine: layer paper at right.

Allowance must be made, with many coil leads, for bulging of the coil at the ends and consequent increase of mean turn length.

The placement, insulation, and soldering of leads constitute perhaps the most important steps in the manufacture of a coil. When coils are wound one at a time, the leads can be placed in the coil while it is being wound. The start lead may be placed on the coil form, suitable insulation may be placed over it, and coil turns may be wound over the insulation. Tap leads can be arranged in the same way. Finish leads must be anchored by means of tape, string, or yarn, because

there are no turns of wire to wind over them. Typical lead anchoring is shown in Fig. 21.

In multiple-wound coils, the leads must be attached after the coils are wound. Extra wire on the start turn is pulled out of the coil and run up the side as shown in Fig. 22, with separator insulation between

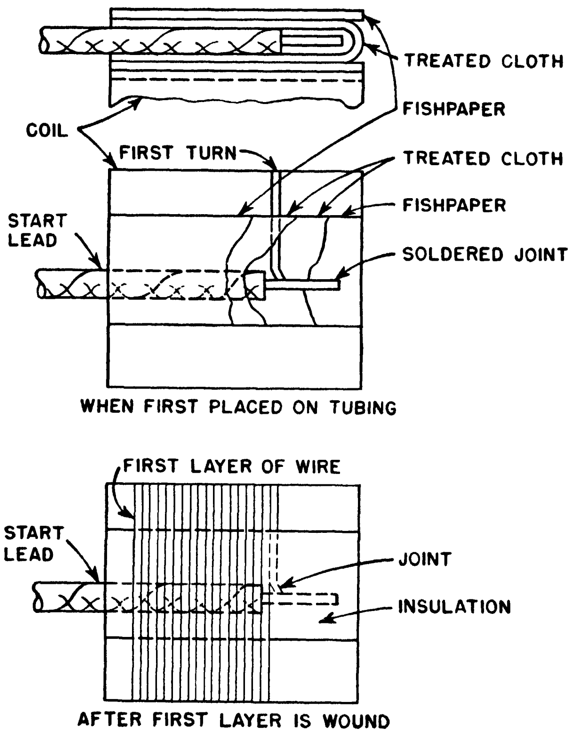


FIG. 21. Start-lead insulation in hand-wound coils.

wire extension and coil. Outer insulation covers the wire extension up to the lead joint. A pad of insulation is placed under the joint and one or more layers of insulation, which insulate and anchor the joint, are wound over the entire coil and the lead insulation. Electrical grade scotch tape is widely used for anchoring leads. It is important to avoid the use of corrosive adhesives.

Leads should be large enough to introduce only a small amount of voltage drop, and should have insulation clearances adequate for the test voltage. These clearances can be found as explained in Section 17. In high voltage transformers it would often be possible to seal the windings if there were no leads; hence lead placement calls for much

care and skill. Leads and joints should also be mechanically strong enough to withstand winding, impregnating, and handling stresses without breakage.

15. Insulation. Two classes of insulation are used in dry-type transformers. Class A insulation is organic material such as paper, cotton, silk, varnish, or wire enamel. Class B insulation is mica, asbestos, glass, porcelain, or other inorganic material with organic binders such as varnish for embedding the insulation. A small amount of other

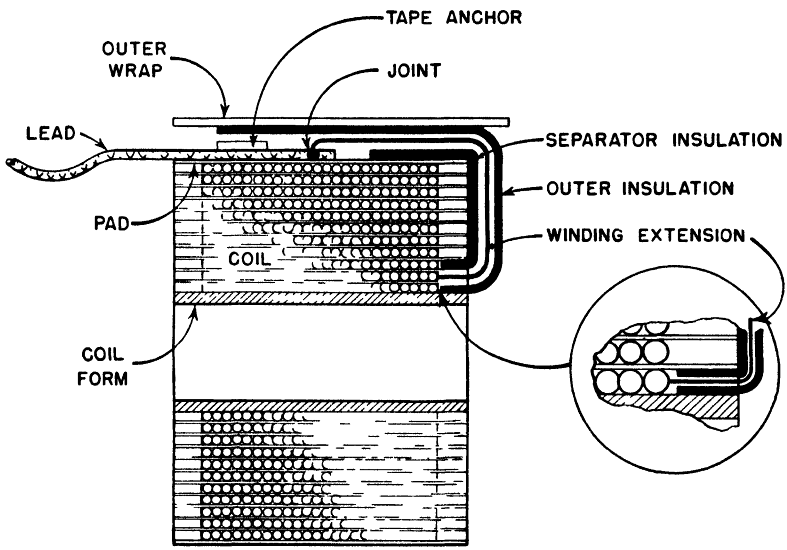


FIG 22 Start-lead insulation in multiple-wound coils.

class A material is permissible in a class B coil "for structural reasons" but it should be kept to a minimum.

In general, the vital difference between the two classes of insulation is one of operating temperature. Glass-covered wire is preferable to asbestos for space reasons; it is available in approximately the same dimensions as cotton-covered wire. Built-up mica is the usual insulation wrapper material. With special bonds it is flexible enough to wind over coils or layers of wire. Stiff mica plate for lead insulation and mica tubing for coil forms are usually bonded with shellac. Class B insulating material is more expensive than class A, and is used only when other advantages outweigh the cost.

The necessity for small size in aircraft or mobile apparatus is continually increasing the tendency to use materials at their fullest capabilities. As size decreases, the ability of a transformer to radiate a

given number of watts loss also decreases. Hence, it operates at higher temperature. Transformers for 400- and 800-cycle power supplies can be made in the smallest feasible dimensions by using class B insulation (glass and mica). As a result, from 30 to 50 per cent decrease in size (as compared with class A insulation), in addition to increased ability to withstand extremes of ambient temperature, humidity, and altitude, are obtained. Class B insulation is thus of special importance in aircraft apparatus. Usually at 60 cycles enough room is available to

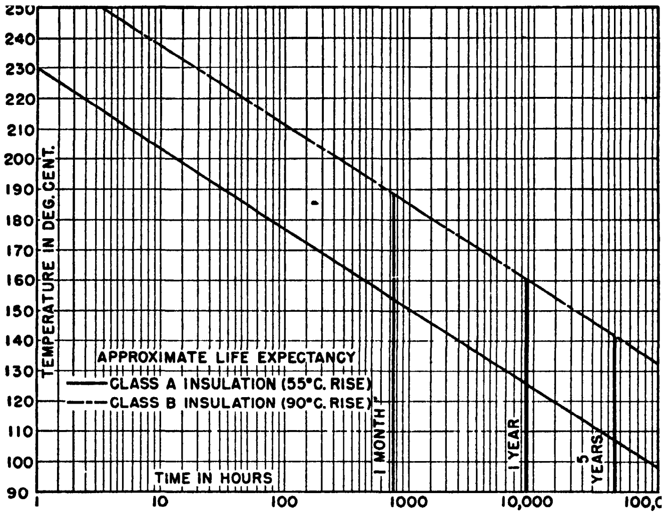


FIG. 23. Class A and class B insulation life. (Courtesy of *Electronics*, Oct. 1939.)

use class A insulation, but mica may be used to reduce size of high voltage units.

The increase in operating temperature made possible by using class B insulation can be seen by referring to Fig. 23. The lower solid line represents the 8° rule for class A insulation (55°C rise, 10°C hot spot, at 40°C ambient) and is taken from data gathered over many years. The class B line has fewer data to support it. Life tests indicate that the average temperature rise could safely be 90 centigrade degrees in a 40°C ambient; this is shown by the dot-and-dash line. Intermittent load temperatures may be high for short periods. The two lines are shown parallel, but experiments on varnish life show that a steeper curve (approximately 10 centigrade degrees for 2:1 difference in life) is more exact for class B.

A third class of insulation comprises the silicones, organic silicates with remarkable thermal and mechanical properties. These materials are coming into use at operating temperatures approaching 200°C.

16. Dielectric Strength. The usual figure given for dielectric strength is the breakdown value in rms volts at 60 cycles in a 1-minute test. It is not possible to operate class A insulation anywhere near this value because of the cellular structure of all organic materials. Even after these materials are treated with varnish, many small holes exist throughout a coil structure which ionize and form corona at

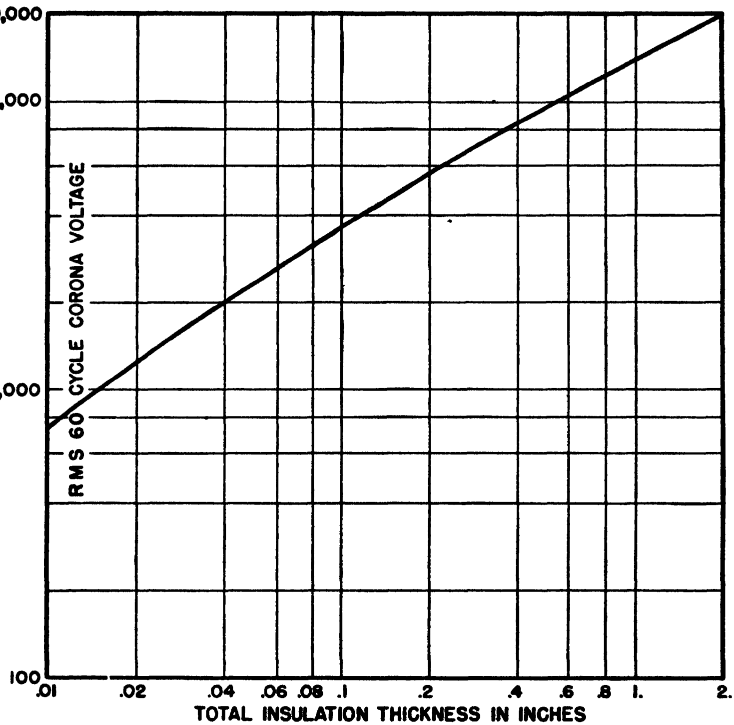
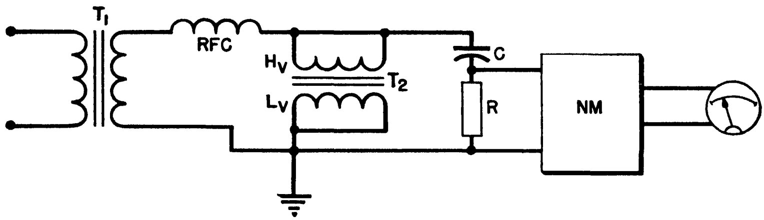


FIG. 24. Corona limit for treated cloth and paper.

voltage far below breakdown. With class A insulation (organic materials), the designer must be governed more by resistance of the insulation to corona over a long period than by breakdown strength of the insulation in a 1-minute test. For example, a 20-mil thickness of treated cloth will withstand 10,000 volts for 1 minute. However, corona starts at 1250 volts, and operation at any higher voltage would puncture the insulation in a few weeks. It is much wiser to keep a reasonable margin, say 20 to 30 per cent, below the corona limit than to use a fraction of the 1-minute breakdown test. Approximate voltages at which corona is audible are plotted in Fig. 24 as a function of insulation thickness.

Differences in hearing ability between persons sometimes make a corona measurement desirable. This is done by means of the standard NEMA circuit of Fig. 25.[1] With the transformer connected as shown, receiver output meter is adjusted to half-scale by a volume control potentiometer in the receiver. Next, the transformer is replaced by a modulated 1-mc signal generator, the output of which is varied until the noise meter output is again half-scale. The signal generator output in microvolts is read on an attenuator; this is then a measurement of the corona present.



T_1	TESTING TRANSFORMER	R	INPUT RESISTOR (600 OHMS)
T_2	TRANSFORMER UNDER TEST	NM	NOISE METER (RECEIVER WITH METER OUTPUT)
C	COUPLING CAPACITOR	RFC	RADIO FREQUENCY CHOKE

FIG. 25. Standard NEMA radio-influence measuring circuit.

Class B insulation can be worked much closer to the ultimate dielectric strength, but the latter is less a factor in determining size than creepage distance to the core. For mica an approximate working voltage rule is 100 volts rms per mil thickness.

Insulated coils in air are subject to a two-dielectric effect that is peculiarly troublesome. If the path of electric stress is partly through solid material and partly through air, the air may be overstressed because it has the lower dielectric constant (unity, compared with 3 to 5 for most coil materials). If this condition exists, it is usually impracticable to increase the air distance, and so reduce the volts per inch to a value below the corona limit. The addition of more solid insulation over the whole coil may make it too large. Often the only feasible remedy is to fill the air space with more solid material, either in the form of filling compound or strips of insulation like micarta or pressboard.

It is important, when dealing with insulation voltage, to make a distinction between test voltage and operating voltage. Of these two, operating voltage is the better value to specify.

[1] See "Radio Influence Characteristics of Electrical Apparatus," by P. L. Bellaschi and C. V. Aggers, *Trans. AIEE*, p. 626, Vol. 57, Nov. 1938.

17. Creepage Distance. While solid insulation dielectric strength is important, the usual bottleneck for high voltage is creepage distance, such as margins between wire and core along the layers of insulation,

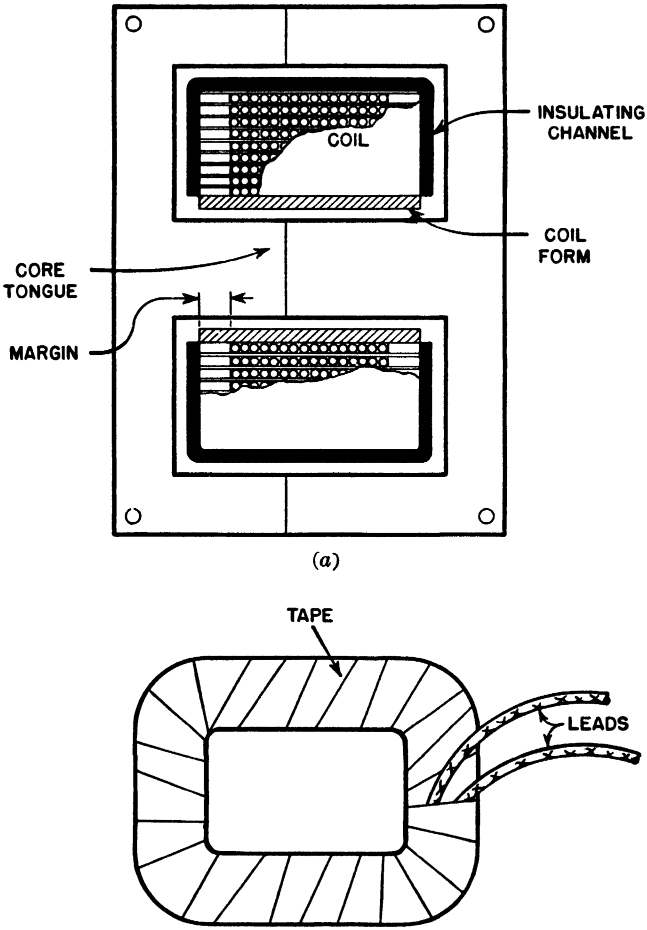


FIG. 26. (a) Use of insulating channel; (b) taped coil.

or margins between lead joints and frame along the leads and coil sides. A common way of increasing the direct creepage distance across the margins is to use an insulating channel as in Fig. 26 (a). This is especially helpful when the part of the coil adjacent to the core tongue is at low potential and the upper part is at high potential, as in some plate transformers. When the whole coil is at high potential

it is often taped as in Fig. 26 (*b*). Lead joint to ground distance then determines breakdown voltage.

Creepage distances over treated cloth or solid organic material in air are shown in Fig. 27 for breakdown voltages up to 100 kv. The primary purpose of these curves is to find the proper margins for coils

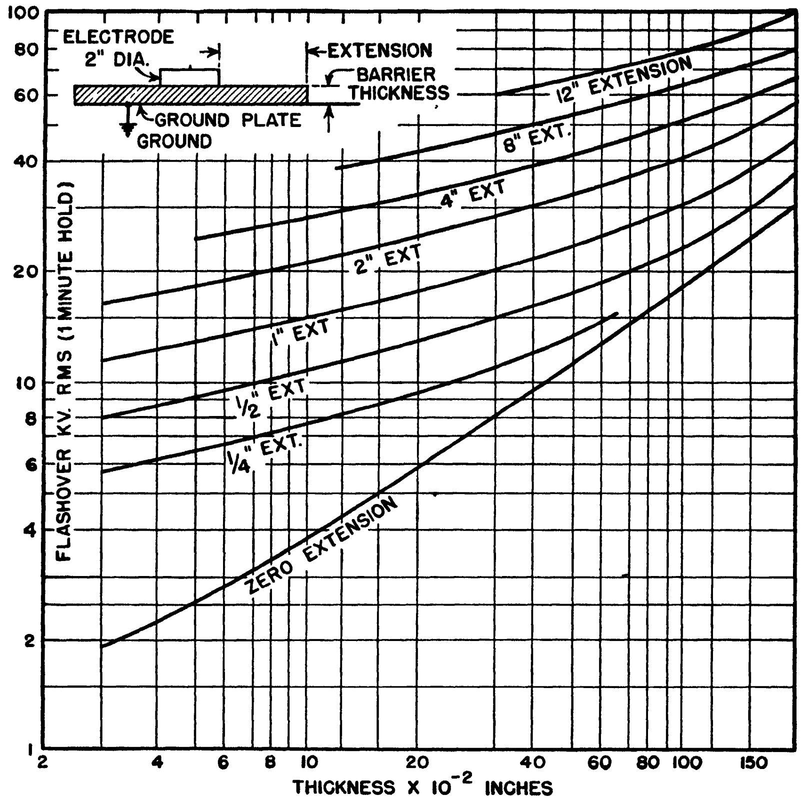


FIG. 27. Creepage curves in air over smooth organic insulation.

adjacent to the core tongue. They may be used for high voltages between adjacent windings as in Fig. 28 if the margins are equal and one-half the insulation thickness is used; the value from Fig. 27 can then be doubled because the voltage stress is symmetrical about the insulation center line.

18. Impregnation. After a coil is wound it is best practice to impregnate it in some sort of insulating liquid which hardens after filling. This is done for several reasons. First, it protects the wire from movement and possible mechanical damage. Second, it prevents the en-

trance of moisture and foreign matter which might corrode the wire or cause insulation deterioration. Third, it increases the dielectric strength of fibrous insulating materials. Fourth, it assists in heat dissipation from the coil. Single-layer coils may be dipped in the liquid, drained, and dried, but deeper, thicker coils require the use of vacuum to remove air from the coil and admit the liquid to all parts of the interior. The best mechanical result is obtained when coils are assembled with cores before treatment.

Chemically neutral mineral wax is used widely for impregnating coils having little or no temperature rise in normal use. The wax is

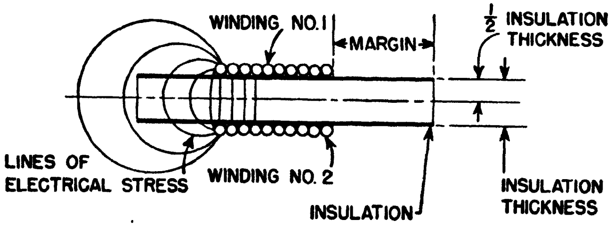


Fig. 28. Adaptation of Fig. 27 for insulation between coils.

melted in a sealed tank, and is drawn into another tank in which preheated coils have been placed, and a vacuum is maintained. Coils are removed from the tank, drained, and allowed to cool. Wax treatment provides good dielectric qualities and moisture protection. It is a quick, simple process.

Varnish is used in transformers having operating temperatures of 65°C or higher. Varnish of good grade and close control is essential to achieve thorough filling and dry coils after impregnation. Oleoresinous varnishes, which polymerize to a hard state by baking, are notably useful for the purpose. A high degree of vacuum, fresh varnish, and accurate baking temperature control are necessary for good results. Plasticizers are sometimes added to the varnish to prevent brittleness in finished coils. Varnish may attack wire enamel (which itself is a kind of varnish), and so the soaking and baking time periods must be regulated carefully.

A recent development is the invention of a radically new varnish. Varnishes used for impregnation of electrical coils have until lately been diluted, to some degree at least, by the use of solvents to lower the viscosity in order to permit full penetration of the windings. When the coils are baked, the varnish dries, but is left with tiny holes through which moisture can penetrate, and in which corona may form. Eventually, this corona destroys the insulation. It is, therefore,

necessary to allow large clearances for high voltages or to immerse the coils in oil. Either of these alternatives increases the size of a high voltage transformer in relation to that of a low voltage transformer.

A new varnish (named Fosterite after the inventor, Mr. Newton C. Foster) changes by heat polymerization from a liquid to a solid state with only slight change in volume. It has very low viscosity, requires no solvent, and with care can be made to produce complete filling of coil interstices. Because of this excellent filling, it is possible to reduce voltage clearance to a smaller value than formerly, to obtain a pronounced reduction in size of high voltage units, to make mechanically strong assemblies, and to protect the windings from moisture penetration. Units with small wire last about 1 week in the tropics unless they are sealed to prevent moisture from entering the coils. Hermetical sealing was formerly necessary to give this protection.

In high voltage units, air around the coils and leads is especially subject to the formation of corona. To reduce this tendency, the containers are filled with asphaltic compound which replaces the air with solid, non-ionizing material. A similar compound is often used to fill containers of low voltage transformers to avoid the need for mechanically fastening the core to the case. This is a permissible practice if the melting point of the compound is higher than the highest operating temperature, and if its cracking point is below the lowest operating temperature.

19. Oil Insulation. Although, in the use of electronic apparatus, there is a tendency toward dry-type transformers, there are many instances in which voltages are so high that air clearances are impracticable and oil-filled containers must be used. In Fig. 29 the curves show rms breakdown voltage versus creepage distance under oil. An example will show the advantage of oil filling. From Figs. 27 and 29 it will be seen that 10-in. creepage distance is required in air to withstand a 1-minute breakdown test of 60 kv on insulation 0.5 in. thick, whereas in oil only 2-in. creepage distance is required.

Only high grades of insulating oil are used for this purpose. Tests are run continually to check condition of the oil. Oil is stored in such a manner as to keep out moisture and dirt and avoid extremes of temperature. Where very high voltages are used, as in X-ray apparatus, oil filling is done under vacuum to remove air bubbles, and containers are sealed afterwards to prevent moisture from entering. Mica insulation is not used in oil because oil dissolves flexible bonds.

20. Size versus Rating. Core area depends upon voltage, induction, frequency, and turns. For a given frequency and grade of core material, core area depends upon the applied voltage. Window area de-

pends upon coil size, or for a given voltage upon the current drawn. Since window area and core area determine size, there is a relation between size and v-a rating.

With other factors, such as frequency and grade of iron, constant, the larger transformers dissipate less heat per unit volume than the

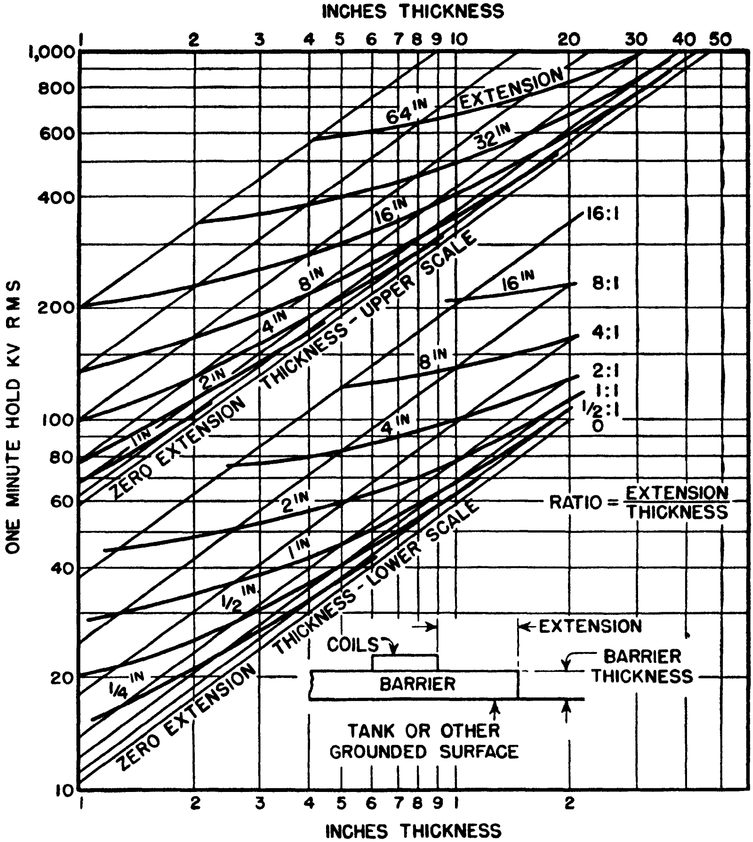


FIG. 29. Creepage curves of solid insulation under oil.

smaller ones. This is true because dissipation area increases as the square of the equivalent spherical radius, whereas volume increases as its cube. Therefore larger units are more commonly of the open type, whereas smaller units are totally enclosed. Where enclosure is feasible, it tends to cause size increase, by limiting the heat dissipation. Figure 30 shows the relation between size and rating for small, enclosed, low voltage, two-winding, 60-cycle transformers having Hipersil cores and class A insulation and operating continuously in a

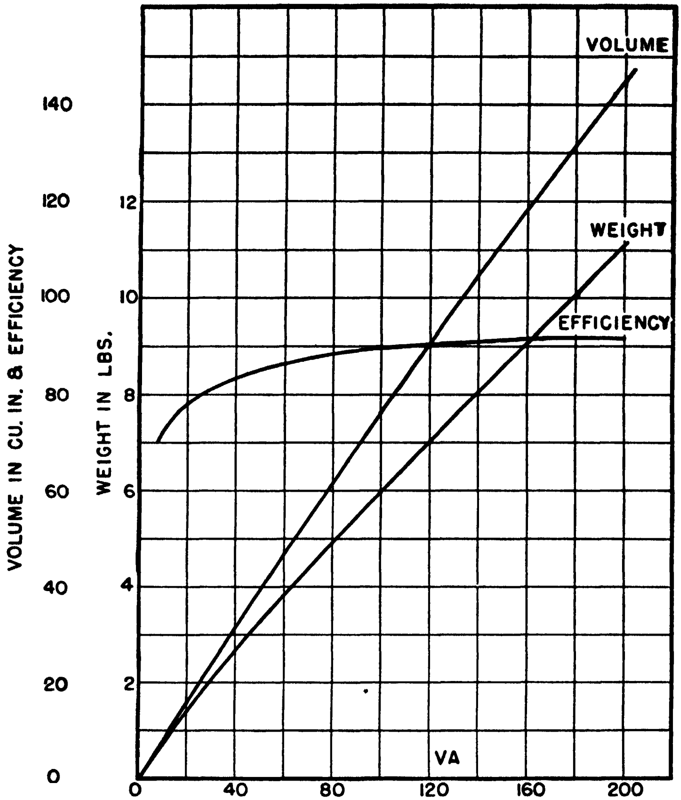


FIG. 30. Size of enclosed 60-cycle transformers.

40°C ambient. The size increases for the same v-a over that in Fig 30 for any of the following reasons:

High voltage	Silicon steel cores
High ambient temperature	Low regulation
Lower frequency	More windings

The size decreases for

Higher frequencies	Open-type units
Class B insulation	Intermittent operation

If low voltage insulation is assumed, two secondary windings reduce the rating of a typical size by 10 per cent; six secondaries by 50 per cent. The decreased rating is due partly to space occupied by insulation, and partly to poorer space factor. The effects of voltage, tem-

perature, and core steel on size have been discussed in preceding sections. Frequency and regulation will be taken up separately in succeeding chapters.

Open-type transformers like those in Fig. 13 have better heat dissipation than enclosed units. The lamination stacking dimension can

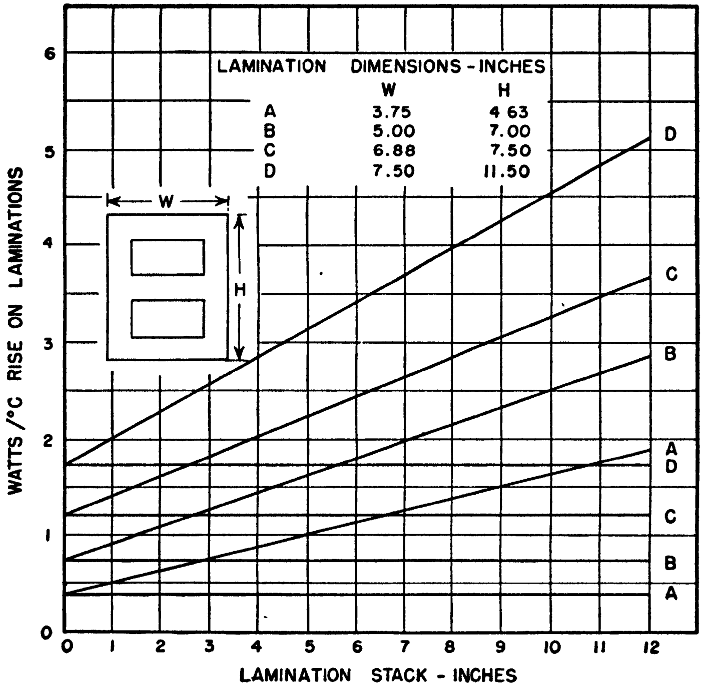


FIG. 31. Heat dissipation from open-type transformers with end cases.

be made to suit the rating, so that one size of lamination may cover a range of v-a ratings. Heat dissipation from the end cases is independent of the stacking dimension, but that from the laminations is directly proportional to it. This is shown in Fig. 31 for several lamination sizes. For each size the horizontal line represents heat dissipation from the end cases; the sloping line represents dissipation from end cases, plus that from the lamination edges which is proportional to the stacking dimension. At ordinary working temperature, heat is dissipated at the rate of 0.008 watt per square inch per centigrade degree rise. In Fig. 31 the watts per centigrade degree of temperature rise are given as a function of lamination stack. This refers to temperature rise at the core surface only. In addition, there is a temperature

gradient between coil and core which is given in similar manner in Fig. 32.

To find the average coil temperature rise, divide the *copper* loss by the watts per centigrade degree from the sloping line of Fig. 32. To this add the *total* of copper and iron losses divided by the appropriate ordinate from Fig. 31. That is, the total coil temperature rise is equal

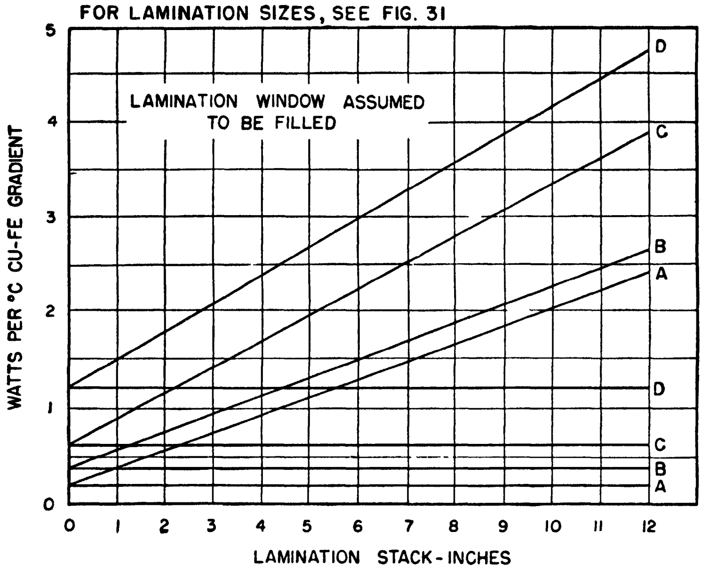


FIG. 32. Winding-to-core gradient for open-type transformers with end cases.

to the sum of the temperature drop across the insulation (marked Cu-Fe gradient in Fig. 32) and the temperature drop from the core to the ambient air. Data like those in Figs. 31 and 32 can be established for any lamination by making a heat run on two transformers, one having a core stack near the minimum and one near the maximum that is likely to be used. Usually stacking dimensions lie between the extremes of ½ to 3 times the lamination tongue width, and poor use of space results from stacking outside these limits. If end cases are omitted, coil dissipation is improved as much as 50 per cent.

The same method can be used for figuring type C Hipersil core designs; here the strip width takes the place of the stacking dimension of punched laminations, and the build-up corresponds to the tongue width. When two cores are used, as in Fig. 11, the heating can be approximated by using data for the nearest punching.

For irregular or unknown heat dissipation surfaces, an approxima-

tion to the temperature rise can be found from the transformer weight, as derived in the next section.

21. Intermittent Ratings. It often happens that electronic equipment is operated for repeated short lengths of time, between which the power is off. In such cases the average power determines the heating and size. Transformers operating intermittently can be built smaller than if they were operated continuously at full rating.

Intermittent operation affects size only if the "on" periods are short compared to the thermal time constant of the transformer; that is, small transformers have less heat storage capacity and hence rise to final temperature more quickly than do large ones. It is important, therefore, to know the relation between size and thermal time constant, or the time which would be required to bring a transformer to 63 per cent of the temperature to which it would finally rise if the power were applied continuously.

The exact determination of temperature rise time in objects such as transformers, having irregular shapes and non-homogeneous materials, has not yet been attempted. Even in simple shapes of homogeneous material, and after further simplifying assumptions have been made, the solution is too complicated[2] for rapid calculation. However, under certain conditions, a spherical object can be shown to cool according to the simple law:[3]

$$\theta = \theta_0 \epsilon^{-\frac{3Et}{pcR}} \tag{16}$$

where θ = temperature above ambient at any instant t

θ_0 = initial temperature above ambient

E = emissivity in calories per second per centigrade degree per square centimeter

p = density of material

c = specific heat of material

R = radius of sphere

ϵ = 2.718.

The conditions involved in this formula are that the sphere is so small or the cooling so slow that the temperature at any time is sensibly uniform throughout the whole volume. Mathematically, this is fulfilled when the expression ER/k (where k is the thermal conductivity of the material) is small compared to unity. Knowing the various properties of the transformer material, we can tell (1) if the required

[2] See *The Mathematical Theory of Heat Conduction*, by L. R. Ingersoll and O. J. Zobel, Ginn and Co., p. 142.

[3] Ingersoll and Zobel, *op. cit.*, p. 143.

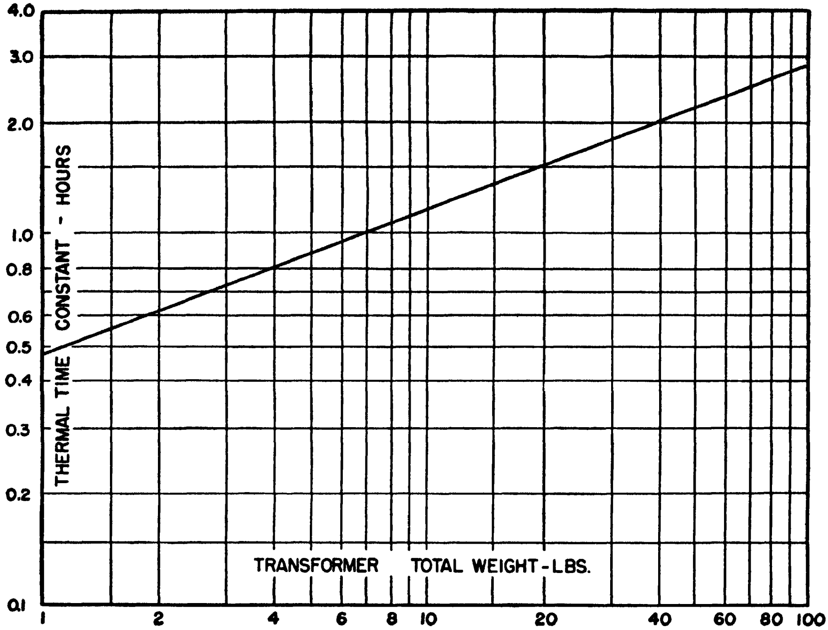


FIG. 33. Transformer time constant, or time required to reach 63 per cent of final temperature.

conditions are met, and (2) what the thermal time constant is. The latter is arrived at by the relation

$$t_c = \frac{pcR_e}{3E} \tag{17}$$

where R_e is the radius of the equivalent sphere.

In order to convert the non-homogeneous transformer into a homogeneous sphere the average product of density and specific heat cp is found. Figures on widely different transformers show a variation from 0.862 to 0.879 in this product; hence an average value of 0.87 can be taken, with only 1 per cent deviation in any individual case.

Since the densities of iron and copper do not differ greatly, and insulation brings the coil density closer to that of iron, it may be further assumed that the transformer has material of uniform density 7.8 throughout. The equivalent spherical radius can then be found from

$$R_e = \left(\frac{\text{Weight}}{1.073}\right)^{1/3} \tag{18}$$

where R_e is in inches and weight is in pounds. The time constant is plotted from equations 17 and 18 in terms of weight in Fig. 33.

The condition that ER/k be small compared to unity is approximated by assuming that k is the conductivity of iron—a safe assumption, because the conductivity of copper is 7 to 10 times that of iron. A transformer weighing as much as 60 lb has $R_e = 5.45$ in., $E = 0.00028$ cal per sec per sq cm/°C, and $k = 0.11$. Changing R_e to metric units gives $ER/k = (0.00028 \times 5.45 \times 2.54)/0.11 = 0.34$, which is small enough to meet the necessary condition of equation 16.

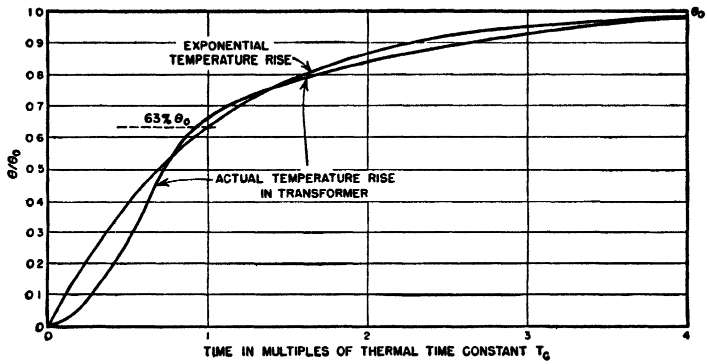


FIG. 34. Transformer temperature rise time.

It will be noticed that equation 16 is a law for cooling, not temperature rise. But if the source of heat is steady (as it nearly is) the equation can be inverted to the form $\theta_0 - \theta$ for temperature rise, and θ_0 becomes the final temperature rise.

Temperature rise of a typical transformer is shown in Fig. 34, together with the exponential law which is $\theta_0 - \theta$, where θ is the temperature of equation 16. The actual rise is less at first than that of the foregoing simplified theory, then more rapid, and with a more pronounced "knee." The 63 per cent of final temperature is reached in about 70 per cent of the theoretical time constant t_c for transformers weighing between 5 and 200 lb. This average correction factor is included in Fig. 33 also.

If a transformer is operated for a short time and then allowed to cool to room temperature before operating again, the temperature rise can be found from Figs. 33 and 34. As an example, suppose that the continuously operated final coil temperature rise is 100 centigrade degrees, the total weight is 5 lb, and operating duty is infrequent periods of 2 hr. From Fig. 33, the transformer has a thermal time constant of 0.85 hr. This corresponds to $t_c = 1$ in Fig. 34. Two hours are therefore $2 \div 0.85 = 2.35$ times t_c, and the transformer rises to 90 per cent of final temperature, or a coil temperature rise of 90 centigrade degrees in 2 hr.

If, on the other hand, the transformer has regular off and on intervals, the average watts dissipated over a long period of time govern the temperature rise. A transformer is never so small that it heats up more in the first operating interval than at the end of many intervals.

From equation 18 can be found a relation between weight, losses, and final temperature rise. For, since heat is dissipated at 0.008 watt per sq in. per deg C rise, and the area A_S of the equivalent sphere is $4\pi R_e^2$,

$$\theta_0 = \frac{\text{Total watts loss}}{0.008A_S} = \frac{\text{Total watts loss}}{0.1\left(\dfrac{\text{Total weight in pounds}}{1.073}\right)^{2/3}} \tag{19}$$

where θ_0 is the final temperature rise in centigrade degrees. This equation is subject to the same approximations as equation 16; test results show it is most reliable for transformers weighing 20 lb or more.

PROBLEMS

1. In the core loss curves of Fig. 15, suppose that each horizontal division is 2 kilogauss and each vertical division is 0.1 watt per lb. Compare iron loss in silicon steel and Hipersil at 12 kilogauss.

2. From Fig. 16, determine the value of magnetizing force necessary to maintain a 60-cycle induction of 10 kilogauss in silicon steel; in grain-oriented steel; 15 kilogauss in grain-oriented steel. *Ans.* 2.2 oersteds; 0.286; 1.5.

3. A paper-insulated coil has 1000 turns of No. 30 single enameled wire and a winding space 1 in. wide. How many layers should it have if wound on a multiple-coil machine? What thickness of layer paper should be used? To what total height does it build, if equal vertical and horizontal space factors are assumed? *Ans.* 13; 0.0015 in.; 0.182 in.

4. If the coil of Prob. 3 were random-wound with an area space factor of 80 per cent, to what coil height would it build? *Ans.* 0.153 in.

5. A dry-type transformer secondary winding operates at 10,000 volts rms above ground, and has a 230-volt primary. What thickness of class A insulation is required between windings? If mica insulation is used instead of class A, what thickness is necessary? *Ans.* 0.55 in.; 0.1 in.

6. What creepage distance is necessary from each winding of Prob. 5 to the core, an insulation test voltage of twice normal voltage plus 1000 volts being assumed? How may this clearance be reduced? *Ans.* Secondary, ¾ in. for class A; 2 in. for mica.

7. A transformer has a 1-in. stack of core laminations A, Fig. 31, a copper loss of 15 watts, and a core loss of 9 watts. If it is allowed to operate continuously, to what final temperature above ambient does the coil rise? *Ans.* 88°C.

8. The core in Prob. 7 weighs 3.2 lb and the coil 1.2 lb. If the transformer operates at infrequent intervals of ½ hr, what is its coil temperature rise at the end of such an interval? *Ans.* 40°C.

9. Check the temperature rise of Prob. 7 by means of equation 19. *Ans.* 94°C

III. RECTIFIERS, TRANSFORMERS, AND REACTORS

Rectifiers are used to convert alternating into direct current. The tubes generally have two electrodes, the cathode and the anode. Both high vacuum and gas-filled tubes are used. Sometimes for control purposes the gas-filled tubes have grids, which are discussed in Chapter VII.

A high vacuum rectifier tube characteristic voltage-current curve is shown in Fig. 35. Current flows only when the anode is positive with respect to the cathode. The voltage on this curve is the internal potential drop in the tube when current is drawn through it. This voltage divided by the current gives effective tube resistance at any point. Tube resistance decreases as current increases, up to the emission limit, where all the electrons available from the cathode are used. Filament voltage governs the emission limit, and must be closely controlled. If the filament voltage is too high, the tube life is shortened; if too low, the tube will not deliver rated current at the proper voltage.

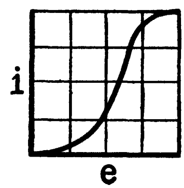


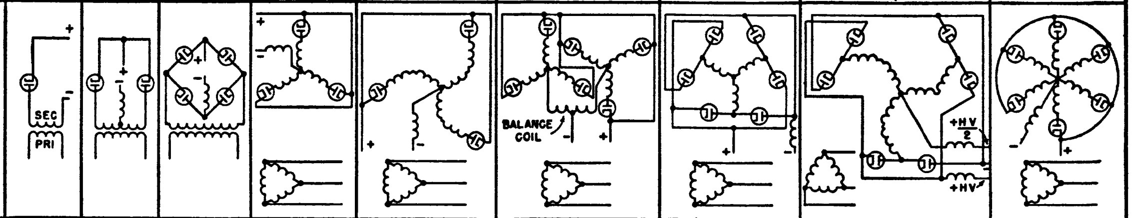
FIG. 35. High vacuum rectifier voltage-current curve.

Gas-filled rectifier tubes have internal voltage drop which is virtually constant and independent of current. Usually this voltage drop is much lower than that of high vacuum tubes. Consequently, gas-filled tubes are used in high power rectifiers, where high efficiency and low regulation are important. In some rectifier circuits, copper oxide or selenium disks are used as the rectifying elements.

In this chapter, the rectifier circuits are summarized and then rectifier transformers and reactors are discussed.

22. Rectifiers with Reactor-Input Filters. Table VII gives commonly used rectifier circuits, together with current and voltage relations in the associated transformers. This table is based on the use of a reactor-input filter to reduce ripple. The inductance of the choke is assumed to be great enough to keep the output direct current constant. With any finite inductance there is always some superposed

TABLE VII – TABLE OF RECTIFIER CIRCUITS

TYPE	SINGLE PHASE HALF WAVE	SINGLE PHASE FULL WAVE	SINGLE PHASE BRIDGE CIRCUIT	3 PHASE HALF WAVE	3 PHASE HALF WAVE ZIG ZAG	DOUBLE 3 PHASE WITH BALANCE COIL ‡	3 PHASE FULL WAVE (SEC MAY BE Δ)	3 PHASE FULL WAVE ZIG ZAG ✱	SIX PHASE HALF WAVE
CIRCUITS									
RECTIFIER PHASES AND NUMBER OF TUBES	1	2	2-4	3	3	6	6	6	6
PHASES OF A C SUPPLY	1	1	1	3	3	3	3	3	3
SECONDARY VOLT PER LEG	2 22	1 11 (HALF SECT)	1 11 (WHOLE)	0 855	0 985 (HALF LEG 493)	0 855	0 428	0 493 HALF LEG 247	0 74
PRIMARY VOLTAGE	2 22	1 11	1 11	0 855	0 855	0 855	0 428	0 428	0 74
SECONDARY CURRENT PER LEG	1 57	0 707	1 00	0 577	0 577	0 289	0 816	0 816	0 408
PRIMARY CURRENT PER LEG	1 21	1 000	1 00	0 471	0 408	0 408	0 816	0 816	0 577
SECONDARY K V A	3 48	1 57	1 11	1 48	1 71	1 48	1 05	1 21	1 81
PRIMARY K V A	2 68	1 11	1 11	1 21	1 05	1 05	1 05	1 05	1 28
AVERAGE OF PRIMARY AND SECONDARY K V A ∮	3 08	1 34	1 11	1 35	1 38	1 26	1 05	1 13	1 55
INVERSE PEAK VOLTAGE	3 14	3 14	1 57	2 09	2 09	2 09	1 05	1 05	2 09
R M S CURRENT PER TUBE	1 57	0 707	0 707	0 577	0 577	0 289	0 577	0 577	0 408
PEAK CURRENT PER TUBE	3 14	1 00	1 00	1 00	1 00	0 500	1 00	1 00	1 00
AVERAGE CURRENT PER TUBE	1 00	0 50	0 50	0 33	0 33	0 167	0 33	0 33	0 167
RIPPLE FREQUENCY	f	2f	2f	3f	3f	6f	6f	6f	6f
R M S RIPPLE VOLTAGE	1 11	0 472	0 472	0 177	0 177	0 04	0 04	0 04	0 042
RIPPLE PEAKS	+2 14 −1 00	+0 57 −1 00	+0 57 −1 00	+0 209 −0 291	+0 209 −0 291	+0 057 −0 077	+0 057 −0 077	+0 057 −0 077	+0 057 −0 077
LINE POWER FACTOR	0 373	0 90	0 90	0 826	0 955	0 955	0 955	0 955	0 955

NOTE THE VALUES OF VOLTAGE AND CURRENT ARE EFFECTIVE OR R M S, UNLESS OTHERWISE STATED, THEY ARE GIVEN IN TERMS OF THE AVERAGE DC VALUES, AND THE KILOVOLT-AMPERES IN TERMS OF DC KILOWATT OUTPUT PERFECT TRANSFORMERS, RECTIFIERS AND DC CHOKE ARE ASSUMED, AND $N_1/N_2 = 1$, EXCEPT IN ZIG ZAG CIRCUITS SECONDARY AND PRIMARY REFER TO ANODE TRANSFORMER

‡ FREQUENCY OF BALANCE COIL VOLTAGE 3f BALANCE COIL VOLTAGE 0 356 PEAK BALANCE COIL VOLTAGE 0 605 BALANCE COIL K V A 0 173

✱ USE COLUMN 5 VALUES FOR HALF VOLTAGE TAP ∮ MAGNETIZING CURRENT ASSUMED NEGLIGIBLE IN ALL CASES

This table is based mostly on "Polyphase Rectification Special Connections," by R. W. Armstrong, *Proc. I.R.E.*, Vol. 19, Jan. 1931.

ripple current which is neglected in the table, and which is considered further in Chapter IV.

The single-phase half-wave rectifier ordinarily has discontinuous output current, and its output voltage is therefore highly dependent upon the inductance of the input filter choke. For this reason, the currents and voltages are given for this rectifier without a filter.

The difference between primary and secondary v-a ratings in several of these rectifiers does not mean that instantaneous v-a values are different; it means that because of differences in current wave form the rms values of current may be different for primary and secondary.

Unbalanced direct current in the half-wave rectifiers requires larger transformers than in the full-wave rectifiers. This is partly overcome in three-phase transformers by the use of zigzag connections. The three-phase full-wave rectifier can be delta-connected on both primary and secondary if desired; the secondary current is multiplied by 0.577 and the secondary voltage by 1.732. Anode windings have more turns of smaller wire in the delta connection. Single-phase bridge and three-phase full-wave rectifiers require notably low a-c voltage for a given d-c output, low inverse peak voltage on the tubes, and small transformers.

23. Rectifiers with Capacitor-Input Filters. When the filter has no reactor intervening between rectifier and first capacitor, rectifier current is not continuous throughout each cycle and the rectified wave form changes. During the voltage peaks of each cycle, the capacitor charges and draws current from the rectifier. During the rest of the time, no current is drawn from the rectifier, and the capacitor discharges into the load.

Comparison between the rectified voltage of reactor-input and capacitor-input filters in a single-phase full-wave rectifier may be seen

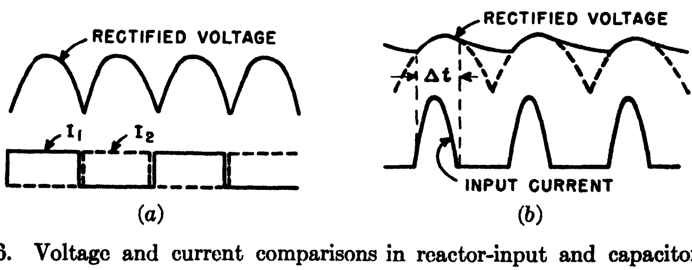


FIG. 36. Voltage and current comparisons in reactor-input and capacitor-input circuits.

in Figs. 36 (*a*) and 36 (*b*) respectively. The two tube currents I_1 and I_2 in (*a*) add to a constant d-c output, whereas in (*b*) the high-peaked tube currents flow only while the rectified voltage is higher than the average d-c voltage. *Average* current per tube in both cases is half the

rectifier output. With large values of capacitance, the rectified voltage in Fig. 36 (*b*) increases to within a few per cent of the peak voltage.

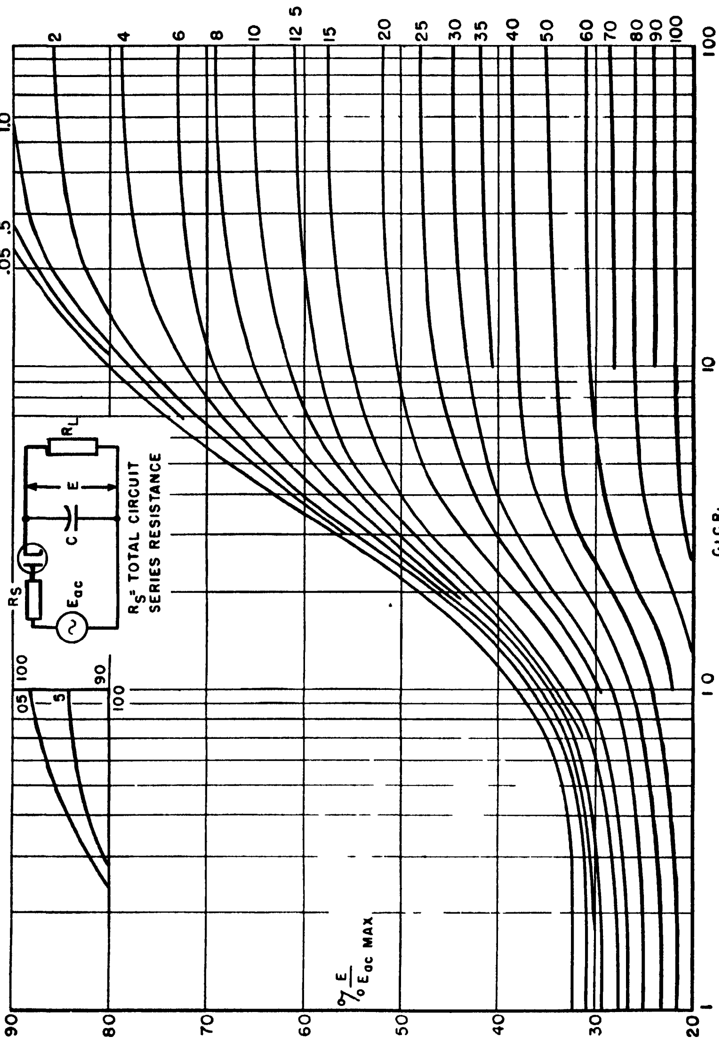


FIG. 37. Relation of peak sine voltage to d-c voltage in half-wave capacitor-input circuits. (From O. H. Schade, *Proc. I.R.E.*, July 1943, p. 341.)

Ripple, average rectified voltage output, and rectifier current are dependent on the capacitance, the supply line frequency, and the load resistance. They are dependent also on rectifier internal resistance because it affects the peak value of current which the filter capacitor can draw during the charging interval Δt.

Analysis of this charge-discharge action involves complicated

Fourier series which require a long time to calculate.[1] Satisfactory voltage and current values have been obtained from experimental

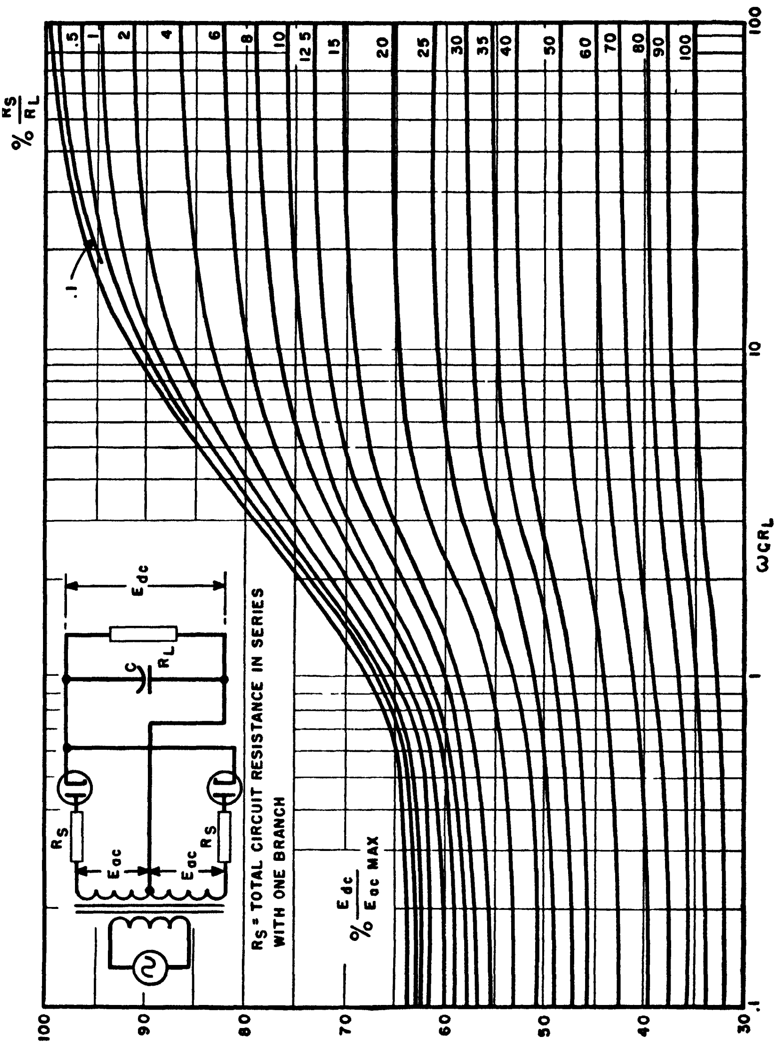


FIG. 38. Relation of peak sine voltage to d-c voltage in full-wave capacitor-input circuits. (From O. H. Schade, *Proc. I.R.E.*, July 1943, p. 341.)

measurements by Schade[2] and are shown in Figs. 37, 38, and 39 for single-phase half-wave and full-wave rectifiers. In these figures R_S is

[1] See "Diode Rectifying Circuits with Capacitance Filters," by D. L. Waidelich, *Trans. AIEE*, Vol. 61, Dec. 1941, p. 1161.

[2] "Analysis of Rectifier Operation," by O. H. Schade, *Proc. I.R.E.*, Vol. 31, July 1943, p. 341.

the rectifier series resistance, including the transformer resistance. Results accurate to within 5 per cent are obtained if the rectifier resistance corresponding to peak current $\hat{I}_P$ is used in finding R_S. The

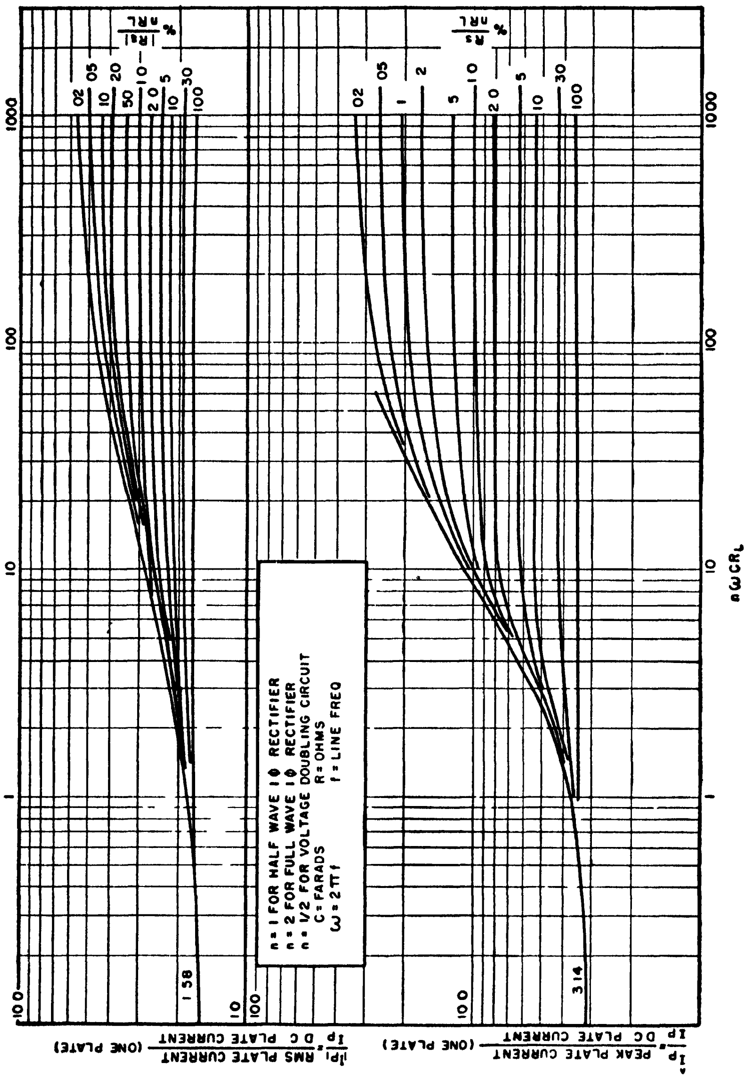


FIG. 39. Relation of peak, average, and rms diode current in capacitor-input circuits. (From O. H. Schade, *Proc. I.R.E.*, July 1943, p. 341.)

process is cut-and-try, because $\hat{I}_P$ depends on R_S and vice versa, but two trials usually suffice. Other variables are explained in the figures. ω is 2π times the supply frequency. Three-phase rectifiers are rarely capacitor-input because of their larger power.

In Fig. 39 the peak current indicates whether the peak current of a given tube is exceeded, and the rms current determines the transformer secondary heating. The v-a ratings are greater, but *ratios* of primary

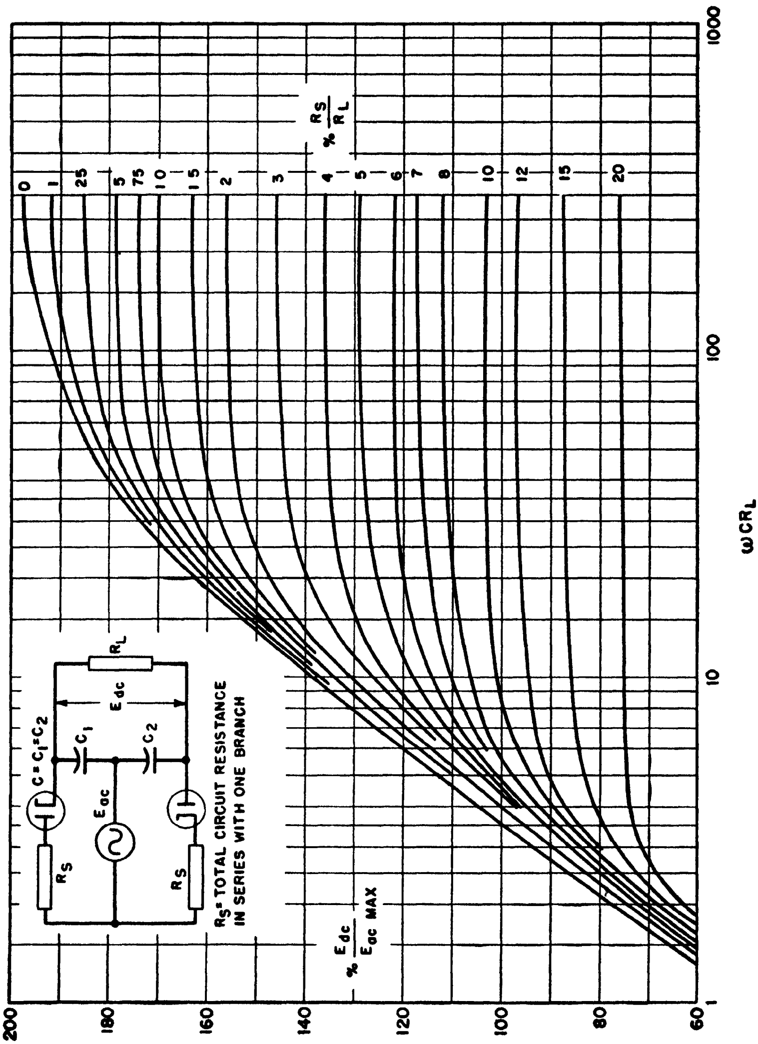


FIG. 40. Relation of peak sine voltage to d-c voltage in voltage-doubling circuit. (From O. H. Schade, *Proc. I.R.E.*, July 1943, p. 341.)

to secondary v-a ratings given in Table VII hold for capacitor-input transformers also.

24. Voltage Doublers. To obtain more d-c output voltage from a rectifier tube, the circuit of Fig. 40 is often used. With proper values

of circuit elements the output is nearly double the a-c peak voltage. Tube inverse peak voltage is little more than the d-c output voltage and no d-c unbalance exists in the anode transformer. Current output available from this circuit is less than from the single-phase full-wave circuit for a given rectifier tube. Current relations are given in Fig. 39.

Voltage tripling and quadrupling circuits also are used, either to increase the d-c voltage or to avoid the use of a transformer.[3]

25. Filament Transformers. Low voltage filament transformers are used for heating tube filaments at or near ground potential. Often the filament windings of several tubes are combined into one transformer. Sometimes this requires several secondary windings. In terms of a single secondary transformer a 5 or 6 secondary unit requires about 50 per cent greater size and weight. But these multi-winding transformers are smaller than five or six separate units; this warrants designing them specially in many instances.

Rectifier tube filaments often operate at high d-c voltages, and require windings with high voltage insulation. It is usually not feasible to combine high voltage windings with low voltage windings when the high voltage is more than 3000 volts d-c because of insulation difficulties, particularly in the leads. Large rectifier filaments are usually heated by separate transformers; in polyphase rectifiers, all tube filaments are at high voltage, and some secondary windings may be combined. See the three-phase full-wave rectifier in Table VII, where the $+HV$ lead connects to a winding which heats the filaments of three tubes.

Low capacitance filament windings are sometimes required for high frequency circuits. The problem is not particularly difficult in small v-a ratings and at moderate voltages. Here air occupies most of the space between windings as in Fig. 41. In larger ratings the problem is more difficult, because the capacitance increases directly as the coil mean turn length for a given spacing between windings. As voltage to ground increases, there comes a point beyond which creepage effects necessitate oil-insulated windings, whereupon the capacitance jumps 3 to 1 for a given size and spacing. There is a value of capacitance below which it is impossible to go because of space limitations in the transformer. What this value is in any given case may be estimated from the fact that the capacitance in $\mu\mu$f of a body in free space is roughly equal to one-half its largest dimension in centimeters.

Except for the differences just mentioned, the design of filament transformers does not differ much from that of small 60-cycle power

[3] See "Analyses of Voltage-tripling and -quadrupling Circuits," by D. L. Waidelich and H. A. Taskin, *Proc. I.R.E.*, Vol. 33, July 1945, p. 449.

transformers. The load is constant and of unity power factor. Leakage reactance plays practically no part, because of its quadrature relationship to the load. Output voltage may therefore be figured as in Fig. 2 (*c*) (p. 7). It should be *accurately* calculated, however, to maintain the proper filament emission and life.

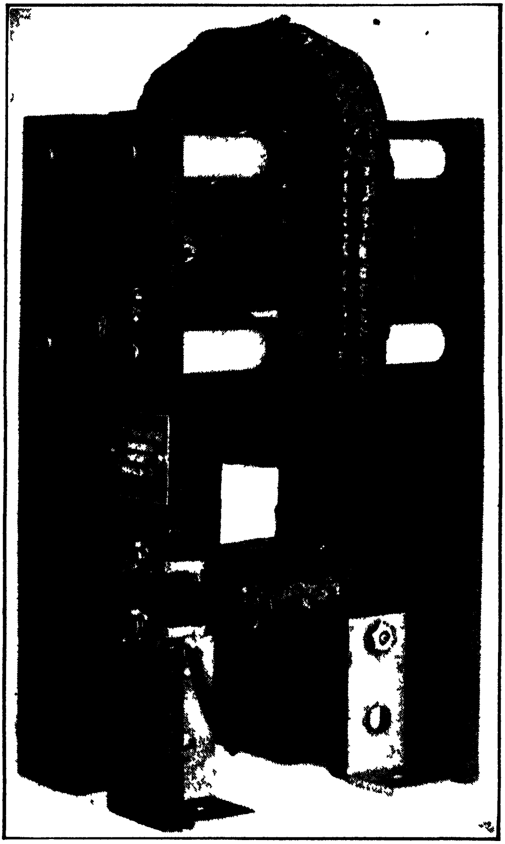
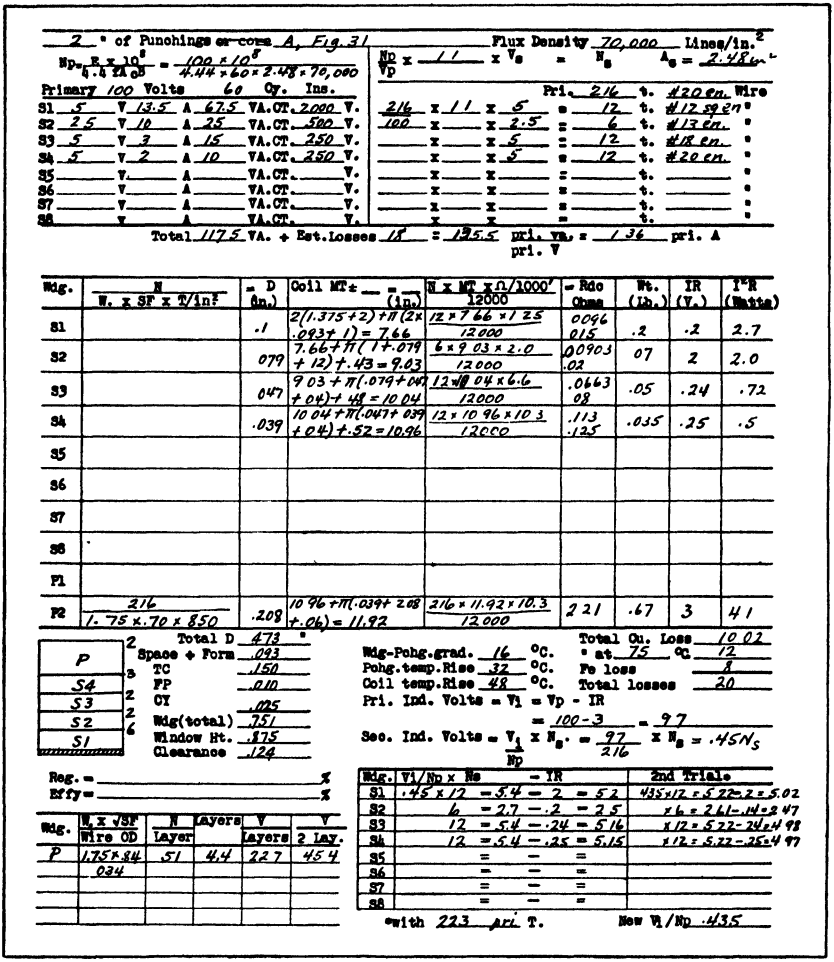


FIG. 41. Low capacitance filament transformer.

When a tube filament is cold, the filament resistance is a small fraction of its operating value. In large tubes it is often necessary to protect the tube filaments against the high initial current they would draw at rated filament voltage. This is done by automatically reducing the starting voltage through the use of a current-limiting transformer having magnetic shunts between primary and secondary windings. The design of these transformers is somewhat special, and is included in Chapter VIII.

26. Filament Transformer Design. It is important that design work be done systematically to save the designer's time and to afford a ready means of finding calculations at a later date. To attain these



2 # of Punchings ~~or core~~ A, Fig. 31 Flux Density 70,000 Lines/in.^{2}

$N_p = \frac{E \times 10^8}{4.44 fA_c B} = \frac{100 \times 10^8}{4.44 \times 60 \times 2.48 \times 70,000}$ $\frac{N_p}{V_p} \times$ 11 $\times V_s = N_s$ $A_c =$ 2.48 in.^{2}

Primary 100 Volts 60 Cy. Ins. Pri. 216 t. #20 en. Wire

	V	A	VA.CT.	V.					t.	Wire
S1	5	13.5	67.5	2000	216	x 11	x 5	=	12	#12 sq en
S2	2.5	10	25	500	100	x	x 2.5	=	6	#13 en.
S3	5	3	15	250		x	x 5	=	12	#18 en.
S4	5	2	10	250		x	x 5	=	12	#20 en.
S5						x	x	=		
S6						x	x	=		
S7						x	x	=		
S8						x	x	=		

Total 117.5 VA. + Est. Losses 18 = 135.5 $\frac{\text{pri. VA.}}{\text{pri. V}}$ = 1.36 pri. A

Wdg.	$\frac{N}{W. \times SF \times T/in.^2}$	= D (in.)	Coil MT = (in.)	$\frac{N \times MT \times \Omega/1000'}{12000}$	= Rdc Ohms	Wt. (Lb.)	IR (V.)	I^2R (Watts)
S1		.1	2(1.375+2)+π(2×.093+1) = 7.66	12×7.66×1.25 / 12000	.0096 .015	.2	.2	2.7
S2		.079	7.66+π(1+.079+.12)+.43 = 9.03	6×9.03×2.0 / 12000	.00903 .02	.07	.2	2.0
S3		.047	9.03+π(.079+.047+.04)+.48 = 10.04	12×10.04×6.6 / 12000	.0663 .08	.05	.24	.72
S4		.039	10.04+π(.047+.039+.04)+.52 = 10.96	12×10.96×10.3 / 12000	.113 .125	.035	.25	.5
S5								
S6								
S7								
S8								
P1								
P2	216 / 1.75×.70×850	.208	10.96+π(.039+.208+.06) = 11.92	216×11.92×10.3 / 12000	2.21	.67	3	4.1

P 2, S4 3, S3 2, S2 2, S1 6

Total D .473"
Space + Form .093
TC .150
FP .010
CY .025
Wdg(total) .751
Window Ht. .875
Clearance .124

Wdg-Pchg. grad. 16 °C.
Pchg. temp. Rise 32 °C.
Coil temp. Rise 48 °C.

Total Cu. Loss 10.02
" at 75 °C 12
Fe loss 8
Total losses 20

Pri. Ind. Volts = $V_1 = V_p - IR$ = 100 − 3 = 97

Sec. Ind. Volts = $\frac{V_1}{N_p} \times N_s = \frac{97}{216} \times N_s = .45 N_s$

Reg. = %
Effy = %

Wdg.	$\frac{W. \times \sqrt{SF}}{\text{Wire OD}}$	$\frac{N}{\text{Layer}}$	Layers	$\frac{V}{\text{Layers}}$	$\frac{V}{\text{2 Lay.}}$
P	1.75×.84 / .034	51	4.4	2.27	4.54

Wdg.	V1/Np × Ns − IR	2nd Trial*
S1	.45×12 = 5.4 − .2 = 5.2	.435×12 = 5.22 − .2 = 5.02
S2	6 = 2.7 − .2 = 2.5	×6 = 2.61 − .14 = 2.47
S3	12 = 5.4 − .24 = 5.16	×12 = 5.22 − .24 = 4.98
S4	12 = 5.4 − .25 = 5.15	×12 = 5.22 − .25 = 4.97
S5	= − =	
S6	= − =	
S7	= − =	
S8	= − =	

*with 223 pri. T. New V1/Np .435

FIG. 42. Filament transformer design calculations.

ends a calculation form, such as that in Fig. 42, is used. The form is usually made to cover several kinds of transformers, and only the spaces applicable to a filament transformer are used.

Suppose that a transformer is required to supply filament power for four single-phase full-wave rectifiers having output voltages of 2000,

500, 250 and 250 volts respectively, with choke input filters, as follows:

Primary voltage 100

Frequency 60 cycles

Four secondaries for the following tube filaments:

2—872 tubes:	5 volts	13.5 amp	Insulated for +2000 v d-c
2—866 tubes:	2.5 volts	10 amp	Insulated for + 500 v d-c
1—5U4G tube:	5 volts	3 amp	Insulated for + 250 v d-c
1—5Y3GT tube:	5 volts	2 amp	Insulated for + 250 v d-c

Ambient temperature: 40°C

First comes the choice of a core. Data such as those in Fig. 30 are helpful in this, and so is design experience in the modification of such

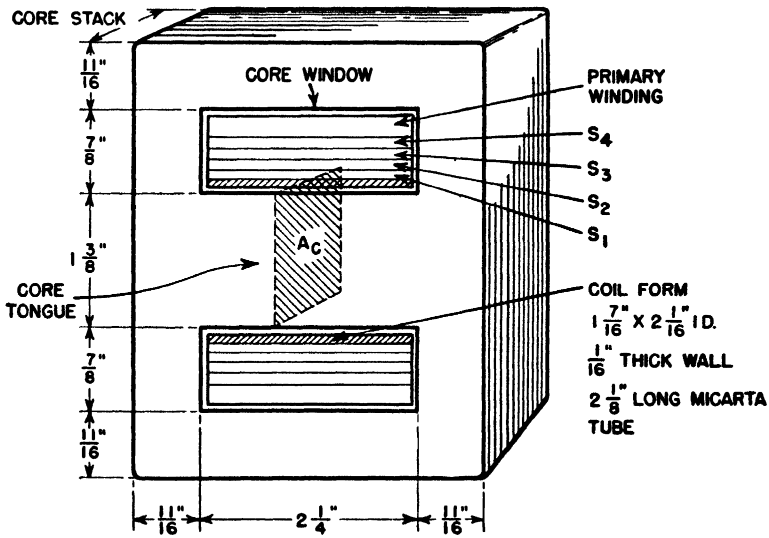


FIG. 43. Dimensions and coil section of filament transformer.

data by the specified requirements. The core used here is a 2-in. stack of laminations A, Fig. 31, which is described more fully in Fig. 43, and has enough heat dissipation surface for this rating. For silicon steel, an induction of 70,000 lines per square inch is practical. The primary turns can be figured from equation 4 by making the substitution $\phi = BA_c$ and transposing to

$$N_1 = \frac{E \times 10^8}{4.44 f A_c B} \tag{20}$$

where A_c is the core cross-sectional area, or product of the core tongue

width and stack dimension, and B is the core induction. In this transformer, with 90 per cent stacking factor, $A_c = 2 \times 0.9 \times 1.375 = 2.48$ sq in., and the primary turns are found to be 216.

Below this calculation are set down the primary voltage and frequency, and the voltage, current, volt-amperes, and insulation voltage for all secondary windings. These are designated S_1 to S_4 for identification. From the sum of the individual v-a figures, the transformer rating is found. To it is added an estimate of losses to obtain the input volt-amperes, and the primary current.

Next an estimate of the regulation is made (10 per cent) and added to unity to obtain the multiplier 1.1 in the estimate of secondary turns near the top of the calculation form. From the currents listed, the wire size for each winding is chosen. Round enameled wire is used for each winding except S_1, and for it No. 12 square wire is used to save space. The largest wire is placed next to the coil form to prevent damage in winding to the smaller wires.

The next task is to find out whether the wire chosen will fit in the core window space. Winding height D is entered for each winding. For each secondary this is the wire diameter, because the wire is wound in a single layer. D for S_1 is slightly larger than the wire dimension to allow for the bulge that occurs when square wire is wound. The twelve turns of S_1 occupy about $1\frac{1}{4}$ in. of horizontal winding space. The core window is $2\frac{1}{4}$ in. wide. From this is subtracted $\frac{1}{8}$ in. for clearance, leaving $2\frac{1}{8}$ in. total coil width. Margins on each side of S_1 are therefore $\frac{1}{2}(2\frac{1}{8} - 1\frac{1}{4}) = \frac{7}{16}$ in. According to Fig. 27 (p. 34) this provides over 8 kv breakdown strength, which is well above the 5-kv test voltage for S_1. Other secondary windings have lower test voltages and wider margins, and hence have more than adequate creepage distances.

The $\frac{1}{16}$-in. thick micarta rectangular tube used for the coil form has a corona voltage of 2700 rms which affords about 23 per cent safety factor over the normal operating voltage at the tube filaments. Over S_1 are wound six wraps of 0.010-in. thick treated cloth, which has 2600-volt corona limit. Winding S_2 supplies a filament at 500 volts of the same polarity as S_1. Hence only 1500 volts direct current or 1660 volts alternating current occur across this insulation. At the right of the small sketch in Fig. 42 are listed the number of wraps of 0.010-in. thick treated cloth over each section of winding. These are added together to give the columnar figure of 0.150 for TC.

The primary winding is wound without layer insulation and with an area space factor of 70 per cent. Cotton is wound in with the wire to form walls $\frac{3}{16}$ in. thick on either side of the primary; this accounts

for the low space factor and for the 1¾-in. winding traverse. The coil is finished with two layers of treated cloth, a layer of 0.010-in. fishpaper for mechanical protection, and a 0.025-in. serving of untreated cotton yarn or tape to hold it together. The total winding adds up to 0.751 in., leaving 0.124 in. clearance, about the right allowance for winding slack for four secondaries.

Mean turns are figured from equations 14 and 15, with 5 per cent incremental increase in S_2, S_3, and S_4 for leads. With the mean turn values the winding resistances, weights of copper, and IR and I^2R for each winding can be found. To S_1, S_2, and S_3 winding resistance is added lead resistance, and the lower figure is the sum of the two in each case. Total copper loss is multiplied by 1.2 to correct for 75°C operating temperature. The core weight is 6.8 lb and the grade of steel used has 1.17 watts per pound at 70,000 lines per square inch. This gives a core loss of 8 watts, and a total of copper and core loss of 20 watts. After these losses are divided by the appropriate ordinates from Figs. 31 and 32 (pp. 39 and 40) the coil temperature rise is figured at 48 centigrade degrees, which is safe for class A insulation.

We know by now that the design is safe, but secondary voltages still must be checked. The method of equation 9 is used. Output voltages on first trial range from 0 to 4 per cent high. S_2 voltage is correct but out of line with the rest. Changing S_2 leads to a larger size makes the per cent voltage drops more nearly alike, and increasing the primary turns to 223 brings all output voltages to correct value within 1.2 per cent. Filament voltage should be kept within 2 per cent for these tubes, to allow for meter error. Primary voltage per layer is checked at the lower left; this is equivalent to 22.7 volts per mil of wire enamel, which is safe practice.

If the design were deficient in any respect, even down to the last things figured, some change would have to be made which would require recalculation of all or part of the transformer, hence the importance of good estimating all the way along.

27. Anode Transformers. Anode transformers differ from filament transformers in several respects.

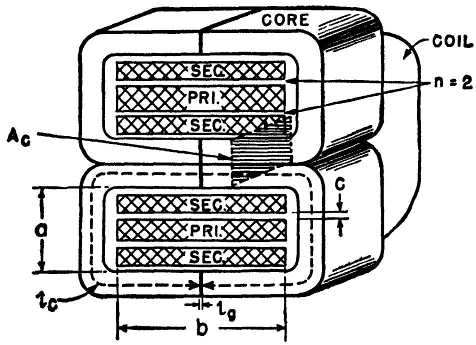
(*a*) *Currents* are non-sinusoidal. In a single-phase full-wave rectifier, for instance, current flows through one half of the secondary during each positive voltage excursion and through the other half during each negative excursion. For half of the time each half-secondary winding is idle.

(*b*) *Leakage Inductance* not only determines output voltage, but also affects rectifier regulation in an entirely different manner than with a straight a-c load. This is discussed in Chapter IV.

(c) *Half-Wave Rectifiers* carry unbalanced direct current; this necessitates less a-c flux density, hence larger transformers, than full-wave rectifiers. Unbalance in the three-phase half-wave type can be avoided by the use of zigzag connections, but an increase in size over full-wave results because of the out-of-phase voltages. These connections are desirable in full-wave rectifiers when half voltage is obtained from a center tap. See Table VII.

(d) *Single-Phase Full-Wave Rectifiers with Two Anodes* have higher secondary volt-amperes for a given primary v-a rating than a filament transformer. Bridge-type (4-anode) rectifiers have equal primary and secondary volt-amperes, as well as balanced direct current, and plate transformers for these rectifiers are smaller than for other types. Three-phase rectifier transformers are smaller in total size, but require more coils. The three-phase full-wave type has equal primary and secondary v-a ratings.

(e) *Induced Secondary Voltage* is much higher. Filament transformers are insulated for this voltage, but have a few secondary turns



NOTE: CONSTUCTION SHOWN IS FOR SHELL TYPE TRANSFORMER WITH 2 HIPERSIL CORES.

FIG. 44. Dimensions and coil section of anode transformer.

of large wire, whereas anode transformers have many turns of small wire. For this reason the volts per layer are higher in anode transformers, and core windows having proportionately greater height and less width than those in Fig. 43 are often preferable. This trend runs counter to the conditions for low leakage inductance and makes it necessary to interleave the windings. Figure 44 shows the windings of a single-phase full-wave rectifier transformer with the primary interleaved between halves of the secondary. This arrangement is especially adaptable to transformers with grounded center tap. The

primary-secondary insulation can be reduced to the amount suitable for primary to ground.

28. Leakage Inductance. Flux set up by the primary winding which does not link the secondary, or vice versa, gives rise to leakage or self-inductance in each winding without contributing to the mutual flux. The greater this leakage flux, the greater the leakage inductance, because the inductance of a winding equals the flux linkages with unit current in the winding. In Fig. 44, all flux which follows the core path l_c is mutual flux. Leakage flux is the relatively small flux which threads the secondary winding sections, enters the core, and returns to the other side of the secondaries, without linking the primary. The same is true of flux linking only the primary winding. But it is almost impossible for flux to leave the primary winding, enter the core, and re-enter the primary without linking part of the secondary also. The more the primary and secondary windings are interleaved, the less leakage flux there is, up to the limit imposed by flux in the spaces c between sections. These spaces contain leakage flux also; indeed, if there is much interleaving or if the spaces c are large, most of the leakage flux flows in them. Large coil mean turn length, short winding traverse b, and tall window height a all increase leakage flux.

Several formulas have been derived for the calculation of leakage inductance. That originated by Fortescue[4] is generally accurate and errs, if at all, on the conservative side:

$$L = \frac{10.6N^2l(2nc + a)}{10^9n^2b} \tag{21}$$

where L = leakage inductance of both windings in henrys, referred to the winding having N turns
l = mean length of turn for whole coil in inches
n = number of dielectrics between windings ($n = 2$ in Fig. 44)
c = thickness of dielectric between windings in inches
a = core window height in inches
b = winding traverse in inches.

The greatest gain from interleaving comes when the dielectric thickness c is small compared to the window height; when nc is comparable to the window height, the leakage inductance does not decrease much as n is increased. A small number of turns, short mean turn, and low, wide core windows all contribute to a low value of leakage inductance.

[4] See *Standard Handbook for Electrical Engineers*, McGraw-Hill Book Co., 1922, fifth ed., p. 413.

29. Anode Transformer Design. Let the requirements of a rectifier be

1200 volts 115 ma rectifier d-c output
Single-phase full-wave circuit with 866 tubes
Primary 115 volts 60 cycles
Rectifier regulation 5 per cent maximum
Ambient 55°C

To fulfill these requirements, a reactor-input filter must be used. If 1 per cent is allowed for reactor IR drop, a maximum of 4 per cent regulation is left in the anode transformer. The approximate secondary output voltage is $1200 \times 2.22 = 2660$, say 2700 volts. The center tap may be grounded. Suppose that a transformer like the one in Fig. 44 is used. The calculations are given in Fig. 45. The various steps are performed in the same order as in filament transformers. The grain-oriented type C core is worked at 38 per cent higher induction, with but 60 per cent of the core loss of Fig. 42; its strip width is $2\frac{1}{4}$ in., build-up $\frac{5}{8}$ in., and window 1 in. by 3 in. for each core loop. Note the difference in primary and secondary volt-amperes and winding heights. Since the primary and secondary are symmetrical about the primary horizontal center line, they have the same mean turn length. Losses and temperature rise are low. Regulation governs size. Secondary layer voltage is high enough to require layer paper. This coil is wound on a multiple-coil machine. Winding height is figured on the basis of layer paper adequate for the voltage instead of from Table VI (p. 25), but turns per layer are taken from this table. Layer insulation is used at 46 volts per mil in the secondary; this counts the 1.7 mils of double enamel, which must withstand impregnation without damage. Anode leads and margins withstand 5 kv rms test voltage. Since the secondary center tap is grounded, two thicknesses of 0.010-in. insulation between windings are sufficient. Clearance of 0.253 in. allows room for in-and-out coil taping.

Secondary leakage inductance, from equation 21 is

$$\frac{10.6 \times 4200^2 \times 10.2(4 \times 0.020 + 1)}{4 \times 2.375 \times 10^9} = 0.218 \text{ henry}$$

At 60 cycles this is $6.28 \times 60 \times 0.218 = 85$ ohms, which would be tripled if a single section were used. The effect of this on regulation is considered in the next chapter. The regulation calculated in Fig. 45 is that due to primary IR calculated in the normal manner, plus I_{dc} times one-half the secondary winding resistance.

30. Combined Anode and Filament Transformers. Anode and filament windings are combined into a single transformer mainly in low power ratings such as those in receivers and grid bias power supplies.

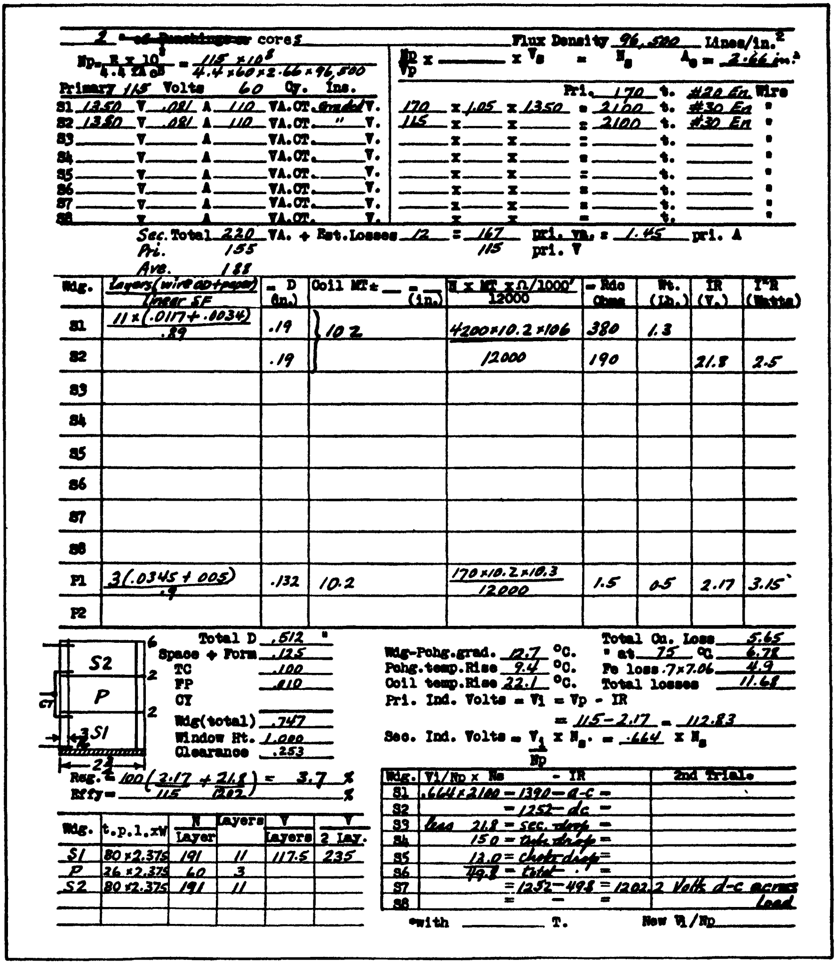


FIG. 45. Plate transformer design calculations.

One widely used combination includes the anode and filament windings for a rectifier and a filament winding for the amplifier tubes. Figures 46 and 47 show how these windings sometimes may be arranged to require a minimum of insulation and space. The high voltage filament winding S_1 is placed over the coil form to take advantage of its thick insulation. Layer insulation is sufficient between S_1 and S_2, and

between S_2 and S_3. Over and under the primary winding is 115-volt insulation. Thus Fig. 47 is a high voltage transformer with no high voltage insulation in it except what is incidental to the coil form.

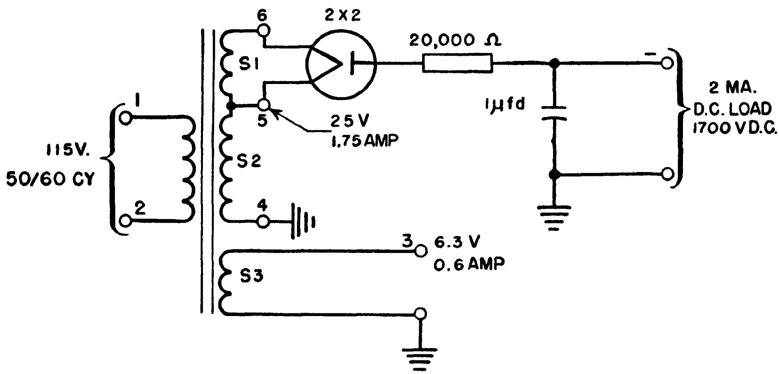


FIG. 46. Power supply transformer.

Combined anode and filament transformers are difficult to test for regulation or output voltage aside from operation in the rectifier circuit itself, because a-c loads do not duplicate rectifier action. Most transformers of this kind are used in rectifiers with capacitor-input filters or with fixed loads in which regulation is not important.

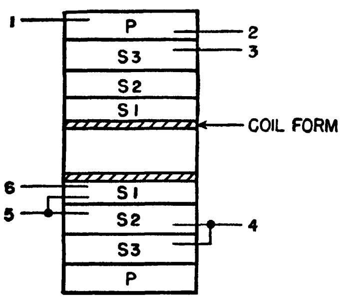


FIG. 47. Winding arrangement to save insulation.

Ratings are easier to predict. Anode secondary v-a rating is the product of rms voltage and current, but the corresponding portion of primary v-a rating depends on the rectifier and is found as mentioned in Sections 22 and 23. To this is added the sum of filament winding v-a ratings, and the primary current can then be calculated from the total volt-amperes.

31. Power Supply Frequency. Foregoing examples were based on a 60-cycle supply. Twenty-five-cycle transformer losses are lower for a given induction. It follows that induction can be increased somewhat over the 60-cycle value, but saturation currents prevent a decided increase. Larger size results, nearly 2:1 in volume. Otherwise 25-cycle transformers are not appreciably different from 60-cycle transformers.

Power supply frequencies of 400 and 800 cycles are used mainly in aircraft and portable equipment to save weight and space. Silicon

steel and nickel alloy core materials 0.005 in. thick are principally used at these frequencies to reduce eddy currents. Losses at 400 and 800 cycles for three core materials are shown in Fig. 48. These losses can be the controlling factors in determining transformer size, because the materials saturate at nearly the same induction whether the frequency is 60 cycles or 800 cycles, but the core loss is so high at 800 cycles that

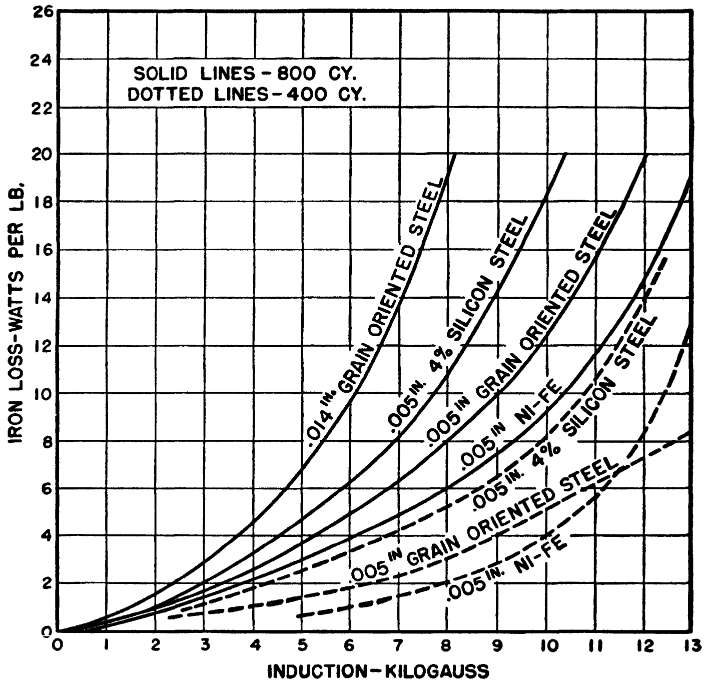


Fig. 48. Core loss at 400 and 800 cycles.

the core material cannot be used near the saturation density. The higher the induction the higher the core heating. For this reason, class B insulation can be used in many 400- and 800-cycle designs to reduce size still further. If advantage is taken of both the core material and insulation, 800-cycle transformers can be reduced to 20 per cent of the size of 60-cycle transformers of the same rating. Typical combinations of grain-oriented core material and insulation are as follows:

Frequency	Strip Thickness	*B*-Gauss	Class of Insulation	Operating Temperature
60	0.014	15,000	A	95°C
400	0.005	12,500	B	140°C
800	0.005	8,500	B	140°C

In very small units, these flux densities may be used at lower temperatures and with class A insulation throughout. The limit has not been reached in either flux density or operating temperature. The necessity for small dimensions, especially in aircraft apparatus, continually increases the tendency to use materials at their fullest capabilities.

32. An 800-Cycle Transformer Design.

Primary 120 volts 800 cycles

Rectifier to deliver 0.2 amp at +450 volts using 5U4G in single-phase full-wave circuit with 0.5μfd capacitor input filter.

Figures 38 and 39 tell whether the product $\omega C R_L$ will produce the necessary d-c output without exceeding the rectifier tube peak inverse voltage rating and peak current rating.

$$\omega C R_L = 6.28 \times 800 \times 0.5 \times 10^{-6} \times \frac{450}{0.2} = 5.65$$

For R_S assume a peak current of 0.5 amp. Average anode characteristics show 97 volts tube drop, or $97 \div 0.5 = 194$ ohms at peak current. $R_S/R_L = 194/2250 = 0.086$. Add 5 per cent for transformer windings; estimated $R_S/R_L = 13.6$ per cent.

Check on Peak Current from Fig. 39.

$$n\omega C R_L = 11.3$$

$$\hat{I}_p = 5I_p = 5 \times 0.1 = 0.5 \text{ amp},$$

the peak value assumed. Rms current in tube plates and secondary windings is $2 \times 0.1 = 0.2$ amp. Output voltage, from Fig. 38, is 0.69 peak a-c voltage per side. Hence secondary rms voltage per side is $450 \times 0.707 \div 0.69 = 460$ volts, and secondary volt-amperes $= 2 \times 460 \times 0.2 = 184$. The anode transformer must deliver $2 \times 460 = 920$ volts at 0.2 amp rms. Primary volt-amperes $= 0.707 \times 184 = 130$.

Inverse peak voltage is the peak value of this voltage plus the d-c output, because the tube filament is at d-c value, plus a small amount of ripple, while one anode has a maximum of peak negative voltage, during the non-conducting interval. Thus peak inverse voltage is $460 \times 1.41 + 450 = 1100$ volts, which is within the tube rating.

Choice of core for this transformer is governed by size and cost considerations. Assume that the core works at 8500 gauss. The loss per pound for 0.005-in. silicon steel, grain-oriented steel, and nickel alloy is 12.2, 8.8, and 6.6 respectively. (See Fig. 48.) But nickel alloy

punchings have 80 per cent stacking factor, whereas the type C core has 90 per cent. For the same number of turns and stack of iron, nickel alloy punchings work at $(8500 \times 90)/80 = 9550$ gauss and the loss per pound is 8.2. This compares with 8.8 for the type C core and does not warrant the extra cost. In this thickness 0.005-in. grain-oriented steel compares still better with ordinary silicon steel and so will be used for the core.

Let two type C cores be used with the following dimensions:

Strip width	¾ in.	Window height	⅝ in.
Build	⅜ in.	Window width	1½ in.
Total net core area	0.506 sq in.	Core weight	0.75 lb

Turns could be figured from equation 20, except that the induction is in gauss. Since many core data are given in gauss, equation 20 is changed for convenience to

$$N_1 = \frac{3.49E \times 10^6}{fA_cB} \tag{22}$$

where dimensions are in inches and B is in gauss. Primary turns are then

$$\frac{3.49 \times 120 \times 10^6}{800 \times 0.506 \times 8500} = 122$$

Final design figures are:

Primary 122 turns	No. 26 glass-covered wire	d-c resistance 1.8 ohms
Secondary 900 turns	No. 29 glass-covered wire	d-c resistance 38 ohms

Primary copper loss at 100°C	=	3.35 watts
Secondary copper loss at 100°C	=	2.04 watts
Core loss 8.8×0.75	=	6.6 watts
Total losses		11.99 watts

With an open-type mounting and mica insulation this transformer had a temperature rise of 85 centigrade degrees.

33. Polyphase Transformers. In large power rectifiers three-phase supplies are generally used. Accurate phase voltages must be maintained to avoid supply frequency ripple in the output. Delta-connected primaries are shown in Table VII for the various rectifiers; these are preferable to open-delta because phase balance is better, and to Y-connections because of possibly high third harmonics. Open-delta connections require only two single-phase transformers instead of three, but a similar saving may be had by using a single core-type

three-phase unit which retains the phase-balance advantage. The main drawback to a three-phase core is its special dimensions. Often, to use standard parts, three single-phase units are employed in the smaller power ratings. But if the power is hundreds or even thousands of kilowatts, the cores are built to order, and the weight saving in a three-phase core is significant.

Two- and three-phase filament transformers are used with output tubes for large broadcast stations to reduce hum in the radio frequency output.

34. Reactors.[5] Reactors are used in electronic power equipment to smooth out ripple voltage in d-c supplies, so they carry direct current in the coils. It is common practice to build such reactors with air gaps in the core to prevent d-c saturation. The air gap, size of the core, and number of turns depend upon three interrelated factors:

Inductance desired
Direct current in the winding
A-c volts across the winding

The number of turns, the direct current, and the air gap determine the d-c flux density, whereas the number of turns, the volts, and the core size determine the a-c flux density. If the sum of these two flux densities exceeds saturation value, noise, low inductance, and non-linearity result. Therefore a reactor must be designed with knowledge of all three of the conditions above.

Magnetic flux through the coil has two component lengths of path: the air gap l_g, and the length of the core l_c. The core length l_c is much greater geometrically than the air gap l_g, as indicated in Fig. 44, but the two components do not add directly because their permeabilities are different. In the air gap, the permeability is unity, whereas in the core its value depends on the degree of saturation of the iron. The effective length of the magnetic path is $l_g + (l_c/\mu)$, where μ is the permeability for the steady or d-c component of flux.

When direct current flows in an iron-core reactor, a fixed magnetizing force H_{dc} is maintained in the core. This is shown in Fig. 49 as the vertical line H_{dc} to the right of zero H in a typical a-c hysteresis loop. H_{dc} intersects the normal magnetization curve OC at point G, and establishes the core flux density B_{dc} which would obtain if no alternating voltage were present. The slope of line OC at point G is the d-c permeability μ.

[5] Sections 34 and 35 are based on the author's "Reactors in D-C Service," *Electronics*, Sept. 1936, p. 19.

A small alternating voltage across the reactor causes a superposed increment of induction ΔB to add to and subtract from B_{dc} alternately, following the small hysteresis loop which touches the large loop at point E and the normal magnetization curve at point F; these points

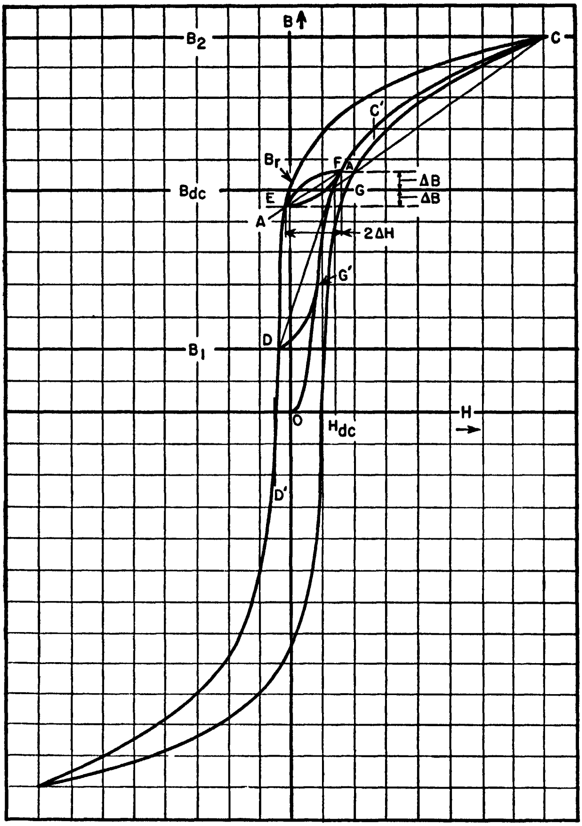


FIG. 49. Reactor hysteresis loop.

are the intersections of the lower and upper horizontal lines ΔB with their respective curves. The *incremental* permeability μ_Δ is $\Delta B/\Delta H$ and is represented by the slope of the line AA. It will be seen that AA has a slope much less than that of the normal magnetization curve at point G. Also, the smaller ΔB is, the flatter the line AA becomes and consequently the smaller the value of permeability μ_Δ. The curve in Fig. 50 marked μ is the normal permeability of 4% silicon steel for steady values of flux—in other words, for the d-c flux in the core. It is 4 to 20 times as great as the incremental permeability μ_Δ for a small

alternating flux superposed upon the d-c flux. The ratio of μ to μ_Δ gradually increases as d-c flux density increases.

Because of the low value of μ_Δ for minute alternating voltages, the effective length of magnetic path $l_g + (l_c/\mu_\Delta)$ is considerably greater for alternating than for steady flux. But the inductance varies inversely as the length of a-c flux path. If, therefore, the incremental permeability is small enough to make l_c/μ_Δ large compared to l_g, it follows that

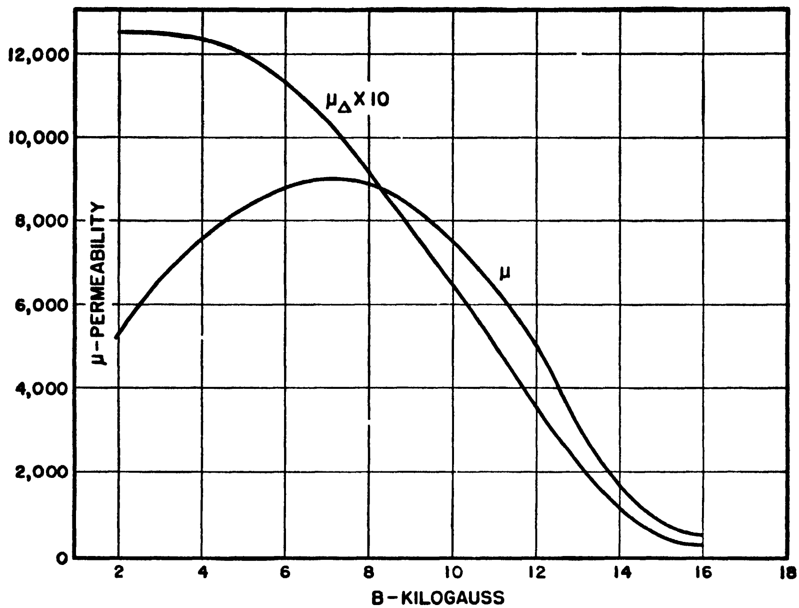


FIG. 50. Normal and incremental permeability of 4% silicon steel. (Courtesy of *Electronics*, Sept. 1936.)

small variations in l_g do not affect the inductance much. For this reason the exact value of the air gap is not important with small alternating voltages.

Reactor size, with a given voltage and ratio of inductance to resistance, is proportional to the stored energy LI^2. For the design of reactors carrying direct current, that is, the selection of the right number of turns, air gap, and so on, a simple method was originated by C. R. Hanna.[6] By this method, magnetic data are reduced to curves such as Fig. 51, plotted between LI^2/V and NI/l_c from which reactors can be designed directly. The various symbols in the coordinates are:

[6] "Design of Reactances and Transformers which Carry Direct Current," by C. R. Hanna, *Jour. A.I.E.E.*, Vol. 46, Feb. 1927, p. 128.

L = a-c inductance in henrys
I = direct current in amperes
V = volume of iron core in cubic inches
= $A_c l_c$ (see Fig. 44 for core dimensions)
A_c = cross section of core in square inches
l_c = length of core in inches
N = number of turns in winding
l_g = air gap in inches

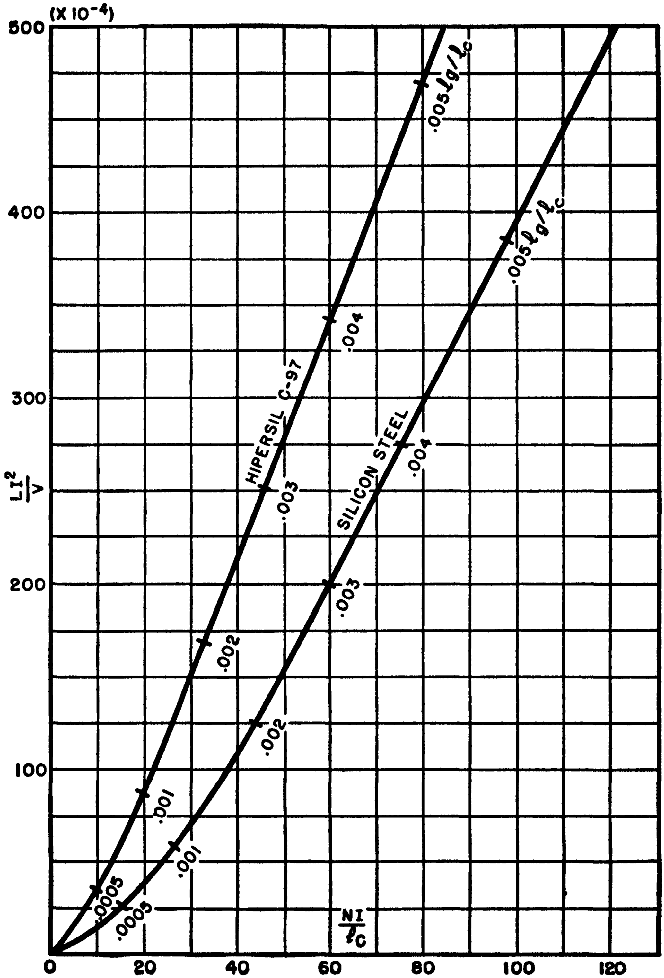


FIG. 51. Reactor energy per unit volume versus ampere-turns per inch of core.

Each curve of Fig. 51 is the envelope of a family of fixed air-gap curves such as those shown in Fig. 52. These curves are plots of data based upon a constant small a-c flux (10 gauss) in the core but a large and variable d-c flux. Each curve has a region of optimum usefulness,

beyond which saturation sets in and its place is taken by a succeeding curve having a larger air gap. A curve tangent to the series of fixed air-gap curves is plotted as in Fig. 51, and the regions of optimum usefulness are indicated by the scale l_g/l_c. Hence Fig. 51 is determined

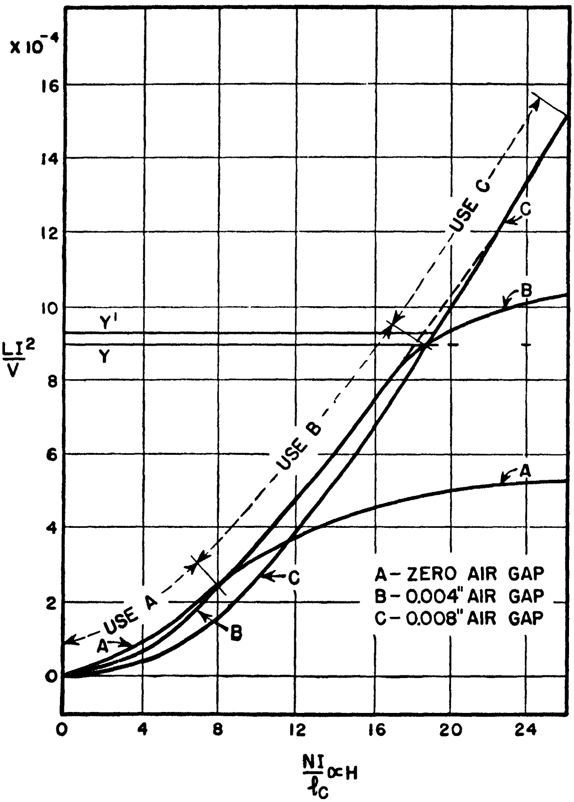


FIG. 52. Fixed air-gap curves. For $B_{dc} \gg B_{ac}$, air gap is not critical.

mainly by the d-c flux conditions in the core and represents the most LI^2 for a given amount of material.

Figure 52 illustrates how the exact value of air gap is of little consequence in the final result. The dotted curve connecting B and C is for a 6-mil gap. Point Y' represents the maximum inductance that could be obtained from a given core for $NI/l_c = 19$. Point Y is the inductance obtained if a gap of either 4 or 8 mils is used. The difference in inductance between Y and Y' is 4 per cent, for a difference in air gap of 33 per cent.

An example will show how easy it is to make a reactor according to this method.

Example. Assume a stack of silicon steel laminations having a cross section ⅞ in. by ⅞ in., and with iron filling 92 per cent of the space. The length of the flux path l_c in this core is 7½ in. It is desired to know how many turns of wire and what air gap are necessary to produce 70 henrys when 20 ma d-c are flowing in the winding.

This problem is solved as follows:

$$A_c = (0.875)^2 \times 0.92 = 0.71 \text{ sq in.}$$

$$V = 0.71 \times 7.5 = 5.3$$

$$\frac{LI^2}{V} = \frac{70 \times 4 \times 10^{-4}}{5.3} = 53 \times 10^{-4}$$

In Fig. 51 the abscissa corresponding to $LI^2/V = 53 \times 10^{-4}$ is $NI/l_c = 25$ for silicon steel. The ratio of air gap to core length l_g/l_c is between 0.0005 and 0.001.

$$\frac{NI}{l_c} = 25$$

$$N = \frac{25 \times 7.5}{0.020} = 9350 \text{ turns}$$

The total air gap is nearly $0.001 \times 7\frac{1}{2}$ or 7.5 mils; the gap at each joint is half of this value, or 3 to 3.5 mils.

The conditions underlying Hanna's method of design are met in most applications. In receivers and amplifiers working at low audio levels, the alternating voltage is small and hence the alternating flux is small compared to the steady flux. Even if the alternating voltage is of the same order as the direct voltage, the alternating flux may be small, especially if a large number of turns is necessary to produce the required inductance; for a given core the alternating flux is inversely proportional to the number of turns. D-c resistance of the coil is usually fixed by the regulation or size requirements. Heating seldom affects size.

35. Large Reactors. With the increasing use of higher voltages, it often happens that the a-c flux is no longer small compared to the d-c flux. This occurs in high impedance circuits where the direct current has a low value and the alternating voltage has a high value. The inductance increases by an amount depending on the values of a-c and d-c fluxes. Typical increase of inductance is shown in Fig. 53 for a reactor working near the saturation point. Increasing a-c flux soon adds to the saturation, which prevents further inductance increase and

accounts for the flattening off in Fig. 53. Saturation of this sort may be avoided by limiting the value of the d-c flux.

To illustrate the effect of these latter conditions, suppose that a reactor has already been designed for negligibly small alternating flux and operates as shown by the small loop of Fig. 49. Without changing anything else, suppose that the alternating voltage across the reactor is greatly increased, so that the total a-c flux change is from B_1 to B_2.

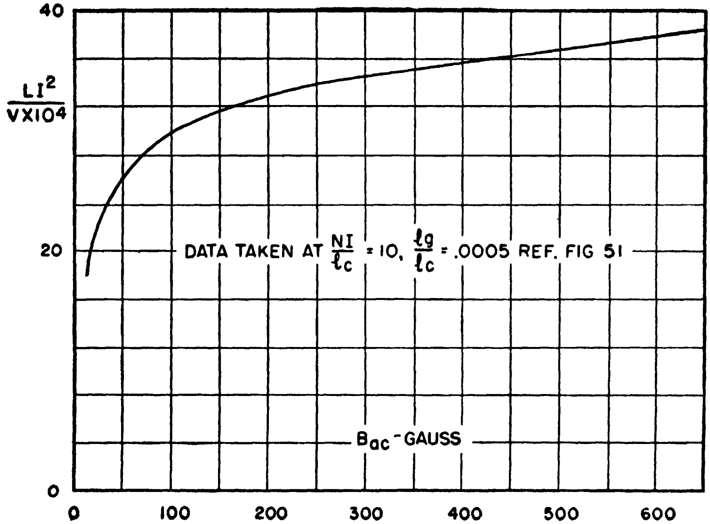


FIG. 53. Increase of inductance with a-c flux.

The reactor still operates about point G. The hysteresis loop, however, becomes the unsymmetrical figure $CFDEC$. The average permeability during the positive flux swing is represented by the line GC, and during the negative flux swing by GD. The slope of GD is greater than that of AA; hence, the first effect exhibited by the reactor is an increase of inductance.

The increase of inductance is non-linear, and this has a decided effect upon the performance of the apparatus. An inductance bridge measuring such a reactor at the higher a-c voltage would show an inductance corresponding to the average slope of lines GD and GC. That is, the average permeability during a whole cycle is the average of the permeabilities which obtain during the positive and negative increments of induction, and is represented by the average of the slopes of lines GD and GC. But if the reactor were put in the filter of a rectifier, the measured ripple would be higher than a calculated value based upon the bridge value of inductance. This occurs because the positive

peaks of ripple have less impedance presented to them than do the negative peaks, and hence they create a greater ripple at the load. Suppose, for example, that the ripple output of the rectifier is 500 volts, and that this would be attenuated to 10 volts across the load by a linear reactor having a value of inductance corresponding to the average slope of lines GC and GD. With the reactor working between B_1 and B_2, suppose that the slope of GD is five times that of GC. The expected average ripple attenuation of 50:1 becomes 16.7:1 for positive flux swings, and 83.3:1 for negative, and the load ripple is

$$\frac{1}{2}\left\{\frac{500}{16.7}+\frac{500}{83.3}\right\} = 18 \text{ volts}$$

or an increase of nearly 2:1 over what would be anticipated from the measured value of inductance.

This non-linearity could be reduced by increasing the air gap somewhat, and thereby reducing H_{dc} so that it passes through point G'. Now B_1 and B_2 drop down to intersect D' and C' respectively. Moreover, the average permeability has increased, and so has the inductance. It will be apparent that to decrease H_{dc} further means another increase in a-c permeability—approaching in value the d-c or normal permeability. This can be done only if the maximum flux density is kept low enough to avoid the saturating effects represented by point C in Fig. 49. Conversely, it follows that if saturation is present in a reactor, it is manifested by a decrease in inductance as the direct current through the winding is increased from zero to full-load value.

In a reactor having high a-c permeability the equivalent length of core l_c/μ is likely to be small compared to the air gap l_g. Hence, it is vitally important to keep the air gap close to its proper value. This is, of course, in marked contrast to reactors not subject to high a-c induction.

If a choke is to be checked to see that no saturation effects are present, access must be had to an inductance bridge. With the proper values of alternating voltage across the reactor, measurements of inductance can be made with various values of direct current through it. If the inductance remains nearly constant up to normal direct current, no saturation is present, and the reactor is suitable for the purpose. If, on the other hand, the inductance drops considerably from zero direct current to normal direct current, the reactor very probably is non-linear. Increasing the air gap may improve it; otherwise, it should be discarded in favor of a reactor which has been correctly designed for the purpose.

Filter reactors subject to the most alternating voltage for a given direct voltage are those used in choke-input filters of single-phase rectifiers. The inductance of this type of reactor influences the following:

Value of ripple in rectified output.

No-load to full-load regulation.

Transient voltage dip when load is suddenly applied, as in keyed loads.

Peak current through tubes during each cycle.

Transient current through rectifier tubes when voltage is first applied to rectifier.

It is important that the inductance be the right value. Several of these effects can be improved by the use of swinging or tuned reactors. In a swinging reactor, saturation is present at full load; therefore the inductance is lower at full load than at no load. The higher inductance at no load is available for the purpose of decreasing voltage regulation. The same result is obtained by shunt-tuning the reactor, but here the inductance should be constant from no load to full load to preserve the tuned condition.

In swinging reactors, all or part of the core is purposely allowed to saturate at the higher values of direct current to obtain high inductance at low values of direct current. They are characterized by smaller gaps, more turns, and larger size than reactors with constant inductance ratings. Sometimes two parallel gaps are used, the smaller of which saturates at full direct current. When the function of the reactor is to control a-c circuits by means of large inductance changes, no air gap at all is used. Design of such reactors is discussed in Chapter VIII.

The insulation of a reactor depends on the type of rectifier and how it is used in the circuit. Three-phase rectifiers, with their low ripple voltage, do not require the turn and layer insulation that single-phase rectifiers do. If the reactor is placed in the ground side of the circuit one terminal requires little or no insulation to ground, but the other terminal may operate at a high voltage to ground. In single-phase rectifiers the peak voltage across the reactor is E_{dc}, so the equivalent rms voltage on the insulation is $0.707E_{dc}$. But for figuring $B_{\max}$ the rms voltage is $0.707 \times 0.57E_{dc}$.

36. Linear Reactor Design. A method of design for linear reactors is based on three assumptions which are justified in the foregoing:

(*a*) The air gap is large compared to l_c/μ, μ being the d-c permeability.

(b) A-c flux density depends on alternating voltage and frequency.
(c) A-c and d-c fluxes can be added or subtracted arithmetically.

From (a) the relation $B = \mu H$ becomes $B = H$. Because of fringing of flux around the gap, an average of $0.85B$ crosses over the gap. Hence $B_{dc} = 0.4\pi N I_{dc}/0.85 l_g$. With l_g in inches this becomes

$$B_{dc} = \frac{0.6 N I_{dc}}{l_g} \text{ gauss} \tag{23}$$

Transposing equation 22,

$$B_{ac} = \frac{3.49E \times 10^6}{f A_c N} \text{ gauss} \tag{24}$$

The sum of B_{ac} and B_{dc} is $B_{\max}$, which should not exceed 11,000 gauss for 4% silicon steel, 16,000 gauss for grain-oriented steel, or 10,000 gauss for a 50% nickel alloy. Curves are obtainable from steel manufacturers which give incremental permeability μ_Δ for various combinations of these two fluxes. Figure 54 shows values for 4% silicon steel.

By definition, inductance is the flux linkages per ampere or, in cgs units,

$$L = \frac{\phi N}{10^8 I_M} = \frac{B_{ac} A_c N}{10^8 I_M} \tag{25}$$

But

$$B_{ac} = \frac{0.4\pi N I_M}{l_g + \dfrac{l_c}{\mu_\Delta}}$$

If this is substituted in equation 25,

$$L = \frac{3.19 N^2 A_c \times 10^{-8}}{l_g + \dfrac{l_c}{\mu_\Delta}} \text{ henrys} \tag{26}$$

provided dimensions are in inches. The term A_c in equation 26 is greater than in equation 24 because of the space factor of the laminations; if the gap is large A_c is greater still because the flux across it fringes. With large gaps, inductance is nearly independent of μ_Δ, at least up to saturation value of $B_{\max}$. Very large gaps may be broken up into several smaller ones to keep fringing down. With small gaps, permeability largely controls. There is always a certain amount of gap even with punchings stacked alternately in groups of 1. Table VIII gives the approximate gap equivalent of various degrees of interleaving laminations for magnetic path l_c of 5.5 in.

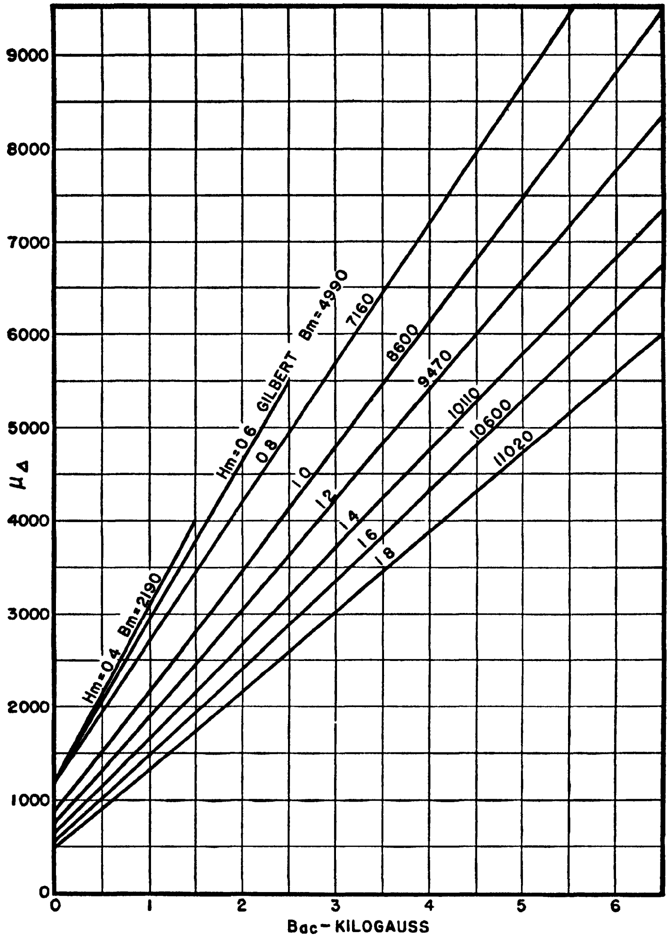


FIG. 54. Incremental permeability for 4% silicon steel with high a-c induction

TABLE VIII. EQUIVALENT GAPS WITH INTERLEAVED LAMINATIONS

0.014-in. Laminations Alternately Stacked	Equivalent Air Gap in Inches (Total) with Careful Stacking
In groups of 1	0.0005
In groups of 4	0.001
In groups of 8	0.002
In groups of 12	0.003
In groups of 16	0.004
Butt stacking with zero gap	0.005

Example. An input reactor is required for the filter of a 1500-volt, ¼-amp, single-phase, full-wave, 60-cycle rectifier. Let N = 2800 turns, net A_c = 2.48 sq in., gross A_c = 2.76 sq in., l_c = 9 in., l_g = 0.050 in. The 120-cycle voltage for figuring B_{ac} is 0.707 × 0.57 × 1500 = 605 volts.

$$B_{dc} = \frac{0.6 \times 2800 \times 0.25}{0.050} = 8{,}400$$

$$B_{ac} = \frac{3.49 \times 605 \times 10^6}{120 \times 2.48 \times 2800} = \underline{2{,}540}$$

$$B_{\max} = \qquad 10{,}940 \text{ gauss}$$

Figure 54 shows

$$\mu_\Delta = 2650$$

$$L = \frac{3.19 \times (2800)^2 \times 2.76 \times 10^{-8}}{0.050 + \frac{9}{2650}} = 13.0 \text{ henrys}$$

37. Transformers with D-C Flux. When there is a net d-c flux in the core, as in single-phase half-wave anode transformers, the choice of core depends on the same principles as in large reactors. The windings carry non-sinusoidal load current, the form of which depends on the circuit. Winding currents may be calculated with the aid of Table I (p. 10). Generally the heating effects of these currents are large. Maximum flux density should be limited as described in Section 36. This precaution is essential in limited power supplies like aircraft or portable generators, lest the generator voltage waveform be badly distorted. On large power systems the rectifier is a minor part of the total load and has no influence on voltage wave form. The chief limitation then is primary winding current, and maximum induction may exceed the usual limits by 25 per cent without excessive heating.

38. Power Transformer Tests. A power transformer is tested to discover whether the transformer will perform as required, or whether it will give reliable service life. Some tests perform both functions.

(*a*) *D-C Resistance.* This test is usually made on transformers at the factory as a check on the correctness of wire size in each winding. Variations are caused by wire tolerances, and by difference in winding tension between two lots of coils or between two coil machine operators. About 10 per cent variation can be expected in the d-c resistance of most coils, but this value increases to 20 per cent rather suddenly in sizes smaller than No. 40. The test is made by means of a resistance bridge or specially calibrated meter.

(*b*) *Turns Ratio.* Once the correct number of turns in each winding is established, correct output voltage can be assured for a coil of given

design by measuring the turns. A simple way of doing this is by use of the turns ratio bridge in Fig. 55. If the turns are correct, the null indicated by the meter occurs at a ratio of resistances:

$$\frac{R_1}{R_2} = \frac{N_1}{N_2} \tag{27}$$

If there is an error in the number of turns of one winding, the null occurs at the wrong value R_1/R_2. A source of 1000 cycles is preferable to one of 60 cycles for this test. The smaller current drawn by the transformer reduces IR and IX errors. Harmonics in the source obscure the null, so the source should be filtered.

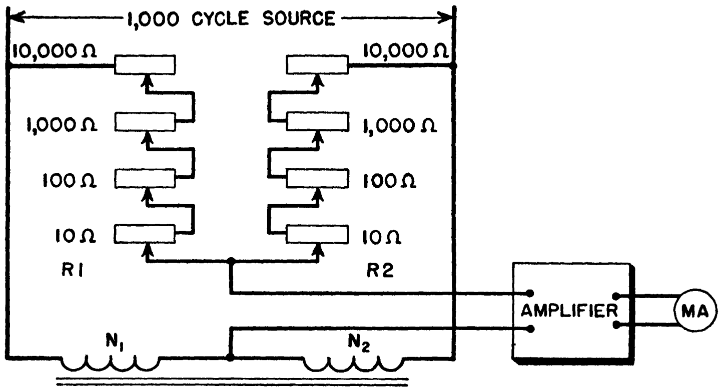


FIG. 55. Turns ratio bridge.

An accuracy of 0.1 per cent can usually be attained with four-decade resistances. Polarity of winding is also checked by this test, because the bridge will not balance if one winding is reversed.

(*c*) *Open-Circuit Inductance* (*OCL*). There are several ways of measuring inductance. If the Q (or ratio of coil reactance to a-c resistance) is high, the check may be made by measuring the current drawn by an appropriate winding connected across a source of known voltage and frequency. This method is limited to those cases where the amount of current drawn can be measured. A more general method makes use of an inductance bridge, of which one form is shown in Fig. 56.

If direct current normally flows in the winding, it can be applied through a large choke as shown. Inductance is then measured under the conditions of use. Source voltage should be adjustable for the same reason, and should be filtered to produce a sharp null. R_c is provided to compensate for coil a-c resistance. Without it an accurate

measurement is rarely attained. Enough data are provided by the test to calculate a-c resistance as well as inductance.

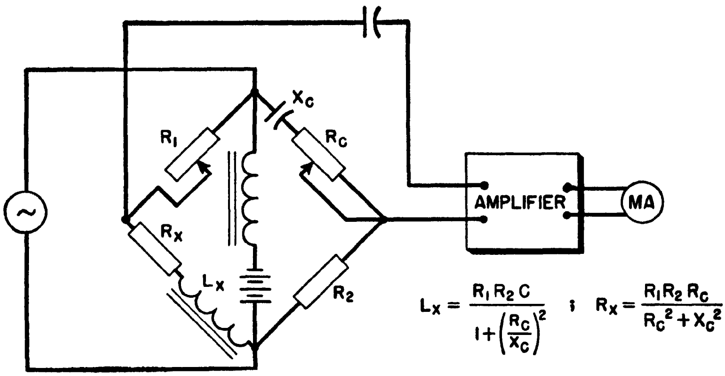


FIG. 56. Modified Hay bridge for measuring inductance.

(*d*) *Temperature Rise.* Tests to determine whether a transformer overheats are made by measuring the winding resistances before and

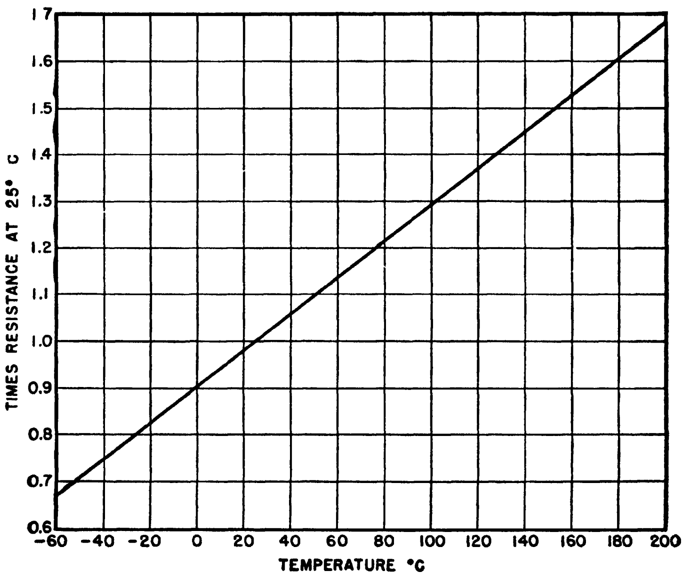


FIG. 57. Copper resistance versus temperature in terms of resistance at 25°C.

after a heat run, during which the transformer is loaded up to its rating. Where several secondaries are involved, each should deliver rated

voltage and current. Power is applied long enough to allow the transformer temperature to become stable; this is indicated by thermometer readings of core or case temperature taken every half hour until successive readings are the same. Ambient temperature at a nearby location should also be measured throughout the test. The average increase in winding resistance furnishes an indication of the average winding temperature. Figure 57 furnishes a convenient means for finding this temperature.

(e) *Regulation.* It is possible to measure voltage regulation by connecting a voltmeter across the output winding and reading the voltage with load off and on. This method is not accurate because the regulation is usually the difference between two relatively large quantities. Better accuracy can be obtained by multiplying the rated winding currents by the measured winding resistances and using equation 9. If the winding reactance drop is small this equation works well for resistive loads. To measure winding reactance drop, a short-circuit test is used. With the secondary short-circuited, sufficient voltage is applied to the primary to cause rated primary current to flow. The quotient E/I is the vector sum of winding resistances and reactances. Reactance is found from

$$X = \sqrt{Z^2 - R^2} \tag{28}$$

where R includes the resistance of both windings and the meter.

Sometimes it is more convenient to measure the leakage inductance with secondary short-circuited on a bridge and multiply by $2\pi f$.

(f) *Output Voltage.* Although the method described under *e* above is accurate for two-winding transformers, it is not applicable to multi-secondary transformers unless they are tested first with newly calibrated meters to see that all windings deliver proper voltage at full load. Once this is established, values of winding resistance and reactance thereafter can be checked to control the voltage. The interdependence of secondary voltages when there is a common primary winding makes such an initial test desirable. This is particularly true in combined filament and plate transformers, for which the best test is the actual rectifier circuit.

(g) *Losses.* Often it is possible to reduce the number of time-consuming heat runs by measuring losses. The copper loss is readily calculated by multiplying the measured values of winding resistance (corrected for operating temperature) by the squares of the respective rated currents. Core loss is measured with open secondary by means

of a low reading wattmeter at rated voltage in the primary circuit. If these losses correspond to the allowable temperature rise, the transformer is safely rated.

(*h*) *Insulation.* There is no test to which a transformer is subjected which has such shaky theoretical basis as the insulation test. Yet it is the one test it must pass to be any good. Large quantities of transformers can be built with little or no insulation trouble, but the empirical nature of standard test voltages does not assure insulation adequacy. It has been found over a period of years that, if insulation withstands the standard rule of twice normal voltage plus 1000 volts rms at 60 cycles for 1 minute, reasonable insulation life is usually obtained. It is possible for a transformer to be extremely underinsulated and still pass this test; conversely, there are conditions under which the rule would be a handicap. Therefore it can only be considered as a rough guide.

The manner of making insulation tests depends upon the transformer. Low voltage windings categorically can be tested by short-circuiting the terminals and applying the test voltage from each winding to core or case with other windings grounded. Filament transformers with secondaries insulated for high voltage may be tested in similar manner. But a high voltage plate transformer with grounded center tap requires unnecessary insulation if it is tested by this method. Instead, a nominal voltage of, say, 1500 volts is applied between the whole winding and ground; after that the center tap is grounded and a voltage is applied across the primary of such value as to test the end terminals at twice normal plus 1000 volts. Similar test values can be calculated for windings operating at d-c voltages other than zero. Such a test is called an induced voltage test. It is performed at higher than normal frequency to avoid saturation. An advantage of induced voltage testing is that it tests the layer insulation.

If insulation tests are repeated one or more times they may destroy the insulation, because insulation breakdown values decrease with time. Successive applications of test voltage are usually made at either decreased voltage or decreased time.

Corona tests are not open to this objection. A voltage about 10 per cent higher than normal is applied to the winding, and the leads are run through blocking capacitors to the input of a sensitive radio receiver as in Fig. 25.[7] NEMA standard noise values for this test are

[7] See "Methods of Measuring Radio Noise," a Report of the Joint Coordination Committee on Radio Reception, E.E.I. Publication No. 69, NEMA No. 107, RMA No. 32.

based primarily on radio reception, but they do indicate whether standard insulation practice is followed. See Table IX.

TABLE IX. NEMA RADIO INFLUENCE VOLTAGE LEVELS

Insulation Class (Operating Kilovolts)	Radio Influence (Microvolts)
1.2, 2.5, 5.0, 8.66, and 15	1,000
23 and 34.5	2,500
46 and 69	5,000
92, 115, and 138	10,000

PROBLEMS

1. In a bridge-type (4-tube), single-phase rectifier, give reason for the difference between tube rms current and transformer current, with reactor-input filter. With capacitor-input filter.

2. In the calculations of Fig. 45, secondary IR is the product of direct current output and one-half the winding resistance, and I^2R is the product of IR and d-c output. Calculate IR and I^2R, using the secondary rms current and the full winding resistance. Compare these figures to those of Fig. 45.

Ans. 30.8 volts; 2.5 watts.

3. Using two type C cores of 0.013 in.-thick grain-oriented steel having same dimensions as the cores in Section 32, but a net $A_c = 0.535$ sq in. and $B_{max} =$ 18,750 gauss at 50 cycles, design a transformer for Fig. 46 in accordance with Fig. 47. Calculate, in order:

(a) Volts and current in anode winding.
(b) B_{ac}, B_{dc}, and μ_Δ.
(c) Turns and wire size for each winding and coil clearance.
(d) Winding IR and losses.
(e) Output voltage of each secondary winding.

Ans. (a) 1660 volts, 4.5 ma rms; (b) 15,600, 3100 gauss; 10,000; (c) primary, 960 turns No. 30; S_1, 25 turns No. 20; S_2, 16,000 turns No. 42; S_3, 62 turns No. 24; 0.060 in.; (d) primary, 12.5 volts, 2.3 watts; S_1, 0.19 volt, 0.33 watt; 56 volts, 0.3 watt; 0.37 volt, 0.22 watt; (e) 6.3, 2.5, 1660 volts.

4. From the relation

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output} + \text{losses}}$$

calculate a volume versus loss curve from Fig. 30 (p. 38). Label this curve for a temperature rise of 55 centigrade degrees. Assuming temperature rise is proportional to loss, calculate another curve for volume versus v-a rating at a temperature rise of 40 centigrade degrees. *Hint*: Draw a new loss curve first, and assume same v-a/efficiency relations as before.

5. Assume a core loss of 0.5 watt per pound for the transformer of Prob. 3. If it is enclosed in a case with a volume of 33 cu in., what is the temperature rise (on the basis of class A insulation and the data of Prob. 4).

Ans. 25 centigrade degrees.

6. Complete the design of Section 32. Use simple primary-secondary arrangement. Note that IR is not so great as anticipated. Recalculate peak and rms currents and revise loss calculations. Calculate secondary leakage inductance. Does it affect output voltage materially? *Ans.* Little change; 0.024 henry; no.

7. Complete the reactor design of Section 36, using laminations similar to those of Fig. 43.

8. Derive the equations in Fig. 56.

9. Using equation 12, derive the rms currents in Table VII.

IV. RECTIFIER PERFORMANCE

39. Ripple. Filters used with rectifiers allow the rectified direct current to pass through to the load without appreciable loss, but ripple in the rectified output is attenuated to the point where it is not objectionable. Filtering sometimes must be carried out to a high degree. From the microphone to the antenna of a high power broadcast station, there may be a power amplification of 2×10^{15}. The introduction of a ripple as great as 0.005 per cent of output voltage at the microphone would produce a noise in the received wave loud enough to spoil the transmitted program. A rectifier used at the low power levels must be unusually well filtered to prevent noticeable hum from being transmitted.

Different types of rectifiers have differing output voltage waves, which affect the filter design to a large extent. Certain assumptions, generally permissible from the standpoint of the filter, will be made in order to simplify the discussion. These assumptions are:

1. The alternating voltage to be rectified is a sine wave.
2. The rectifying device passes current in one direction, but prevents any current flow in the other direction.
3. Transformer and rectifier voltage drops are negligibly small.
4. Filter condenser and reactor losses are negligible.

40. Single-Phase Rectifiers. Single-phase half-wave rectified voltage across a resistive load R is shown in Fig. 58. It may be resolved by Fourier analysis into the direct component whose value is $0.318E_{pk}$ or $0.45E_{ac}$, and a series of alternating components. The fundamental alternating component has the same frequency as that of the supply.

Single-phase half-wave rectifiers are used only when the low average value of load voltage and the presence of large variations in this voltage are permissible. The chief advantage of this type of rectifier is its simplicity. A method of overcoming both its disadvantages is illustrated in Fig. 59 where a capacitor C shunts the load. By using the proper capacitor, it is often possible to increase the value of E_{dc} to within a few per cent of the peak voltage E_{pk}. The principal disadvantage of this method of filtering is the large current drawn by the capaci-

tor during the charging interval as shown in Fig. 36 (b) (p. 47). This current is limited only by transformer and rectifier regulation; yet it must not be so large as to cause damage to the rectifier. The higher the value of E_{dc} with respect to E_{ac}, the larger is the charging current

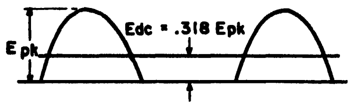


FIG. 58. Half-wave rectifier voltage.

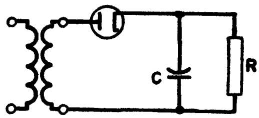


FIG. 59. Capacitor filter.

taken by C. (See Figs. 37 and 39, pp. 48 and 50.) Therefore, if a smooth current wave is desired, some other method of filtering must be used.

To obtain less voltage variation or ripple amplitude, after the limiting capacitor size has been reached, an inductive reactor may be employed. It may be placed on either the rectifier or the load side of the capacitor, depending on whether the load resistance R is high or low respectively. See Figs. 60 (a) and 60 (b). In the former, the voltage E_{dc} has less than the average value $0.45E_{ac}$, because the inductor delays the build-up of current during the positive half-cycle of voltage, and yet the inductor in this case should have a high value of reactance X_L, compared to the capacitive reactance X_C, in order to filter effectively. When R is low, reactance X_L should be high compared to R. In Fig. 60 (a) the ripple amplitude across R is $-X_C/(X_L - X_C)$ times the amplitude generated by the rectifier, if R is high compared to X_C. Also, in Fig. 60 (b), the ripple amplitude across R is R/X_L times the ripple obtained with capacitor only. R here is small compared to X_L.

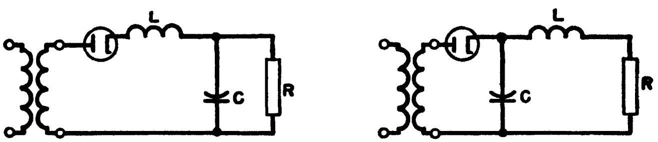


FIG. 60. (a) Inductor-input filter; (b) capacitor-input filter.

Large values of inductance are required to cause continuous current flow when the inductor is on the rectifier side of the capacitor in a half-wave rectifier circuit. Since current tends to flow only half the time, the rectified output is reduced accordingly. This difficulty is eliminated by the use of the full-wave rectifier of Fig. 61. The alternating components of the output voltage have a fundamental frequency

double that of the supply and the amplitudes of these components are much less than for the half-wave rectifier. The higher ripple frequency causes L and C to be doubly effective; the smaller amplitude

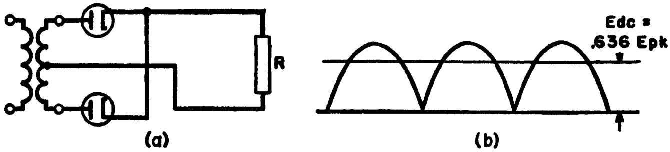


FIG. 61. (*a*) Single-phase full-wave rectifier; (*b*) rectified full-wave voltage.

results in smaller percentage of ripple input to the filter. Current flow is continuous and E_{dc} has double the value that it had in Fig. 58. For these reasons, this type of rectifier is widely used.

A full-wave rectifier uses only one half of the transformer winding at a time; that is, E_{ac} is only half the transformer secondary voltage. A circuit which utilizes the whole of this voltage in producing E_{dc} is the single-phase bridge rectifier shown in Fig. 62. The output voltage relations are the same as those of Fig. 61 (*b*). Although this circuit requires more rectifying tubes, it eliminates the need for a transformer mid-tap.

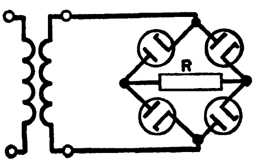


FIG. 62. Bridge rectifier.

41. Polyphase Rectifiers. The effect of rectifying more than one phase is to superpose more voltages of the same peak value, but in different time relation to each other. Figures 63 (*a*) and 63 (*b*) give a comparative picture of the rectified output voltage for three-phase half-wave and full-wave rectifiers. Increasing the number of phases increases the value of E_{dc}, increases the frequency of the alternating components, and decreases the amplitude of these components. Ripple frequency is p times that of the unrectified alternating voltage,

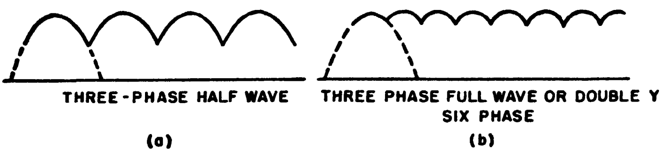


FIG. 63. Polyphase rectifier output waves.

p being 1, 2, 3, and 6 for the respective waves. Roughly speaking, p may be taken to represent the number of phases, provided that due allowance is made for the type of circuit, as in Fig. 64. Rectifiers with $p = 3$ or 6 are derived from three-phase supply lines, and, by

special connections, rectifiers with $p = 9$, 12 or more are obtained. The frequency of any ripple harmonic is mp, where m is the order of the harmonic.

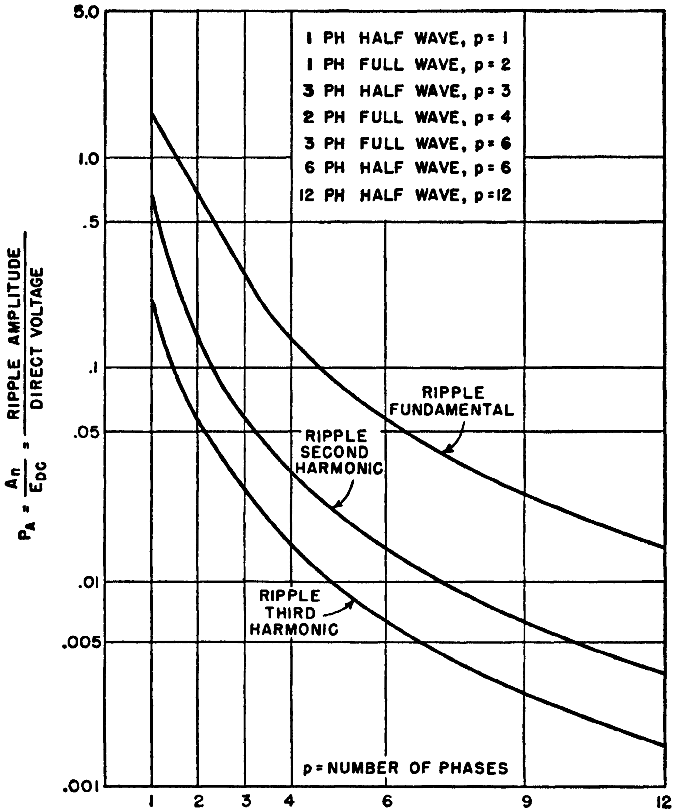


FIG. 64. Rectifier ripple voltage.

Ripple voltage for any of these rectifiers can be found by the Fourier relation:

$$A_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos n\omega t \, dt \tag{29}$$

where A_n = amplitude of nth ripple harmonic
T = ripple fundamental period
t = time (with peak of rectified wave as $t = 0$)
$\omega = 2\pi/Tp = 2\pi \times$ supply line frequency
$f(t)$ = ripple as a function of time
$= E_{pk} \cos \omega t,\ T/2 > \omega t > -T/2.$

The voltage peak is chosen as $t = 0$ to obtain a symmetrical function $f(t)$ and eliminate a second set of harmonic terms like those in equation 29, but with $\sin n\omega t$ under the integral.

Ripple amplitude is given in Fig. 64 for the ripple fundamental, and second and third harmonics with reactor-input filters. In this curve, the ratio P_A of ripple amplitude to direct output voltage is plotted against the number of phases p. It should be noted that P_A diminishes by a considerable amount for the second and third harmonics. In general, if a filter effectively reduces the percentage of fundamental ripple across the load, the harmonics may be considered negligibly small.

42. Multi-Stage Filters. In the inductor-input filter shown in Fig. 60 (*a*), the rectifier is a source of non-sinusoidal alternating voltage

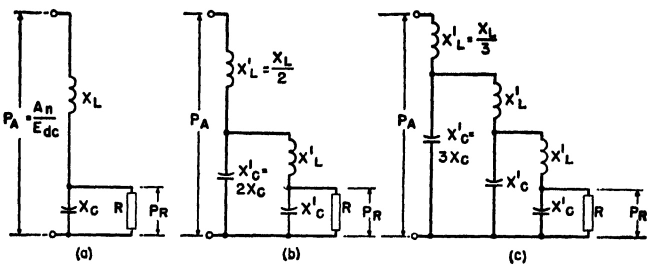


FIG. 65. Inductor-input filter circuits.

connected across the filter. It is possible to replace the usual circuit representation by Fig. 65 (*a*). For any harmonic, say the nth, the voltage across the whole circuit is the harmonic amplitude A_n, and the voltage across the load is $P_R E_{dc}$, P_R being ripple allowable across the load, expressed as a fraction of the average voltage. Since the load resistance R is high compared to X_C, the two voltages are nearly in phase, and they bear the same ratio to each other as their respective reactances, or

$$\frac{P_A}{P_R} = \frac{X_L - X_C}{X_C} = \frac{X_L}{X_C} - 1 \qquad (30)$$

From the type of rectifier to be used, and the permissible amount of ripple in the load voltage, it is possible to determine the ratio of inductive to capacitive reactance.

When the magnitude P_R must be kept very small, the single-stage filter of Fig. 65 (*a*) may require the inductor and the capacitor to be abnormally large. It is preferable under this condition to split both the inductor and the capacitor into two separate equal units, and

connect them like the two-stage filter of Fig. 65 (*b*). A much smaller total amount of inductance and of capacitance will then be necessary. For this filter

$$\frac{P_A}{P_R} = \left(\frac{X'_L - X'_C}{X'_C}\right)^2 \tag{31}$$

X'_L and X'_C being the reactances of each inductor and capacitor in the circuit. Likewise, the three-stage filter of Fig. 65 (*c*) may be more practicable for still smaller values of P_R. In the latter filter,

$$\frac{P_A}{P_R} = \left(\frac{X'_L - X'_C}{X'_C}\right)^3 \tag{32}$$

and, in general, for an n-stage filter,

$$\frac{P_A}{P_R} = \left(\frac{X'_L - X'_C}{X'_C}\right)^n \tag{33}$$

It is advantageous to use more than one stage only if the ratio P_A/P_R is high. That the gain from multi-stage filters is realized only for certain values of P_A/P_R is shown by Fig. 66. The lower curve shows the relation between P_A/P_R and X_L/X_C for a single-stage filter. The second curve shows the increase in P_A/P_R gained by splitting up the same amount of L and C into a two-stage filter; as indicated in Fig. 65 (*b*), the inductor and capacitor both have one-half their "lumped" value. The upper curve indicates the same increase for a three-stage filter, each inductor and capacitor of which have one third of their "lumped" or single-stage filter value. The attenuation in multi-staging is enormous for high X_L/X_C. For lower ratios there may be a loss instead of a gain, as shown by the intersection of the two upper curves. These curves intersect the lower curve if all are prolonged further to the left. This may be a puzzling condition; but consider that, for $X_L/X_C = 50$ in the single-stage filter, the ratio is $\frac{1}{3}X_L/3X_C$ or $^{50}/_{9}$ in the three-stage filter; the rather small advantage in the latter is not difficult to account for.

Other factors may influence the number of filter stages. In some applications, modulation or keying may require that a definite size of filter capacitor be used across the load. Usually these conditions result in a single-stage filter, where otherwise more stages might be most economical.

Table VII (p. 46) shows filter reactors in the negative lead, which may be either at ground or high potential. If low ripple is required in the filtered output, it is usually preferable to locate the filter reac-

tors in the high voltage lead. Otherwise, there is a ripple current path through the anode transformer winding capacitance to ground which bypasses the filter reactor. Ripple then has a residual value

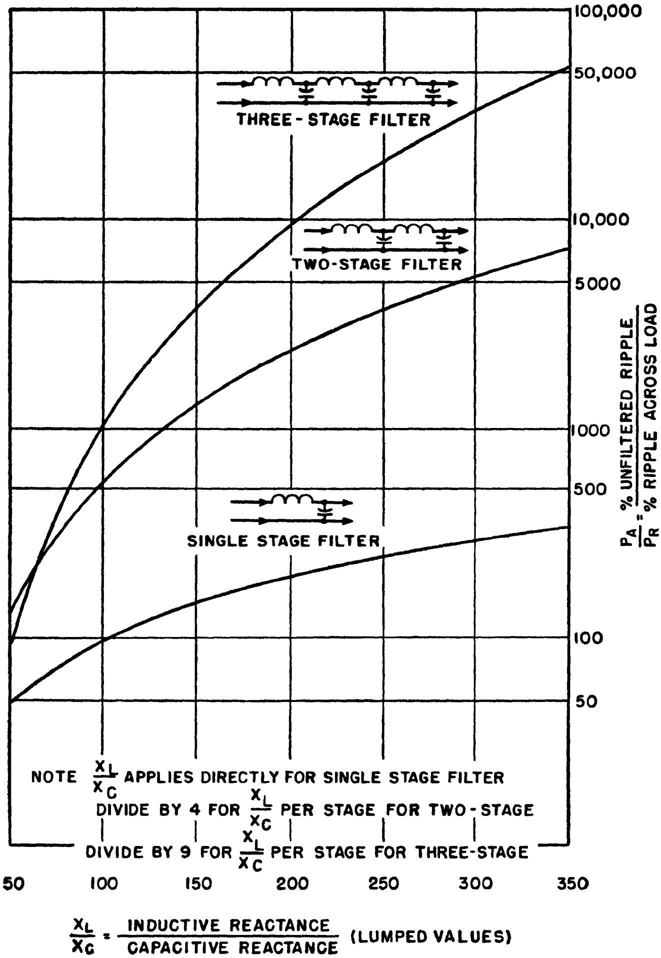


FIG. 66. Attenuation in one-, two-, and three-stage filters.

which cannot be reduced by additional filtering. In the three-phase, zigzag, full-wave circuit, with center tap used for half-voltage output, separate reactors should be used in the positive leads; placing a common reactor in the negative lead introduces high amplitude ripple in the high voltage output.

In rectifiers with low ripple requirements, both filament and anode windings should be accurately center-tapped to avoid low frequency

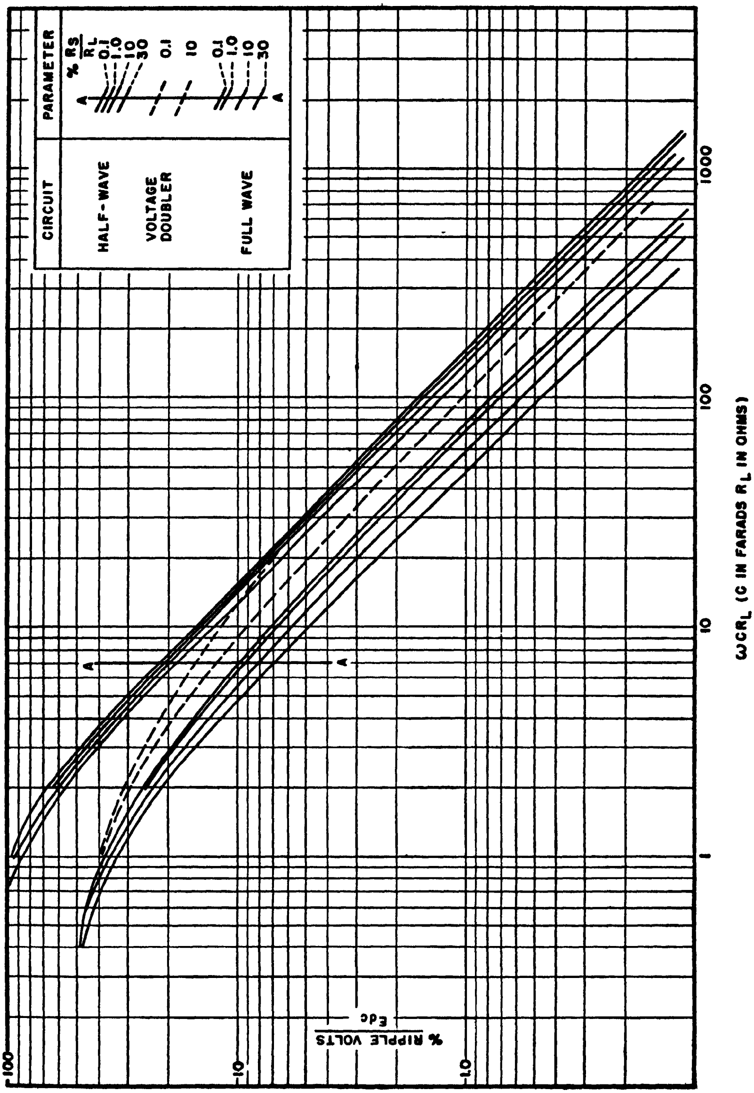


FIG. 67. Rms ripple voltage of capacitor-input filters. (From O. H. Schade, *Proc. I.R.E.*, July 1943, p. 341.)

ripple, which is difficult to filter. Three-phase leg voltages should be balanced for the same reason.

43. Capacitor-Input Filters. One of the assumptions implied at the beginning of this chapter, namely, that transformer and rectifier volt-

age drops are negligibly small, cannot usually be made when capacitor-input filters are used, because of the large peak currents drawn by the capacitor during the charging interval. Such charging currents drawn through finite resistances affect both the d-c output voltage and the ripple in a complicated manner, and simple analysis such as that given for inductor-input filters is no longer possible. Figure 67 is a plot of the ripple in the load of capacitor-input filters with various ratios of source to load resistance, and for three types of single-phase rectifiers. These curves are useful also when resistance is used in place of an inductor at the input of a filter. ω is 2π times the a-c supply frequency, C is the capacitance, R_L is the load resistance, and R_S the source resistance.

When L-C filter stages follow a capacitor-input filter, the ripple of the latter is reduced as in Fig. 66, except that the value of P_A must be taken from Fig. 67. When an R-C filter stage follows any type of filter, the ripple is reduced in the ratio R/X_C represented by the R-C stage.

44. Rectifier Regulation. The regulation of a rectifier comprises three distinct components:

1. The d-c resistance or IR drop.
2. The commutation reactance or IX drop.
3. The capacitor charging effect.

The first component can be reduced to a small value by the use of tubes, transformers, and inductors having low resistance. Mercury vapor tubes are of noteworthy usefulness in this respect, as the internal voltage drop is low and almost independent of load current variations.

Commutation reactance can be kept to a low value by proper transformer design, particularly in those cases where the ratio of short-circuit current to normal load is high.

During part of each cycle, both tubes of a single-phase full-wave rectifier are conducting. During this interval one tube loses its current and the other one builds up to normal current. Because of the inevitable reactance in the transformer, this change does not take place immediately, but during an angle θ as in Fig. 68. Short-circuit current is initiated which would rise as shown by the dotted lines of Fig. 68, if it could pass through the rectifier tubes; it prevents the rectified voltage wave from retaining its normal shape, so that for a portion of each cycle the rectified output is zero.

Let the transformer winding resistance be temporarily neglected; if the current could rise to maximum, the short-circuit value would be $2E_{pk}/X$; but it is limited by the rectifier to I_{dc}. The short-circuit current rises to $(1 - \cos\theta)$ times maximum in the commutation period,

or $[2E_{pk}(1 - \cos\theta)]/X = I_{dc}$. The average voltage from zero to the re-ignition point V is $(E_{pk}/\pi)(1 - \cos\theta)$.

Combining these relations gives, for the *average* voltage cut out of the rectified voltage wave by commutation,

$$V_{av} = \frac{I_{dc}X}{2\pi} \tag{34}$$

By similar reasoning, the commutation reactance drop for polyphase rectifiers is

$$\text{Per cent commutation reactance drop} = \frac{100 I_{dc} X}{2E_{pk} \sin \dfrac{\pi}{p}} \tag{35}$$

In equations 34 and 35, X = the commutation secondary reactance, E_{pk} = the peak value of the a-c phase voltage, p = the number of

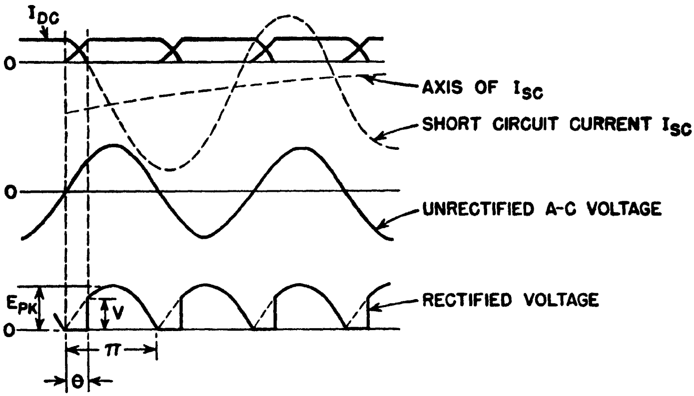


FIG. 68. Commutation current effect on rectifier voltage.

phases in Fig. 64. When high winding resistance limits short-circuit current, commutation reactance has less effect than equation 35 indicates.

Example. The leakage inductance of the plate transformer in Fig. 45 (see p. 61) is

$$\frac{10.6 \times (4200)^2 \times 10.2(4 \times 0.020 + 1.0)}{4 \times 2.375 \times 10^9} = 0.218 \text{ henry}$$

The commutation reactance drop is, from equation 34,

$$\frac{0.115 \times 0.218 \times 2\pi \times 60}{2\pi} = 1.5 \text{ volts}$$

or 0.125 per cent. This added to the 3.7 per cent calculated in Fig. 45 is less than the 4 per cent regulation allotted to the transformer. In this case the short-circuit current would be limited by winding resistance rather than by leakage inductance.

If the rectifier had no filter capacitor, the rectifier would deliver the average value of the rectified voltage wave, less regulation components 1 and 2. But with a filter capacitor, there is a tendency at light loads for the capacitor to charge up to the peak value of the rectified wave. At zero load, this amounts to 1.57 times the average value, or a possible regulation of 57 per cent in addition to the IR and IX components, for single-phase full-wave rectifiers. This effect is smaller in magnitude for polyphase rectifiers, although it is present in all rectifiers to some extent.

45. Capacitor Effect.[1] Suppose that the rectifier circuit shown in Fig. 61 (*a*) delivers single-phase full-wave rectifier output as shown in Fig. 61 (*b*) to an inductor input filter and thence to a variable load. In such a circuit, the filter inductor keeps the capacitor from charging to a value greater than the average E_{dc} of the rectified voltage wave at heavy loads. At light loads the d-c output voltage rises above the average of the rectified wave, as shown by the typical regulation curve of Fig. 69.

Starting at zero load, the d-c output voltage E_0 is 1.57 times the average of the rectified wave. As the load increases, the output voltage falls rapidly to E_1 as the current I_1 is reached. For any load greater than I_1, the regulation is composed only of the two components IR and IX. It is good practice to use a bleeder load I_1 so that the rectifier operates between I_1 and I_2.

Filter elements X_L and X_C determine the load I_1 below which voltage rises rapidly. The filter, if it is effective, attenuates the a-c ripple voltage so that across the load there exists a d-c voltage with a small ripple voltage superposed. A choke-input filter attenuates the harmonic voltages much more than the fundamental and, since the harmonics are smaller to begin with, the main function of the filter is to take out the fundamental ripple voltage. This has a peak value, according to Fig. 64, of 66.7 per cent of the average rectified d-c voltage for a single-phase full-wave rectifier. Since this ripple is purely a-c it encounters a-c impedances in its circuit. If we designate the choke impedance as X_L, and the condenser impedance as X_C, both at the fundamental ripple frequency, the impedance to the fundamental

[1] This section is based on the author's "Solving a Rectifier Problem," *Electronics* April 1938, p. 39.

component is $X_L - X_C$, the load resistance being negligibly high compared to X_C in an effective filter. The d-c voltage, on the other hand, produces a current limited mainly by the load resistance, provided the choke IR drop is small.

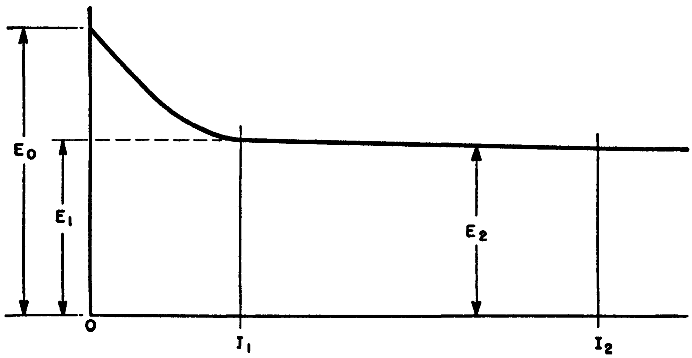


FIG. 69. Rectifier regulation curve. (Courtesy of *Electronics*, April 1938.)

A-c and d-c components are shown in Fig. 70, with the ripple current I_{ac} superposed on the load direct current I_{dc}. If the direct current is made smaller by increased load resistance, the a-c component is not affected because load resistance has practically no influence in determining its value. Hence a point will be reached, as the d-c load current is diminished, where the peak value of ripple current just equals the load direct current. Such a condition is given by d-c load I_1 which is equal to I_{ac}. If the d-c load is reduced further, say to the value I_x, no current flows from the rectifier in the interval A-B of each

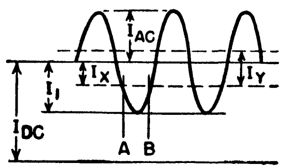


FIG. 70. A-c and d-c components of filter current.

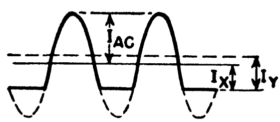


FIG. 71. Capacitor effect at light load.

(Courtesy of *Electronics*, April 1938.)

ripple cycle. The ripple current is not a sine wave, but is cut off on the lower halves, as in the heavy line of Fig. 71. Now the average value of this current is not I_x but a somewhat higher current I_y. That is, the load direct current is higher than the average value of the rectified sine wave voltage divided by the load resistance. This increased

current is caused by the tendency of the capacitor to charge up to the peak of the voltage wave between such intervals as A-B; hence the term capacitor effect which is applied to the voltage increase. The limiting value of voltage is the peak value of the rectified voltage, which is 1.57 times the sine-wave average, at zero load current.

To prevent capacitor effect the choke must be large enough so that I_{ac} is equal to or less than the bleeder current I_1. This consideration leads directly to the value of choke inductance. The bleeder current I_1 is E_1/R_1, where R_1 is the value of bleeder resistance. The ripple current is the fundamental ripple voltage divided by the ripple circuit impedance, or

$$I_{ac} = \frac{0.667E_1}{X_L - X_C}$$

Equating I_1 and I_{ac} we have, for a single-phase full-wave rectifier,

$$R_1 = \frac{X_L - X_C}{0.667} \tag{36}$$

Here we see that the value of capacitance also has an effect, but it is minor relative to that of the choke. In a well-designed filter, the choke reactance X_L is high compared to X_C. Therefore, the predominant element in fixing the value of R_1 (and of I_1) is the filter reactor.

Polyphase rectifiers have similar effects, but the rise in voltage is not so great because of the smaller difference between peak and average d-c output. The bleeder resistor for eliminating capacitor effect can be found in general from

$$R_1 = \frac{X_L - X_C}{A_1} \tag{37}$$

where A_1 is the fundamental ripple peak amplitude from Fig. 64, and X_L and X_C are the filter reactances at fundamental ripple frequency.

Between load I_1 and zero load, the rate of voltage rise depends upon the filter. Figure 72 shows the voltage rise as a function of the ratio $(X_L - X_C)/R_L$ for a single-phase full-wave rectifier. A curve of ripple in terms of ripple at full load is given. Figure 72 is a plot of experimental data taken on a rectifier with $IR + IX$ regulation of 5 per cent. Reactances X_L and X_C are computed for the fundamental ripple frequency.

Capacitor input filters have the voltage regulation curves shown in Figs. 37, 38, and 40 (pp. 48, 49, 51) for their respective circuits. At light loads these filters may give reasonably good regulation, but it is

possible to get very poor regulation at heavier loads, as can be seen from the curves. Rectifier series resistance plays an important part in the voltage regulation of this type of filter. The effect of anode transformer leakage inductance can be found from Fig. 72.

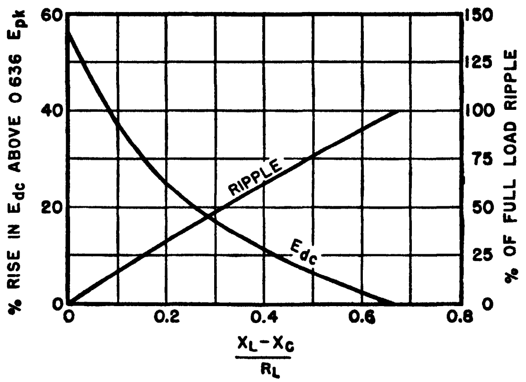


FIG. 72. Voltage rise in single-phase full-wave rectifier at light loads.

46. Tuned Rectifier Filters. Sometimes an inductor-input filter is tuned as in Fig. 73. The addition of capacitor C_1 increases the effective reactance of the inductor to the fundamental ripple frequency. Both regulation and ripple of this type of filter are improved. The filter is not tuned for the ripple harmonics, so the use of high Q filter inductors is unnecessary. An increase in effectiveness of the filter inductor of about three to one can be realized in a single-phase full-wave rectifier circuit. Tuned filters are less effective with three-phase rectifiers because slight phase unbalance introduces low frequency ripple which the filter does not attenuate.

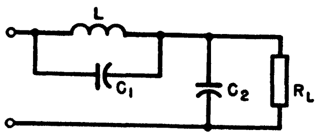


FIG. 73. Shunt-tuned filter.

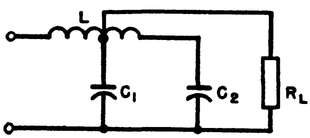


FIG. 74. Series-tuned filter.

Filters may be tuned as in Fig. 74 where the filter capacitor C_1 is connected to a tap near the right end of inductor L, and the other filter capacitor C_2 is chosen to give series resonance and hence zero reactance across the load at the fundamental ripple frequency. Because of choke losses, the impedance across R_L is not zero, but the resulting ripple across load resistor R_L can be made lower than without

the use of capacitor C_1. Ripple is attenuated more than in the usual inductor-input filter, but regulation is not substantially different.

47. Rectifier Currents. If the inductor in an inductor-input filter were infinitely large, the current through it would remain constant. If the commutation reactance effect is not considered, the current through each tube of a single-phase rectifier would be a square wave, as shown by I_1 and I_2 of Fig. 36 (*a*) (p. 47). The peak value of this current wave is the same as the d-c output of the rectifier, and the rms value is $0.707I_{dc}$. With finite values of inductance, an appreciable amount of ripple current flows through the inductor and effectively modulates I_1 and I_2, thus producing a larger rms inductor current like the first wave of Table I (p. 10).

Capacitor-input filters draw current from the rectifier only during certain portions of the cycle, as shown in Fig. 36 (*b*). For a given average direct current, the peak and rms values of these current waves are much higher than for inductor-input filters. Values for the single-phase rectifiers are given in Fig. 39 (p. 50). If an *L-C* filter stage follows the input capacitor, the inductor rms current is the output direct current plus the ripple current in quadrature.

Polyphase rectifiers are ordinarily of the choke-input type, because they are used mostly for larger power, and therefore any appreciable amount of series resistance cannot be tolerated. For this reason, the low *IR* drop tubes, such as mercury vapor rectifiers, are commonly used. Such tubes do not possess sufficient internal drop to restrict the peak currents drawn by capacitor-input filters to the proper values.

In a shunt-tuned power supply filter such as shown in Fig. 73, the current drawn from the rectifier is likely to be peaked because two capacitors C_1 and C_2 are in series, without intervening resistance or inductance. This peak quickly subsides because of the influence of inductor L, but an oscillation may take place on top of the tube current wave as shown in Fig. 75. The rectifier tube must be rated to

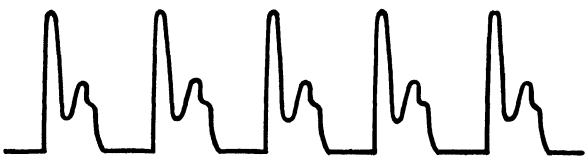


FIG. 75. Anode current with shunt-tuned filter.

withstand this peak current. At the end of commutation the voltage jumps suddenly from zero to V (Fig. 68). Peak rectifier current may be as much as

$$I_{pk} = \frac{V}{\omega L_s} \qquad (38)$$

L_s is half the transformer leakage inductance, and $\omega = 2\pi \times$ frequency of oscillation determined by L_s in series with C_1 and C_2. This peak current is superposed on I_{dc}. It flows through the anode transformer and tube, but the current in choke L (Fig. 73), is determined by ripple voltage amplitude and choke reactance. Series resistance R_s reduces this peak current to the value

$$I_{pk} = \frac{V}{\omega L_s} \epsilon^{-\frac{\pi R_s}{4\omega L_s}} \tag{39}$$

It is obtained by applying a step function voltage to the series R_sL_sC circuit. The criterion for oscillations is

$$R_s < 2\sqrt{\frac{L_s}{C}} \tag{40}$$

where C is the capacitance of C_1 and C_2 in series. Many rectifier tubes have peak current ratings which must not be exceeded by such currents.

Currents shown in Table VII (p. 46) and Figs. 36 and 75 are reflected back into the a-c power supply line, except that alternate

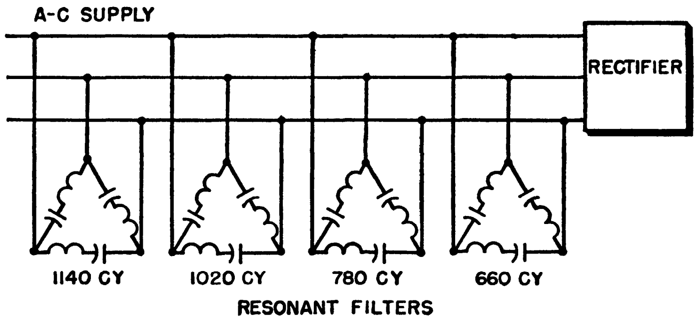


FIG. 76. A-c line filter for large power rectifier.

current waves are of reverse polarity. Small rectifiers have little effect on the power system, but large rectifiers may produce excessive interference in nearby telephone lines because of the large harmonic currents inherent in rectifier loads. High values of commutation reactance reduce these line current harmonics, but since good regulation requires low commutation reactance, there is a limit to the control possible by this means. A-c line filters are used to attenuate the line current harmonics. A large rectifier, with three-phase series resonant circuits designed to eliminate the eleventh, thirteenth, seventeenth, and nineteenth harmonics of a 60-cycle system, is shown in Fig. 76. Smaller rectifiers sometimes have filter sections such as

those in Fig. 77; these are rarely used in large installations because of the excessive voltage regulation introduced by the line inductors.

Filters designed to keep r-f currents out of the a-c lines are often used with high voltage rectifiers. Even if the anode transformer has low radio influence, commutation may cause r-f currents to flow in the supply lines unless there is a filter.

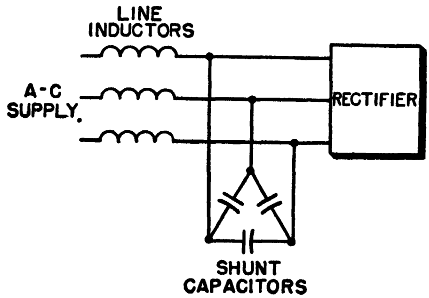


FIG. 77. A-c line filter for medium-sized power rectifier.

48. Rectifier Transients. The shunt-tuned filter currents mentioned in the preceding section are transient. Since the tube current is cut off during each cycle, a transient current may occur in each cycle. When power is first applied to the rectifier, another transient occurs, which may be smaller or larger than the cyclic transient, depending on the filter elements. In reactor-input filters the transient current can be approximated by the formula given in Section 47 for a step function applied to the series circuit comprising filter L and C plus R_s. This circuit is valid because the shunting effect of the load is slight in a well-proportioned filter. In capacitor-input filters, the same method can be used, but here the inductance is the leakage inductance of the anode transformer. Therefore, equation 39 applies, except that the maximum step function voltage is E_{pk}.

Transients which occur when power is first applied differ from cyclic transients in that they are spasmodic. Power may be applied at any instant of the alternating voltage cycle, and the suddenly impressed rectifier voltage ranges from zero to E_{pk}. Starting transients are difficult to observe on an oscilloscope because of their random character. It is necessary to start the rectifier several times for one observation of maximum amplitude, and the trace is faint because it appears for a very brief time.

Excessive current inrush, which occurs when a power transformer is connected to a supply line, plagues rectifier design. The phenomenon is associated with core saturation. For example, suppose that

the core induction is at the top of the hysteresis loop in Fig. 49 (p. 67) at the instant when power is removed from the rectifier, and that it decreases to remanent value B_r for $H = 0$. Suppose that the next application of power is at such a point in the voltage cycle that the

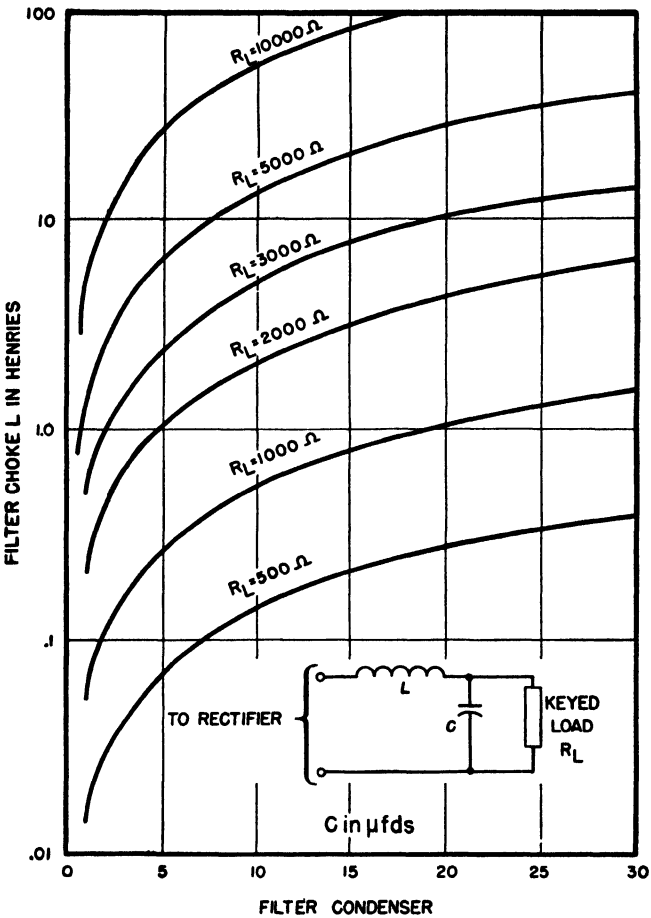


FIG. 78. Capacitor required for 20 per cent transient dip in filter output voltage.

normal induction would be B_2. This added to B_r requires a total induction far above saturation value; therefore heavy initial magnetizing current is drawn from the line, limited only by primary winding resistance and leakage inductance. This heavy current has a peaked wave form which may induce momentary high voltages by internal resonance in the secondary coils and damage the rectifier tubes. Or it

may trip a-c overload relays. The problem is especially acute in large transformers with low regulation. A common remedy is to start the rectifier with external resistors in the primary circuit and short-circuit them a few cycles later. Some rectifiers are equipped with voltage regulators which reduce the primary voltage to a low value before restarting.

In some applications the load is varied or removed periodically. Examples of this are keyed or modulated amplifiers. Transients occur when the load is applied (key down) or removed (key up), causing respectively a momentary drop or rise in plate voltage. If the load is a device which transmits intelligence, the variation in filter output voltage produced by these transients results in the following undesirable effects:

1. Modulation of the transmitted signal.
2. Frequency variation in oscillators, if they are connected to the same plate supply.
3. Greater tendency for key clicks, especially if the transient initial dip is sharp.
4. Loss of signal power.

A filter which attenuates ripple effectively is normally oscillatory; hence damping out the oscillations is not practicable. Nor would it remedy the transient dip in voltage, which may increase with non-oscillatory circuits. The filter capacitor next to the load should be large enough to keep the voltage dip reasonably small. Relations between L, C, and R_L in a single-stage filter are given in Fig. 78 for 20 per cent transient dip in load voltage.[2] An approximation which neglects the damping effect of load and series resistance is

$$\Delta e_B = \frac{1}{R_L}\sqrt{\frac{L}{C}} \tag{41}$$

where Δe_B is the transient dip expressed as a fraction of the steady-state voltage across R_L, and L, C, and R_L are as shown in Fig. 78.

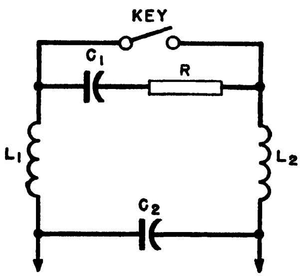


FIG. 79. Key click filter.

Although the tendency for key clicks in the signal may be reduced by attention to the d-c supply filter elements, the clicks may not be entirely eliminated. Where key click elimination is necessary, some

[2] Equations for these curves are derived in the author's "Radiotelegraph Keying Transients," *Proc. I.R.E.*, Vol. 22, Feb. 1934, p. 213.

sort of key click filter is used, of which Fig. 79 is an example. This filter has inductance and capacitance enough to round off the top and back of a wave and eliminate sharp, click-producing corners. Figure 80 is an oscillogram showing a keyed wave shape with and without such a filter.

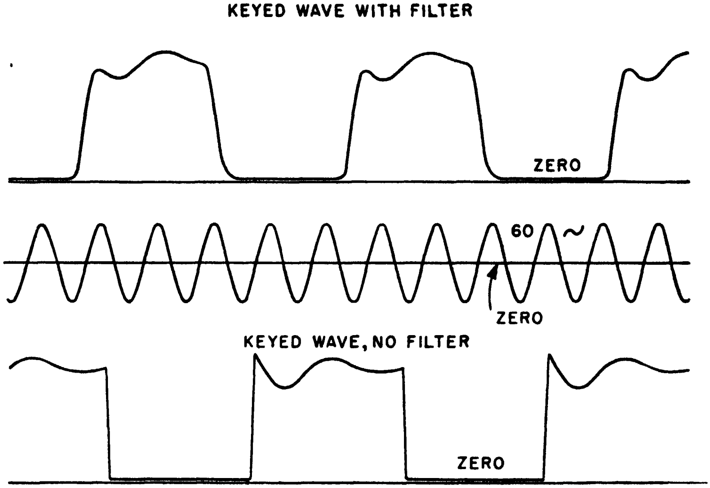


FIG. 80. Transmitter wave shape with and without key click filter.

49. Rectifier Efficiency. Losses in a rectifier consist of transformer, tube, and filter losses. Filament power should be counted as loss, especially when a tube rectifier is compared with a rotating machine or metal disk rectifier. Even in spite of this loss, a high voltage polyphase rectifier of the mercury vapor or pool type may have 95 per cent efficiency at full load. In contrast, the rectifier for a radio receiver rarely has more than 60 per cent efficiency. Reasons for this low figure are the high tube and reactor *IR* drops and low transformer efficiency. The filament power, too, is a greater portion of the total.

50. Rectifier Tests. Even though the transformers, chokes, tubes, and capacitors have been tested before assembly of the rectifier, performance tests of the rectifier are desirable. These generally include tests of output, regulation, efficiency, ripple, and input kilovolt-amperes or power factor. Accurate meters should be used, and polyphase rectifiers should have balanced supply voltages. Wiring is tested at some voltage higher than normal, preferably with transformers, tubes, and capacitors disconnected to avoid damage during the test. Ordinary care in testing is sufficient except for regulation tests. If the regulation is low, the difference in meter readings at no

load and full load may be inaccurate. Differential measurements are sometimes used, such as a voltmeter connected between the rectifier and a fixed source of the same polarity and voltage. Artificial loading of a high voltage rectifier is often a problem. Water rheostats have been used for this purpose.

PROBLEMS

1. A three-phase full-wave rectifier delivers 1 amp at 3000 volts. Rms output ripple must not exceed ½ per cent. What ratio of choke to capacitor reactance in a single-stage filter is necessary? With a 2-μf capacitor, how many henrys should the choke have for a 60-cycle supply? *Ans.* 9; 0.89 henry.

2. A 60-cycle, single-phase, full-wave rectifier delivers 500 ma at 500 volts. A two-stage filter with 3 henrys and 1 μf per stage is used. Is this a good filter? What better disposition could be made of the same filter parts?

Ans. No; single-stage.

3. Check suitability of two 1616 rectifier tubes (peak inverse voltage 5500 max, plate current 0.13 amp average, 0.8 amp peak) in a voltage doubler circuit for 100 ma at 4 kv d-c output, assuming 400 ohms plate resistance. What plate transformer minimum secondary voltage is required? *Ans.* Not suitable; 4800 volts rms.

4. What bleeder current is necessary in the filter of Prob. 1 to prevent capacitor effect? *Ans.* 96 ma.

5. If the load in Prob. 1 is a telegraph transmitter, approximately what transient dip in voltage occurs when the key is pressed? *Ans.* 22 per cent.

6. In a single-phase, full-wave 60-cycle rectifier delivering 1.5 amp at 1200 volts, the filter is a 1.75-henry reactor shunt-tuned with 1 μf capacitor, and a 15-μf capacitor across the d-c output. The anode transformer has 0.1-henry secondary leakage inductance. Neglecting *IR* drops, determine the peak anode current at the beginning of each cycle. What is it with 30 ohms total series resistance?

Ans. 2.9 amp; 2.76 amp.

7. Assume that 10 ohms of the series resistance in Prob. 6 is in the mercury vapor tubes, type 872A, two of which have 75 watts filament power. Neglect transformer core loss. What are the ripple, efficiency, regulation, and input volt-amperes of this rectifier at full load?

Ans. 19 volts rms; 92.7 per cent; 3.25 per cent; 2150 v-a.

V. AMPLIFIER TRANSFORMERS

An amplifier is a device for increasing voltage, current, or power in a circuit. The original wave form may or may not be maintained; the frequency usually is. An amplifier may be mechanical, electromechanical, electromagnetic, or electronic in form, or it may be a combination of these. In this chapter the electronic amplifier is considered. The circuit usually consists of a vacuum tube in conjunction with capacitors, transformers, or resistors. Input voltage or current is impressed on some element of the tube; this causes higher voltage or current to appear in the output circuit.

51. Amplifier Potentials. Amplifiers are characterized by the use of tubes having three or more elements. In triodes or three-element tubes, the addition of the third element, the grid, alters the voltage gradient between cathode and anode as shown in Fig. 81. The grid

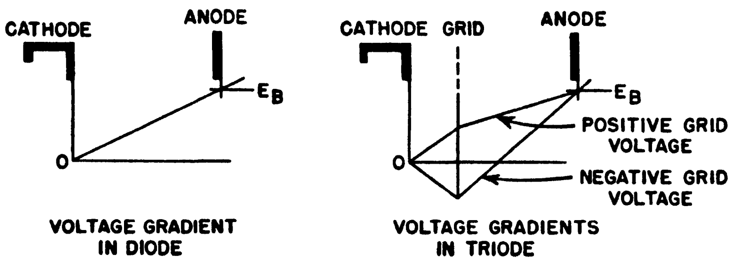


FIG. 81. Diode and triode voltage gradient.

either aids or opposes the flow of electrons from cathode to anode, depending on whether the grid voltage is positive or negative respectively, compared to the cathode, which is shown at zero voltage in Fig. 81.

As the grid voltage is made more and more negative, electron flow is diminished and finally stops. At this point the anode current is zero; the condition is called *anode current cut-off*.

If the grid voltage is made more and more positive, eventually further increase in grid voltage causes no additional anode current increase. This condition is called *grid saturation*.

Tetrodes and pentodes have respectively two and three grids. The voltage gradient between cathode and anode is more complex than that indicated in Fig. 81. The advantages to be gained from the additional grids are mentioned below.

52. Transformer-Coupled Amplifiers. Amplifier circuits in which transformers are used can be represented by a circuit similar to that of Fig. 82 (*a*). Here a triode is shown with a voltage e_c impressed upon the grid, which comprises the grid bias (a constant negative direct voltage) and a superimposed alternating voltage e_g. Anode

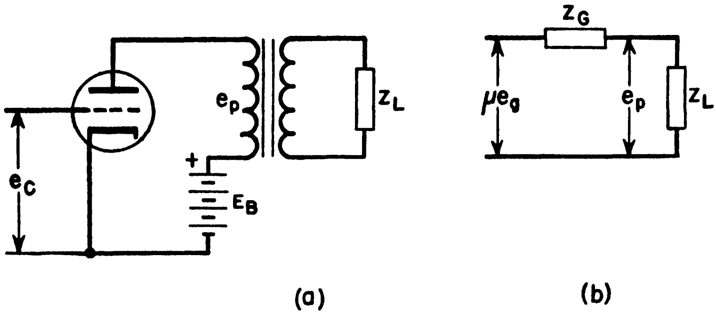


FIG. 82. (*a*) Transformer-coupled amplifier; (*b*) equivalent circuit.

voltage E_B is supplied from some source through the primary of the transformer, across which appears an alternating voltage e_p. The secondary of the transformer is connected to a load Z_L. Under certain conditions, which will be defined below, this circuit may be simplified to that of Fig. 82 (*b*). A fictitious alternating voltage μe_g is impressed on the circuit, where μ is the tube amplification factor. Internal tube resistance Z_G is in series with the load Z_L, which is reflected by the transformer to the proper value in the primary circuit for tube operation. That is, Z_L in Fig. 82 (*b*) is equal to that in Fig. 82 (*a*) only if the transformer has a 1:1 ratio. For any turns ratio, the quotient of two Z's is equal to the (turns ratio)^2 as in equation 7 (p. 8). Note that the winding resistances are regarded as zero, so that, in the absence of a grid signal, full voltage E_B appears on the plate of the vacuum tube.

Alternating voltage μe_g causes voltage e_p to appear across the load Z_L. The voltage e_p is not μ times e_g but is related by the following equation:

$$e_p = \mu e_g \frac{Z_L}{Z_G + Z_L} \tag{42}$$

Although transformer-coupled amplifiers are used sometimes for volt-

age amplification, they are used mostly where power output is required of the amplifier, and where a good reproduction of the grid voltage is required in the plate circuit.

53. Tuned Amplifiers. Figure 83 shows the circuit for an amplifier in which the output voltage appears across a parallel-tuned circuit. This circuit is shown coupled to a load Z_L. This type of amplifier may be used where large outputs are required, but the voltage e_p is not necessarily a reproduction of e_g, and they are not related as in equation 42.

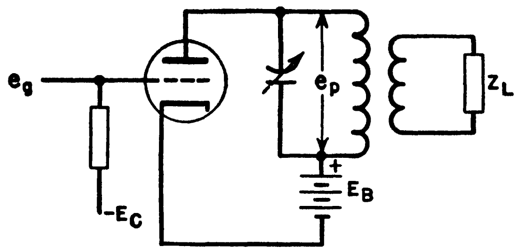


FIG. 83. Tuned amplifier.

54. Amplifier Classification. Amplifiers can be divided into classes, depending upon the mode of operation. A class A amplifier is one in which the grid bias and alternating grid voltage are such that anode current flows continuously. In a class B amplifier the grid bias is almost equal to the cut-off value, so that plate current is nearly zero when no exciting grid voltage is applied. When full alternating grid voltage is applied, plate current flows for approximately one half of each cycle. A class C amplifier has a grid bias greater than the cut-off value, so that the plate current is zero when no alternating grid voltage is applied and it flows for appreciably less than one half of each cycle when an alternating grid voltage is applied. These classes are illustrated in Fig. 84, in which the alternating plate current, plate voltage, grid voltage, and grid current are shown with the steady or average values which are respectively I_B, E_B, E_C, and I_G. Relative plate and grid voltage amplitudes for these three types of amplifiers are shown in Fig. 84, and other properties are summarized in Table X.

Class A amplifiers are characterized by comparatively high no-signal anode current. Usually the grid never swings positive. Anode current remains comparatively constant, when averaged over a whole a-c cycle. In class B amplifiers, the grid is biased at a greater negative potential so that current is nearly cut off in the absence of a signal. Positive swings of grid voltage result in anode current being drawn; this causes a dip in the residual voltage on the plate of the amplifier.

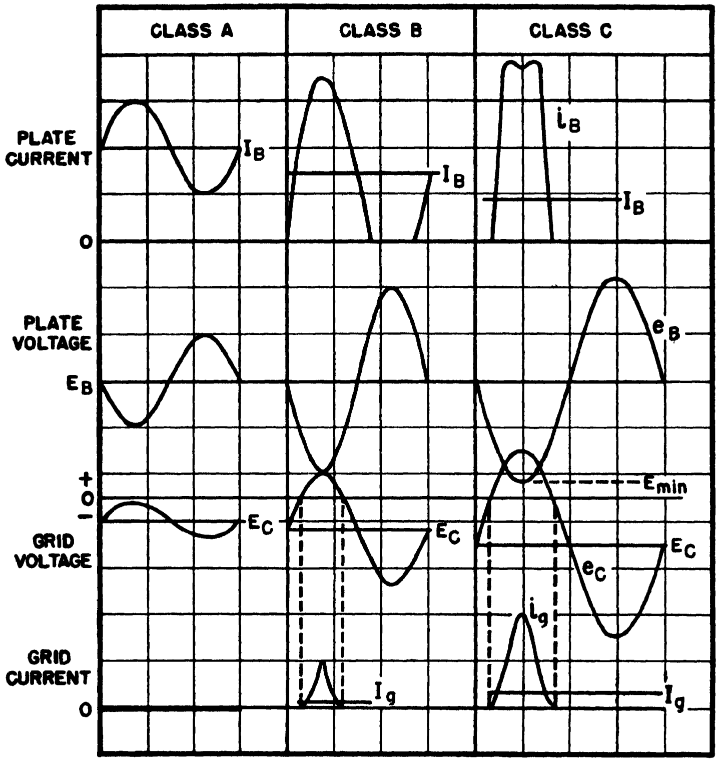


FIG. 84. Amplifier voltages and currents.

TABLE X. AMPLIFIER CLASSES

Amplifier Class	A	B	C
Anode efficiency			
a. Theoretical maximum	50%	78.5% *	100%
b. Practical value for low distortion	Up to 30%	40–67% *	70–85% †
Output proportional to	e_g^2	e_g^2	E_B^2 (grid saturated)
Grid current I_g	None	Small	Large (may $\approx I_B$)
Anode current I_B	Fairly constant	$e_g = 0$, I_B low $e_g =$ max, I_B high	$e_g = 0$, $I_B = 0$ $e_g =$ max, I_B high

* These values are for push-pull amplifiers.

† With a high-Q tank circuit, the efficiency depends on excitation power.

Negative grid swings cause no plate current to flow, but do cause a positive plate voltage swing. In class C amplifiers, the grid is biased more negatively still, with the result that plate current flows for less than half a cycle, and mostly when the plate voltage on the tube is at a relatively low value. Grid current in this class of amplifier reaches values comparable to the plate current. Output voltage wave form is maintained by a tuned plate circuit.

Operation may sometimes be improved by the use of two tubes connected push-pull, as shown in Fig. 85. This is the most common connection for class B amplifiers; also, it is used in class A amplifiers in many cases. Intermediate between class A and class B amplifiers are those known as class AB with grid bias and efficiency intermediate between class A and class B amplifiers. Such amplifiers are further subdivided into class AB_1 and class AB_2. Class AB_1 amplifiers draw no grid current, but the bias voltage is somewhat higher than the class A value and the plate current may be discontinuous during the cycle when grid signal is applied. Class AB_2 amplifiers draw grid current, but are not biased as close to cut-off as class B amplifiers. Both class AB_1 and AB_2 amplifiers are commonly used with the push-pull connection.

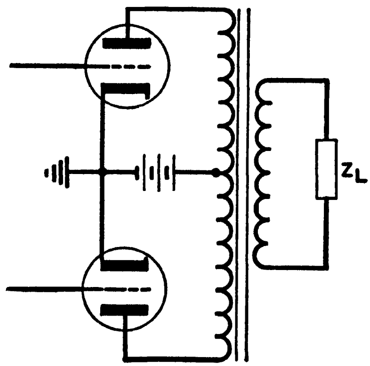


FIG. 85. Push-pull amplifier.

55. Anode Characteristics. Anode characteristics of vacuum tubes are commonly given in the form shown in Fig. 86, which applies specifically to the 851 tube, a large air-cooled triode. From such characteristics three common properties of a tube can be found: the plate resistance r_p, amplification factor μ, and mutual conductance g_m. Reference to Fig. 82 (*b*) shows that the plate resistance is the impedance Z_G which represents the impedance of the source. It may be found from the curves by measuring the change in plate voltage for a given change in plate current, with the grid voltage maintained at a constant value. The amplification factor is the change in plate voltage which occurs for a given change in grid voltage, with plate current maintained constant. Mutual conductance is the change of plate current for a given change in grid voltage, plate voltage being maintained constant.

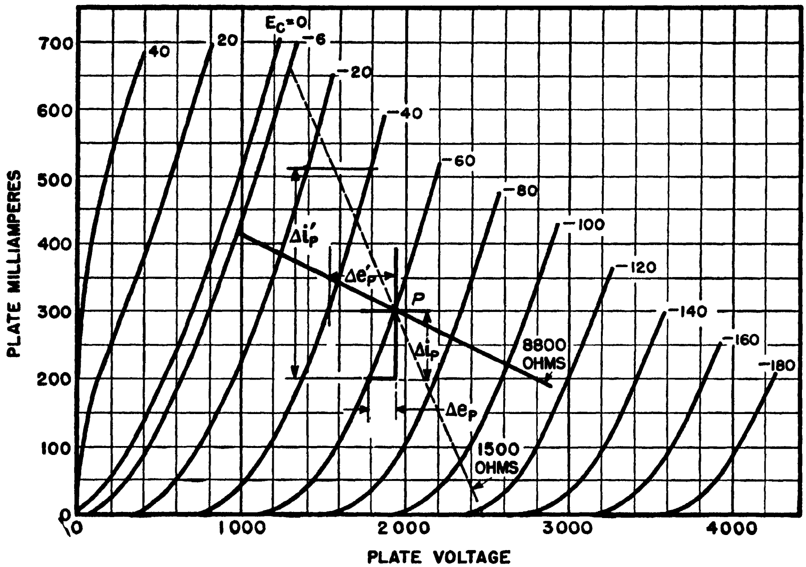


FIG. 86. Anode characteristics of 851 tube.

All three properties are measured under class A conditions at small a-c grid voltage amplitudes. In Fig. 86 the three properties can be found as follows:

$$r_p = \frac{\Delta e_p}{\Delta i_p} = \frac{1}{i - e \text{ slope}} = \frac{1930 - 1780}{0.300 - 0.200} = 1500 \text{ ohms}$$

$$\mu = \frac{\Delta e'_p}{\Delta e_g} = \frac{1930 - 1530}{20} = 20$$

$$g_m = \frac{\Delta i'_p}{\Delta e_g} = \frac{0.510 - 0.200}{20} = .0155 \text{ mho}$$

$$= 15{,}500 \text{ micromhos}$$

Consider the operation of this tube in a class A amplifier. The static, or no-signal, condition consists of $E_B = 1920$ volts, $E_C = -60$ volts, and $I_B = 300$ ma. These values establish the operating point P in Fig. 86. A light load of 8800 ohms will be used for the first example. A maximum grid swing of 54 volts will be assumed. That is, the grid swings from -6 to -114 volts on alternate half-cycles. The 8800-ohm load line can be constructed in Fig. 86 as follows: 100 ma plate current swing times 8800 ohms gives 880 volts plate voltage

swing. Adding 100 ma to 300 gives 400 plate ma. Subtracting 880 volts from 1920 gives 1040 instantaneous plate voltage corresponding to the 100-ma increase in plate current. This gives one point on the load line. Now a straight line is drawn between this point and the static point P. The ends of the load line are determined by the peak grid swings of -6 and -114. If a sinusoidal grid voltage is assumed, the plate current swing is sinusoidal also, and so is the plate voltage swing. That is, all points lie on the straight 8800-ohm load line. This may be confirmed by Table XI.

TABLE XI. 851 TRIODE OPERATION WITH 8800-OHM LOAD

$E_B = 1920$ volts
Peak grid voltage swing $= 54$ volts $= e_g$
$E_C = -60$ volts
$I_B = 0.300$ amp

θ (deg)	$\sin\theta$	$e_g \sin\theta$	e_c	e_B	i_B
0	0	0	−60	1920	0.300
30	0.5	27	−33	1440	0.355
60	0.866	47	−13	1090	0.395
90	1.0	54	−6	960	0.410
120	0.866	47	−13	1090	0.395
150	0.5	27	−33	1440	0.355
180	0	0	−60	1920	0.300
210	−0.5	−27	−87	2400	0.245
240	−0.866	−47	−107	2750	0.205
270	−1.0	−54	−114	2880	0.190
300	−0.866	−47	−107	2750	0.205
330	−0.5	−27	−87	2400	0.245
360	0	0	−60	1920	0.300

Equal grid voltage swings give equal plate current and plate voltage swings on this load line; therefore, the tube is acting as a linear impedance. For such a load line, the circuit of Fig. 82 (*b*) correctly represents the amplifier performance. In spite of the large voltage swing, the power output is relatively low because of the small plate current swing. The peak a-c output is 0.11 times 960 or 105 watts. Multiplying each of these peak values by 0.707 to obtain rms values gives 52.5 watts average output from the tube. This is far below its capabilities. The power input is 300 ma times 1920 volts or 576 watts. The efficiency is therefore only 9.1 per cent.

Next consider the 1500-ohm load line. Use the same peak-to-peak grid voltage; the positive voltage swing is $2400 - 1920 = 480$ volts, Δi_p is 275 ma, and $\Delta e_p \Delta i_p = 132$ watts peak output. Negative $\Delta e_p = 1920 - 1300 = 620$ volts, $\Delta i_p = 330$ ma, and $\Delta e_p \Delta i_p = 205$ watts

peak output. Average output is then $(205 + 132)/4 = 84$ watts, and efficiency = 14.6 per cent. But the large difference in positive and negative plate voltage swings gives rise to harmonic distortion. Lower distortion is obtained at 3000-ohm load or $2r_p$. This is the load impedance generally given for triodes, and is known as the maximum undistorted power output (*UPO*) load.

Suppose that the tube of Fig. 86 is biased at -114 volts, the plate voltage is raised to 2400, and the grid signal voltage is increased to double, that is, to 108 volts. If now we use the 1500-ohm load line, the positive plate current peak is 665 ma and $\Delta e_p = 1070$. This gives a peak power of 710 watts or an average of 355. This is only for one half-cycle, because anode current is cut off in the other half-cycle, but if for the second half-cycle another tube is operating in the push-pull connection, then the average power for the whole cycle would be 355 watts. The average current in one tube would be $665/\pi = 212$ ma and, in the two tubes, 424 ma. This is a power input of 1020 watts; therefore the efficiency is 35 per cent. This efficiency and increased power input are still below the capabilities of the two tubes when used in push-pull class B, but they indicate how the efficiency of a pair of tubes can be increased by departing from the class A conditions.

Class B push-pull amplifiers may be readily visualized if inverted plate characteristics are plotted as in Fig. 87. A wider range of plate current is used. Plate voltage may swing as low as the "diode" line ($E_C = E_B$). The operating points P and P' are on the same vertical line, which represents the same plate voltage. Because the tubes are not quite biased to cut-off, the composite load line has a somewhat greater slope than either tube. While one tube is giving up the load and the other taking it on, there is a short period in each half-cycle similar to the commutation period of a rectifier, during which both tubes are active. The slope of the load line of the first tube decreases, and that of the second tube increases until it assumes the whole load. During this period tubes change from class B to class A operation and vice versa.

Plate current swing is zero to 2.5 amp for each tube, and plate voltage swing 1700 volts. Power output is $2.5 \times (1700/2) = 2100$ watts. Plate power input is approximately $2 \times (2.5/\pi)$ amp or 1.59 amp $\times$ 2000 volts = 3180 watts. Efficiency is $2100/3180 = 66$ per cent.

The output transformer turns ratio is different for class A and B amplifiers. For example, suppose the load Z_L in Fig. 85 is 1500 ohms, the amplifier is class A, and each tube works into 1500 ohms. Both tubes work all the time; hence the plate-to-plate primary load is 3000 ohms and, from equation 7 (p. 8), the turns ratio is $\sqrt{3000/1500}$

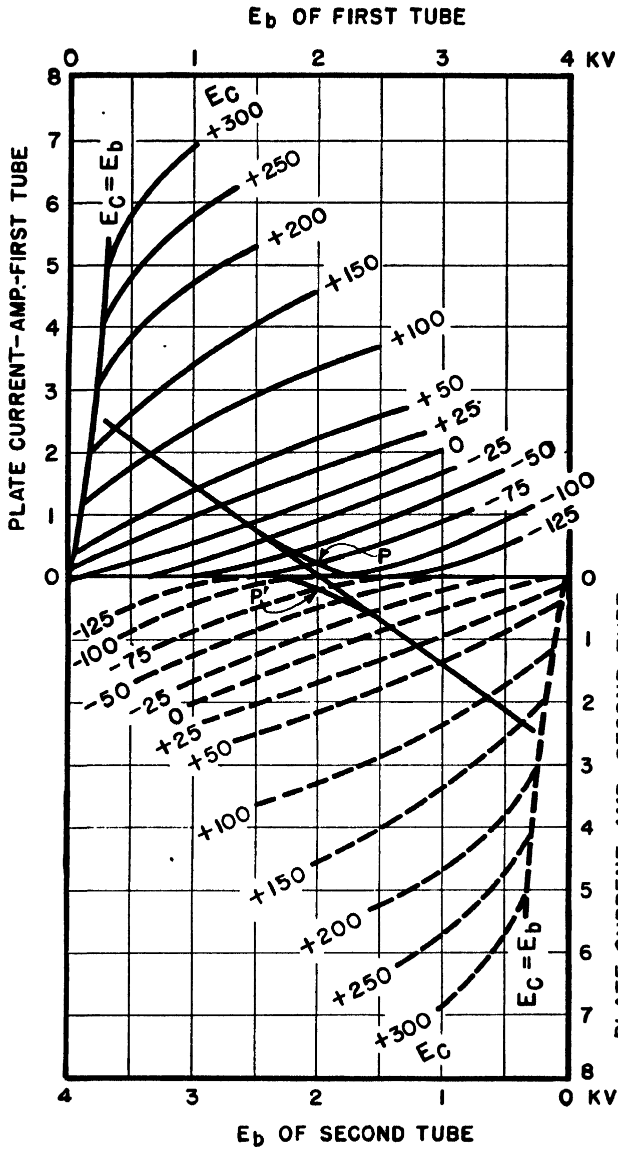


FIG. 87. Class B push-pull operation of 851 tubes.

= 1.41, step-down. In a class B amplifier, only one tube works at a time and, for a tube load of 1500 ohms in this example, the turns ratio of each half-primary to the secondary is 1:1. Thus the turns ratio of

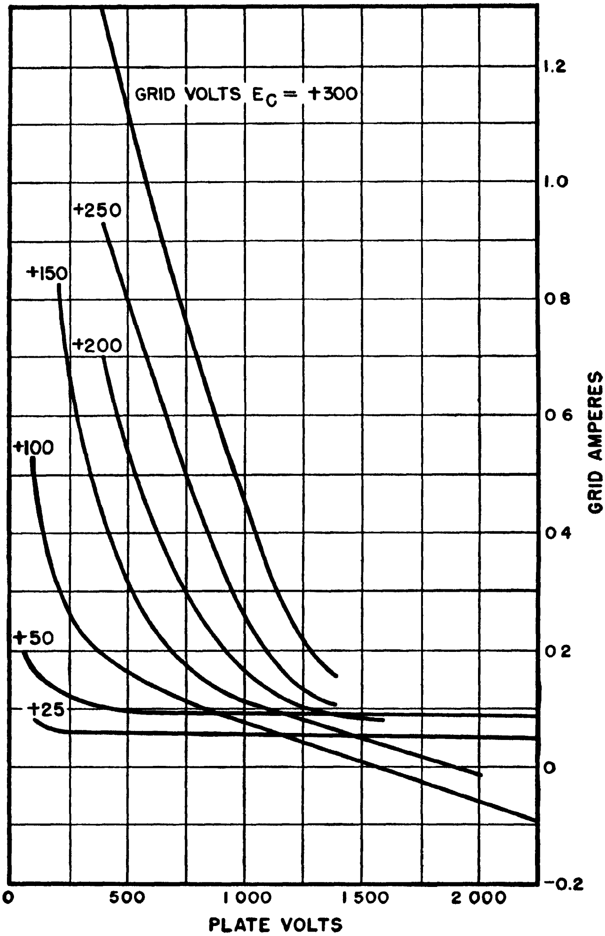


FIG. 88. Grid current curves of 851 tube.

the whole primary to the secondary is 2:1. Sometimes the load impedance of a pair of class B tubes is stated as so many ohms plate-to-plate (6000 ohms in this example). This is misleading because only during the short change-over period do both tubes work simultaneously; while one tube is delivering full power the other is idle, and a better statement of operation is the load impedance per tube.

In a class C amplifier it is not possible to draw load lines like those for class A or class B operation. The load changes from infinity at zero plate current to a low value at E_{min} in Fig. 84. On the assumption that the tank circuit maintains sinusoidal alternating plate voltage, and the alternating component of grid voltage is also sinusoidal, it is possible to find the instantaneous plate currents from the plate current characteristics in a manner similar to that used in the example given above for class A amplifiers. The same procedure can be used for finding grid current and grid power. See Table XII.

TABLE XII. 851 TRIODE CLASS C AMPLIFIER

(Data taken from Fig. 87 and Fig. 88)

E_B = 2000 volts e_g = 425 volts peak E_C = −200 volts

θ (deg)	$\sin\theta$	$e_g \sin\theta$	e_c	e_p	e_B	i_B	$e_p i_B$	i_g
0	0	0	−200	0	2,000	0	0	0
10	0.174	73	−127	− 278	1,722	0	0	0
20	0.342	145	− 55	− 547	1,453	0.10	55	0
30	0.500	213	13	− 800	1,200	0.85	680	0.02
40	0.643	273	73	−1,028	972	1.85	1,900	0.10
50	0.766	326	126	−1,225	775	2.60	3,180	0.14
60	0.866	368	168	−1,385	615	3.20	4,430	0.30
70	0.940	400	200	−1,505	495	3.85	5,790	0.55
80	0.985	419	219	−1,575	425	4.00	6,300	0.76
90	1.0	425	225	−1,600	400	4.00	6,400	0.80
						20.45	28,735	2.67

During the next 90 degrees the same figures could be put down in reverse order, but for negative e_g the output is zero. Since there are 10 ordinates in the table, averages are the sums divided by 20.

$$\frac{20.45}{20} = 1.023 \text{ amp} = I_B$$

$$\text{Power input} = E_B I_B = 2000 \times 1.023 = 2045 \text{ watts}$$

$$\frac{28{,}735}{20} = 1436 \text{ average watts output}$$

$$\text{Plate dissipation} = 2045 - 1436 = 609 \text{ watts}$$

$$\text{Plate efficiency} = \frac{1436}{2045} = 70.4 \text{ per cent}$$

$$\frac{2.67}{20} = 0.134 \text{ amp} = I_g$$

$$\text{Excitation power} = e_g(\text{max})\, I_g = 425 \times 0.134 = 57 \text{ watts}$$

The smaller $E_{\min}$ in Fig. 84, the higher is the plate efficiency, because practically all the plate current is drawn at or near this value of plate voltage. However, the difficulty of making it too low is that the positive grid voltage swing becomes higher than $E_{\min}$, and so the grid draws heavy current which subtracts from the plate current, thus causing the saddle in the top of the plate current wave which is shown in Fig. 84. This not only decreases plate efficiency, but increases the required amount of grid excitation, which in turn requires more power from the driver. Ordinarily, $E_{\min}$ is approximately 10 to 25 per cent of E_B.

In plate-modulated class C amplifiers, sufficient excitation must be applied so that grid saturation still obtains at 100 per cent modulation; otherwise output would not be proportional to plate voltage. In such a case, the modulator would not modulate linearly.

56. Decibels; Impedance Matching. In amplifier work, the ratio of two voltages E_1 and E_2 at the same impedance level is often stated in decibels (db) according to the definition

$$\text{db} = 20 \log_{10} \frac{E_1}{E_2} \tag{43}$$

Amplifier voltage gain, transformer ratio, frequency response, and noise levels all may be expressed in decibels. Volume, voltage, or power in decibels must be compared to a reference level; otherwise the term is meaningless.

Transmission lines at audio and higher frequencies exhibit properties commonly ignored at 60 cycles. Line wave length, characteristic impedance, and attenuation are important at audio frequencies; so is the matter of matching impedance. If a long transmission line has no attenuation, its characteristic impedance is given by

$$Z_0 = \sqrt{\frac{L}{C}} \tag{44}$$

where L and C are the inductance and capacitance per unit length. If such a line terminates in a pure resistance load equal in ohmic value to Z_0, all the power fed into the line appears in the load without attenuation or reflection. This is called matching the impedance of the line. It is very desirable to save audio power and avoid reflections; therefore impedance matching of lines is the usual practice wherever possible. The notion has been extended to include the loading of vacuum tubes, but this is stretching the meaning of the term

matching. A vacuum tube has its optimum load impedance, but the value depends upon the conditions of tube operation, and is not necessarily the same as the tube internal impedance.

57. Amplifier Transformers. The major problem of amplifier transformer design is obtaining proper output when the transformer is operated in conjunction with the apparatus for which it is intended. Several factors external to the transformer affect its performance, namely,

1. Impedance of the source.
2. Linearity of this impedance.
3. Impedance of the load.
4. Frequency.

The simplest method of dealing with amplifier transformers is an adaptation of the so-called equivalent network which has long been used for power transformers. The transformer that connects the source to its load in Fig. 82 (*a*) may be represented more fully by the diagram of Fig. 89 (*a*).

58. Low Frequency Response. At low frequencies, the leakage reactances are negligibly small. Resistance R_P may then be combined with Z_g to form R_1 for a pure resistance source, and R_S with Z_L to form R_2 for a resistance load. At low frequencies both source and load are pure resistance, and the circuit may be simplified to that of Fig. 89 (*b*). Here the a^2 has been dropped; in other words, a transformer with a 1:1 ratio is shown, referred to the primary side. X_N is the primary open-circuit reactance, or $2\pi f$ times the primary open-circuit inductance (*OCL*) as measured at low frequencies.

If shunt resistance R_N is included in load resistance R_2, the circuit becomes like that of Fig. 89 (*c*). Winding resistances are small compared with source and load resistances in well-designed transformers. Likewise, R_N is high compared with load resistance, especially if core material of good quality is used.

Therefore, *to a good approximation*, in Fig. 89 (*c*), R_1 may represent the source impedance and R_2 the load impedance. On a 1:1 turns ratio basis, the voltages E_2 and E_1 are proportional to the impedances across which they appear or

$$\frac{E_2}{E_1} = \frac{\dfrac{jX_N R_2}{jX_N + R_2}}{R_1 + \dfrac{jX_N R_2}{jX_N + R_2}} \tag{45}$$

The scalar value of this ratio is found by taking the square root of the sum of quadrature terms:

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{\left(1 + \frac{R_1}{R_2}\right)^2 + \left(\frac{R_1}{X_N}\right)^2}} \tag{46}$$

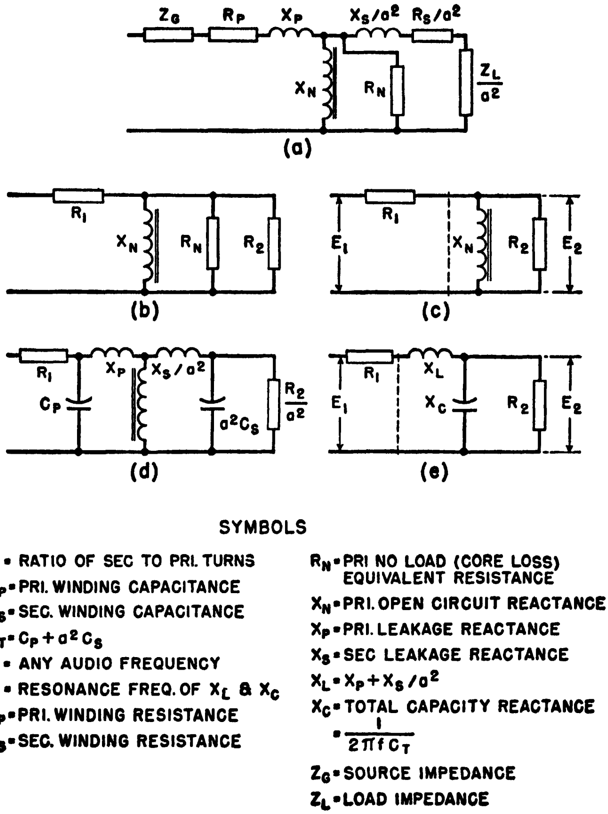


FIG. 89. (*a*) Transformer equivalent circuit; (*b*) low frequency equivalent circuit; (*c*) simplified low frequency circuit; (*d*) high frequency equivalent circuit; (*e*) simplified high frequency circuit. (Courtesy of *Radio Engineering*, June 1937.)

Equation 46 holds for any values of R_1, R_2, and X_N whatsoever, but there are three cases that deserve particular attention: (*a*) $R_2 = R_1$; (*b*) $R_2 = 2R_1$; and (*c*) $R_2 = \infty$. Of these, (*a*) corresponds to the usual line-matching transformer with the source and load impedances equal; (*b*) is often recommended for maximum undistorted output of

triodes; (c) is realized practically when the load is the grid of a class A amplifier. For these cases, equation 46 becomes

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{4 + \left(\frac{R_1}{X_N}\right)^2}} \tag{46a}$$

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{2.25 + \left(\frac{R_1}{X_N}\right)^2}} \tag{46b}$$

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{1 + \left(\frac{R_1}{X_N}\right)^2}} \tag{46c}$$

These three equations are plotted in Fig. 90 to show low frequency response as "db down" from median. The median frequency in an audio transformer is the geometric mean of the audio range; for other transformers it is a frequency at which the ratio X_N/R_1 is very large. At median frequency the circuit is properly represented by Fig. 82 (b). The equivalent voltage ratio E_2/E_1 has maxima of 0.5, 0.667, and 1.0 for cases (a), (b) and (c) respectively at the median frequency, or for $X_N/R_1 = \infty$ in Fig. 90. The higher *OCL*, the nearer the transformer voltage ratio approaches median frequency value. The lower the value of loading resistance R_2, the lower the equivalent voltage ratio is. The factors 0.5, 0.667, and 1.0 multiplied by the turns ratio, *a*, give the actual voltage ratio at median frequency. At lower frequencies, the factors diminish.

The transformer loaded by the lowest resistance has the best low frequency characteristic. A transformer having an open-circuit secondary has twice the voltage ratio and gives the same response at twice the "low end" frequency of a line-matching transformer of the same turns ratio.

Figure 90 is of direct use in determining the proper value of primary *OCL*. Permissible response deviation at the lowest operating frequency fixes X_N/R_1 and therefore X_N. At the corresponding frequency, this represents a certain value of primary *OCL*. As this inductance determines the size and weight of the transformer, the importance of Fig. 90 is evident.

If the primary and equivalent (1:1) secondary winding resistance each are 5 per cent of R_1, the total effect will be a decrease of 10 per

cent in the median frequency voltage ratio, in the case of the line-matching transformer, with corresponding decreases at lower frequencies. On the other hand, the primary resistance of an open-secondary transformer has no effect upon the median frequency voltage ratio, but has some effect at lower frequencies, whereas the secondary resistance has no effect either at median or at lower frequencies. Hence it is important in the open-secondary case, for the sake of low

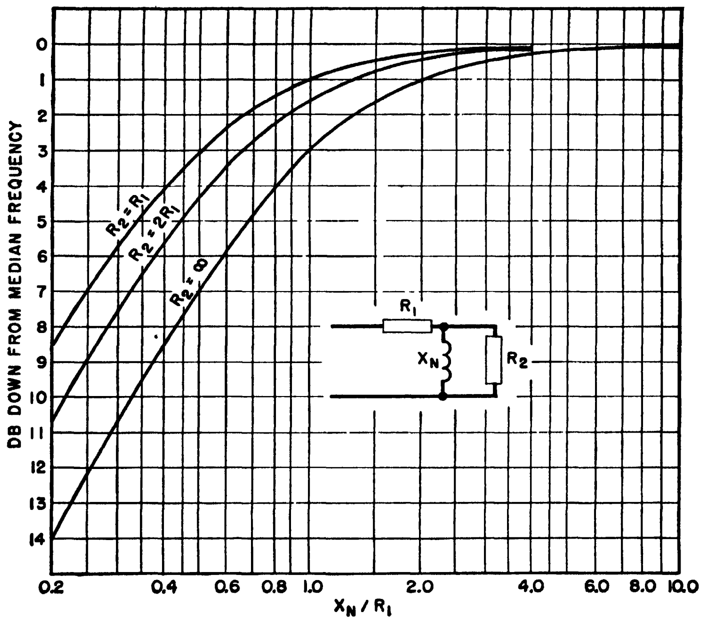


FIG. 90. Transformer characteristics at low frequencies.

frequency response, to keep the primary winding resistance low, but the secondary winding resistance may be any value. The maximum number of secondary turns may be determined by the smallest practicable wire size rather than by winding resistance.

As the frequency increases, the primary inductive reactance X_N also increases until it has almost no effect upon frequency response. This is true for median frequency in Fig. 90. It is also true for higher frequencies; in other words, the *OCL* has an influence only on the low frequency end of the frequency response curve. The ratio of R_2 to R_1 still limits the voltage ratio, however. If the amplifier works at one frequency only, *OCL* is determined by the deficiency in voltage gain that can be tolerated in the amplifier design. This can be found in Fig. 90.

In an amplifier with a band of operating frequencies, e.g., the audio band, a well-designed transformer has uniform voltage ratio for a

frequency range extending from the frequency at which X_N ceases to exert any appreciable influence, upward to a zone designated as the high frequency end of the transformer frequency range.

59. High Frequency Response. The factors which influence the high frequency response of a transformer are leakage inductance, winding capacitance, source impedance, and load impedance. Hence a new equivalent diagram, Fig. 89 (*d*), is necessary for the high frequency end. Winding resistances are omitted or combined as in Fig. 89 (*b*). Winding capacitances are shown across the windings. If primary and secondary leakage inductances and capacitances are combined, X_N is omitted as if it were infinitely large, and a^2 is dropped as before, the circuit becomes that shown in Fig. 89 (*e*). X_L is the leakage reactance of both windings, X_C the capacitive reactance of both windings, and R_2 the load resistance, all referred to the primary side on a 1:1 turns ratio basis.

At any frequency, the equivalent voltage ratio in the circuit of Fig. 89 (*e*) can be found by the ratio of impedances, as for the low frequency response. The scalar value is

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{\left(\frac{R_1}{X_C} + \frac{X_L}{R_2}\right)^2 + \left(\frac{X_L}{X_C} - \frac{R_1}{R_2} - 1\right)^2}} \tag{47}$$

In equation 47 the term X_L/X_C may be written $4\pi^2 f^2 LC = f^2/f_r^2$, where $1/(2\pi\sqrt{LC}) = f_r$, the resonance frequency of the leakage inductance and winding capacitance, considered as lumped and without resistance. Also $X_L/R_2 = X_C f^2/R_2 f_r^2$. Assign to the ratio X_C/R_1 a value B at frequency f_r. Then at any frequency f, $X_C/R_1 = Bf_r/f$. In the three cases considered at the low frequencies,

$R_2 = R_1$,

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{\left(\frac{f}{Bf_r} + \frac{Bf}{f_r}\right)^2 + \left(\frac{f^2}{f_r^2} - 2.0\right)^2}} \tag{47a}$$

$R_2 = 2R_1$,

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{\left(\frac{f}{Bf_r} + \frac{Bf}{2f_r}\right)^2 + \left(\frac{f^2}{f_r^2} - 1.5\right)^2}} \tag{47b}$$

$R_2 = \infty$,

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{\left(\frac{f}{Bf_r}\right)^2 + \left(\frac{f^2}{f_r^2} - 1\right)^2}} \tag{47c}$$

Equations 47*a*, *b*, and *c* are plotted in Figs. 91, 92, and 93. If X_C/R_1 has certain values at frequency f_r, the frequency characteristic is relatively flat up to frequencies approaching f_r. In particular, performance is good at $B = 1.0$ in all three figures.

When leakage inductance and winding capacitance are regarded as "lumped" quantities, current distribution in the windings is assumed to be uniform throughout the range of frequencies considered. As

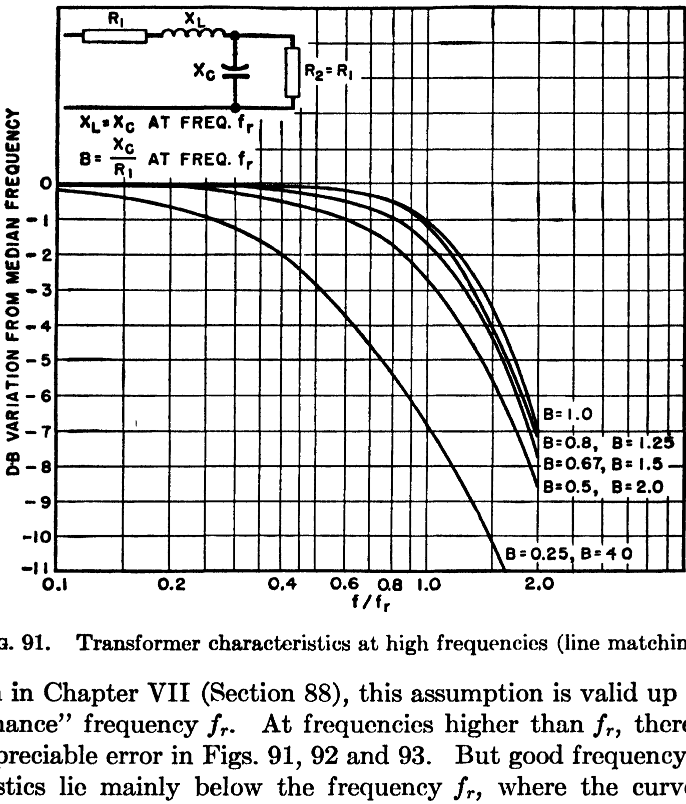


FIG. 91. Transformer characteristics at high frequencies (line matching).

shown in Chapter VII (Section 88), this assumption is valid up to the "resonance" frequency f_r. At frequencies higher than f_r, there may be appreciable error in Figs. 91, 92 and 93. But good frequency characteristics lie mainly below the frequency f_r, where the curves are correct within the assumed limits.

To use these curves in design work, choose the most desirable characteristic curve and, from a knowledge of the source impedance, find the proper value of capacitive reactance X_C at frequency f_r. The value of f_r should be such that the highest frequency to be covered lies on the flat part of the curve. X_C and f_r determine the values of winding capacitance and leakage inductance which must not be exceeded in order to give the required performance.

In Fig. 89 (*e*) the capacitance is shown across the load. This is correct if the main body of capacitance is greater on the secondary

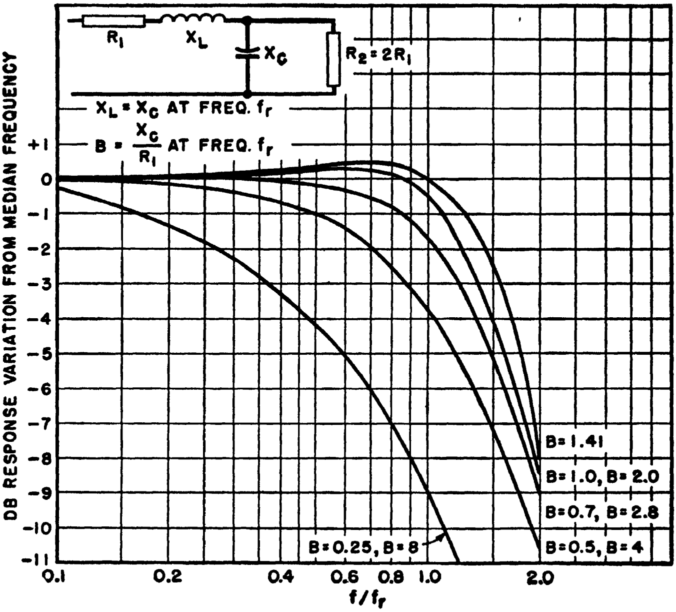


FIG. 92. Transformer characteristics at high frequencies (triode output.) (Courtesy of *Radio Engineering*, June 1937.)

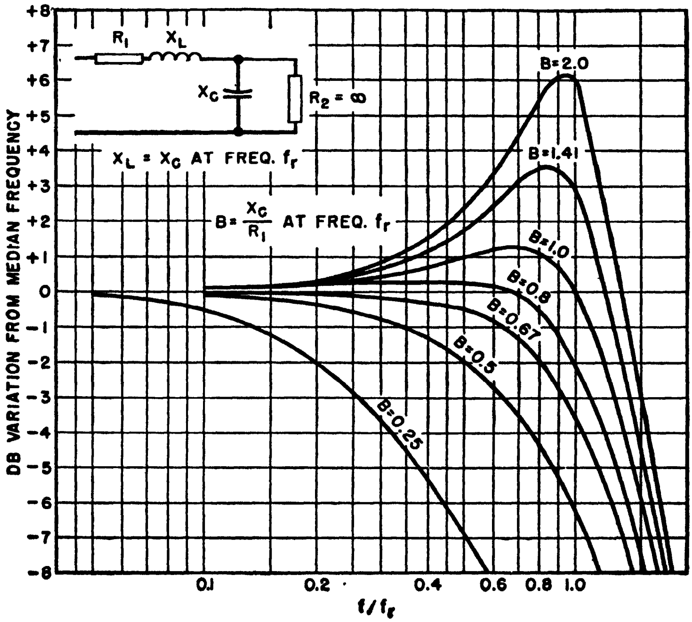


FIG. 93. Transformer characteristics at high frequencies (class A grid).

than on the primary side. Normally this is true if the secondary winding has the greater number of turns. Figures 91, 92, and 93 are thus plotted specifically for *step-up* transformers. Modifications are necessary for *step-down* transformers, the equivalent circuit for which is shown in Fig. 95. Analysis shows the scalar voltage ratio to be

$$\frac{E_2}{E_1} \qquad \sqrt{\left(\frac{R_1}{X_C} + \frac{X_L}{R_2}\right)^2 + \left(\frac{R_1}{R_2} \cdot \frac{X_L}{X_C} - \frac{R_1}{R_2} - 1\right)^2} \tag{48}$$

Notice the similarity to equation 47. In fact, if $R_1 = R_2$, equation 48 reduces to equation 47; for this case the response is the same for step-down and step-up transformers, and is given by Fig. 91.

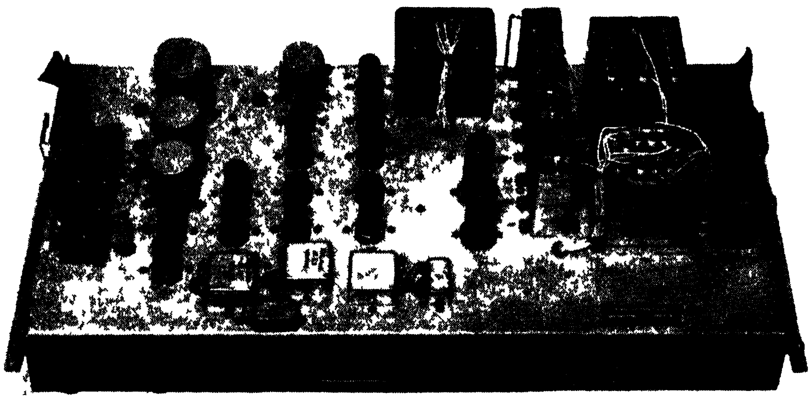


FIG. 94. Audio Amplifier. Audio transformers are inverted on chassis at left. Power supply is at right.

For $R_2 = 2R_1$, equation 48 becomes, after substitution in terms of frequency,

$$\frac{E_2}{E_1} = \frac{1}{\sqrt{\left(\frac{f}{Bf_r} + \frac{Bf}{2f_r}\right)^2 + \left(\frac{f^2}{2f_r^2} - 1.5\right)^2}} \tag{49}$$

which is plotted in Fig. 95. Non-uniform response comes at somewhat lower frequency than in Fig. 92.

The case of $R_2 = \infty$ for step-down transformers is not important. By inspection it can be seen to be the response of R_1 and X_C in series, because X_L carries no current. This case rarely occurs in practice.

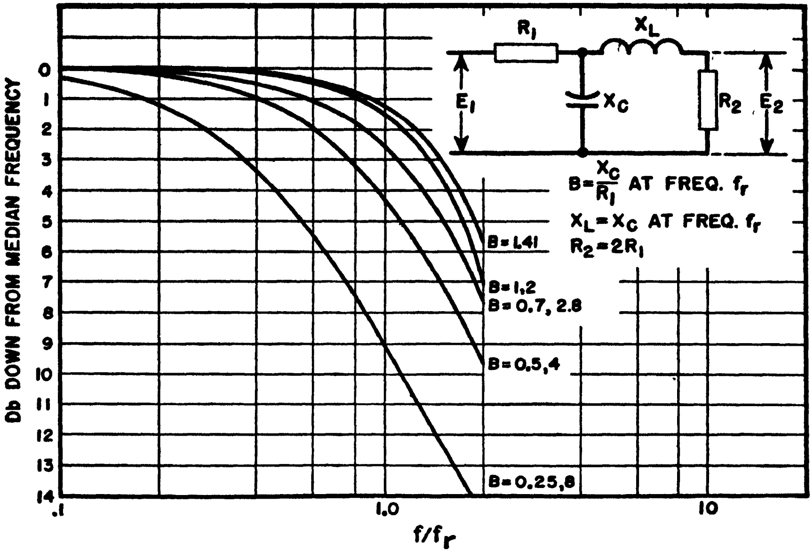


FIG. 95. High frequency response of step-down transformers.

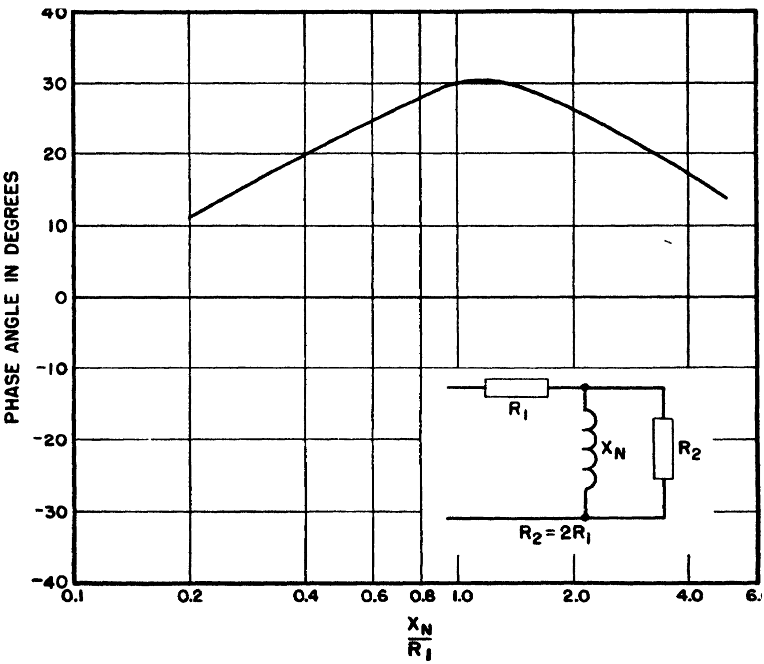


FIG. 96. Variation of amplifier phase angle at low frequencies.

60. Harmonic Distortion.[1] Audio response may be good according to Figs. 91, 92, 93, and 95, but at the same time the output may be badly distorted because of changes in load impedance or phase angle. This possibility is considered here for the case in which the load impedance is twice the source impedance.

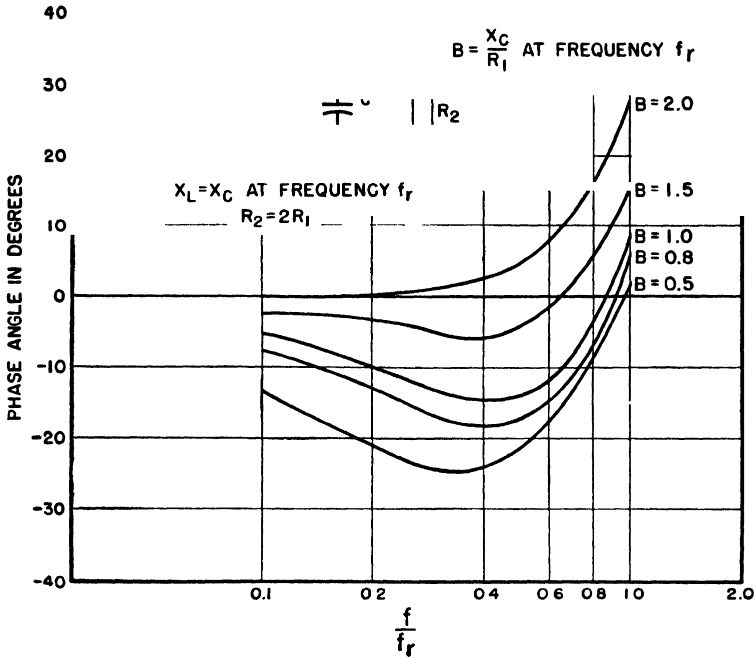


FIG. 97. Variation of amplifier phase angle at high frequencies.

The phase angle of the equivalent circuits of Figs. 89 (*c*) and 89 (*e*) is found by taking the angle whose tangent is the ratio of imaginary to real components of the total circuit impedance in each case. This angle is plotted in Figs. 96 and 97 for the low and high frequency ranges respectively, with the same abscissas as in Figs. 90 and 92. It is the angle between the voltage E_1 and the current entering the equivalent circuits of Figs. 89 (*c*) and 89 (*e*) and therefore represents the angle between a-c grid voltage and plate current. Positive angle indicates lagging plate current.

The phase angle exhibited by a transformer over the range considered in Figs. 96 and 97 does not exceed 30°, whereas for the most

[1] This section is based on the author's "Distortion in High Fidelity Audio Amplifiers," *Radio Engineering*, June 1937, p. 16.

favorable curve in Fig. 97 ($B = 1.0$) it does not exceed 15°. To study the effect of phase angle alone upon distortion, the light load of 8800 ohms calculated in Table XI is plotted upon the plate characteristics of triode type 851 in Fig. 98. The result is a true sine wave of plate

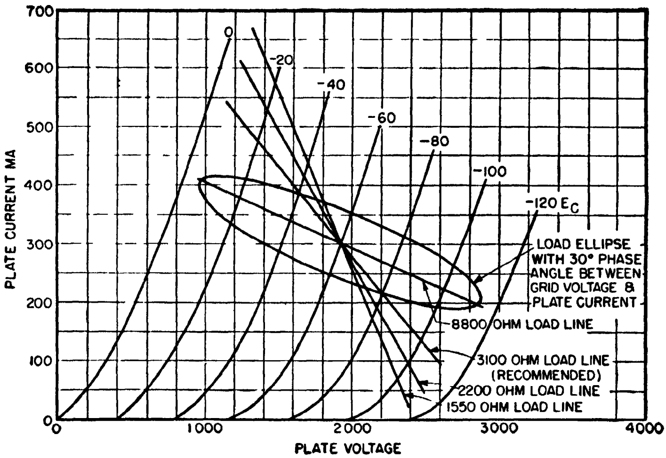


FIG. 98. Triode type 851 with reactive load.

voltage. If the phase angle between grid voltage and plate current waves is then arbitrarily made 30°, as in Table XIII, the elliptical load curve obtains. The wave of plate voltage is plotted for both zero

TABLE XIII. 851 TRIODE OPERATION WITH 8800-OHM 30° PHASE ANGLE LOAD

θ (deg)	e_c	i_B	e_B
0	− 60	0.245	1850
30	− 33	0.300	1400
60	− 13	0.355	1080
90	− 6	0.395	960
120	− 13	0.410	1150
150	− 33	0.395	1520
180	− 60	0.355	2000
210	− 87	0.300	2460
240	−107	0.245	2790
270	−114	0.205	2880
300	−107	0.190	2720
330	− 87	0.205	2350
360	− 60	0.245	1850

and 30° phase angle in Fig. 99. These waves indicate that the phase angle encountered in audio transformers does not of itself introduce much distortion in a lightly loaded triode.

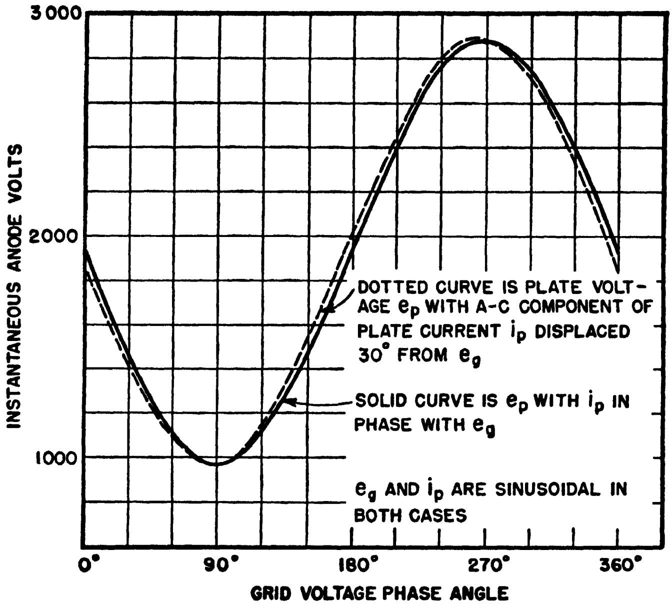


FIG. 99. Plate voltage wave forms with zero and 30° phase angles.

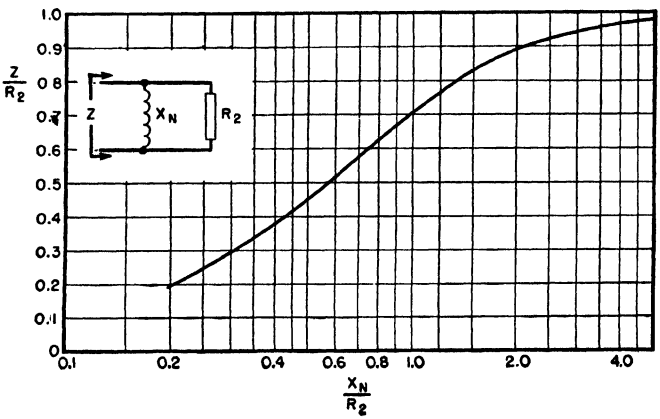


FIG. 100. Variation of load impedance with transformer characteristics at low frequencies. (Courtesy of *Radio Engineering*, June 1937.)

The remaining factor, load impedance, will be considered as distinct from the impedance of the entire circuit.

In Fig. 89 (c) the load impedance, to the right of the dotted line, is

$$Z = \frac{jR_2X_N}{R_2 + jX_N}$$

Hence

$$\frac{Z}{R_2} = \frac{\sqrt{1 + \left(\frac{X_N}{R_2}\right)^2}}{\frac{R_2}{X_N} + \frac{X_N}{R_2}} \tag{50}$$

Equation 50 is plotted in Fig. 100. It shows the change in load Z from its median frequency value R_2, as the frequency is lowered. It should be observed that the abscissas are necessarily X_N/R_2 instead of X_N/R_1 as in Fig. 90.

For the higher audio frequencies, the load impedance at the right of the dotted line in Fig. 89 (e) is

$$Z = \frac{jX_LR_2 + X_LX_C - jX_CR_2}{R_2 - jX_C}$$

$$\frac{Z}{R_2} = \frac{\sqrt{\left(\frac{X_C}{R_2}\right) + \left(\frac{X_LX_C}{R_2{}^2} + \frac{X_L}{X_C} - 1\right)^2}}{\frac{R_2}{X_C} + \frac{X_C}{R_2}} \tag{51}$$

If we let $X_C/R_2 = D$ at frequency f_r, then, at any frequency f, $X_C/R_2 = Df_r/f$. If this substitution is made in equation 51 and also if $X_L/X_C = f^2/f_r{}^2$,

$$\frac{Z}{R_2} = \frac{\sqrt{\frac{D^2f_r{}^2}{f^2} + \left(D^2 + \frac{f^2}{f_r{}^2} - 1\right)^2}}{\frac{f}{Df_r} + \frac{Df_r}{f}} \tag{51a}$$

Equation 51a is plotted in Fig. 101 for several values of D. The impedance varies widely from its median frequency value, especially at lower values of D.

From Figs. 100 and 101 it is possible to compare the change in impedance with the frequency characteristics of Figs. 90 and 92. If

the amplifier response is allowed to fall off 1.0 db at the lowest frequency, the corresponding value of X_N/R_1 from Fig. 90 is 1.3. This means that X_N/R_2 is 0.65. The corresponding load impedance in Fig. 100 is only 0.55 of its median frequency value. Likewise, for 0.5 db droop of the frequency characteristic, the load impedance falls to $0.7R_2$; while for a good load impedance of $0.9R_2$, the frequency characteristic can fall off only 0.1 db. It is thus evident that load

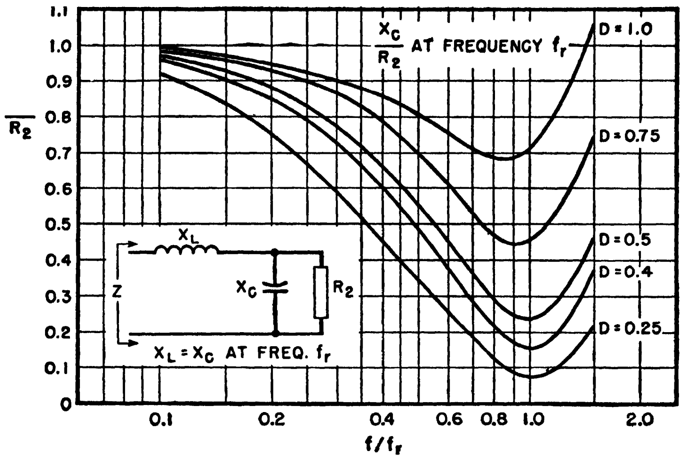


FIG. 101. Variation of load impedance with transformer characteristics at high frequencies. (Courtesy of *Radio Engineering*, June 1937).

impedance may vary widely even with comparatively flat frequency characteristics.

At high audio frequencies the divergences are still greater. Suppose, for example, that a transformer has been designed so that X_C/R_1 is 1.0 at f_r (that is, $B = 1.0$ in Fig. 92). Suppose further that the highest audio frequency at which the transformer operates is $0.75f_r$. The amplifier then has a relatively flat characteristic, with a slight rise near its upper limit of frequency. In Fig. 101, the curve corresponding to $B = 1.0$ is marked $D = 0.5$, for which at $0.75f_r$ the load impedance has dropped to 32 per cent of R_2, an extremely poor match for the tube.

It might be thought that since $0.75f_r$ is the upper frequency limit, the harmonics resulting from the low value of load impedance would not be amplified, and no harm would be done. But at the frequency $0.375f_r$, whose second harmonic would be amplified, the load impedance is only $0.69R_2$.

Between $0.375f_r$ and $0.75f_r$ (over half of the amplifier frequency range) the load impedance gradually drops from $0.69R_2$ to $0.32R_2$.

Thus distortion is large over a wide frequency range. It would be much better to design the transformer so that $B = 2.0$; the change in impedance is much less, and the rise in response is slight.

To ascertain how much distortion these low load impedances produce, a series of loads was plotted on 851 plate characteristics: 100, 70, and 50 per cent of the class A *UPO* value of twice the plate resistance (3100, 2200, and 1550 ohms respectively). The distortion is tabulated below for 54 volts grid swing.

Load	Percentage of 2nd Harmonic	Percentage of 3rd Harmonic	Plate Voltage Swing (Peak to Peak)
3100 ohms	4	1	1500
2200 ohms	10	4	1270
1550 ohms	19	6	1100

The plate voltage amplitude decrease with low impedance loads means that the combination of tube and transformer has a characteristic which droops instead of remaining flat as indicated by the curve $B = 1.0$ in Fig. 92.

This droop modifies the upper ends of the curves of Fig. 92. Although these curves were intended specifically for vacuum tubes, they were derived on the basis of a constant sinusoidal voltage in the source. Figure 101 demonstrates one important fact: For vacuum tubes operating into loads of twice the tube plate resistance, it is better to design transformers so that $B = 2$ or more. Then the output voltage and distortion are less affected by impedance variations at high frequencies. The actual frequency characteristics for triodes lie somewhere between the curves of Fig. 92 and the corresponding curves of Fig. 101.

Designing transformers for $B \geqq 2.0$ means keeping the effective capacitance lower, but the leakage inductance may be proportionately greater than for transformers having $B = 1.0$.

Besides the harmonic distortion caused by variations in load impedance, at low frequencies additional distortion is caused by nonlinear magnetizing current. If a transformer is connected to a 60-cycle supply line, the no-load current contains large harmonics, but the voltage wave form remains sinusoidal because the line impedance is low. But if distorted magnetizing current is drawn from an amplifier tube, the plate resistance is high enough to produce a distorted voltage wave form across the transformer primary winding, caused mainly by the third harmonic. If the harmonic current amplitude I_H in the magnetizing current is found by connecting the transformer across a

low impedance source, the amplitude of harmonic voltage appearing in the output with a higher impedance source is

$$\frac{E_H}{E_f} = \frac{I_H R}{I_f X_N}\left(1 - \frac{R}{4X_N}\right) \tag{52}$$

where E_H = harmonic voltage amplitude
E_f = fundamental voltage amplitude
I_H = harmonic current amplitude
I_f = fundamental current amplitude
$R = R_1R_2/(R_1 + R_2)$. (R_1, R_2, and X_N are as shown in Fig. 89 (*c*).)[2]

If flux density is below the knee of the saturation curve, and if $X_N = 3R_2$ at the lowest operating frequency, the harmonic amplitude is less than 5 per cent. An air gap in the core reduces this figure. Table XIV gives typical harmonic currents for silicon steel.

TABLE XIV. TYPICAL SILICON STEEL MAGNETIZING CURRENT HARMONIC COMPONENTS WITH ZERO IMPEDANCE SOURCE

B_m Gauss	Percentage of 3rd Harmonic	Percentage of 5th Harmonic
100	4	1
500	7	1.5
1,000	9	2.0
3,000	15	2.5
5,000	20	3.0
10,000	30	5.0

Output voltage distortion may be analyzed to find harmonic content by the usual Fourier method. Several simplifications have been devised to reduce the labor and increase accuracy.[3] In general, if the recommended tube load impedances are maintained, harmonic percentages will be as given in the tube manuals. If other load impedances obtain at some frequencies, to predict the harmonic output requires harmonic analysis.

61. Push-Pull Amplifier Transformers. The analysis of single-side amplifiers in Section 60 applies to class A push-pull, except that the second harmonic components in the amplifier output are due to unlike tubes, rather than to low impedance distortion.

[2] For a discussion of equation 52 and magnetizing currents in general, see "Harmonic Distortion in Audio-Frequency Transformers," by N. Partridge, *Wireless Engineer*, Vol. 19, Sept., Oct., and Nov., 1942.

[3] For example, "Graphical Harmonic Analysis," by J. A. Hutcheson, *Electronics* Vol. 9, January 1936, p. 16.

The internal tube resistance of a class B amplifier varies so much with the amount of signal voltage on the grids, power output, and plate voltage that it is not practicable to draw curves similar to Fig. 92 for class B operation. If the tubes are loaded lightly, the plate current may rise at the higher frequencies; if loaded heavily, the output voltage falls off. Qualitatively, the characteristic curves may be expected to follow the same general trend as for class A amplifiers. A basis for class B amplifier design is to make the transformer constants such that the load impedance does not fall below a given percentage of the load resistance R_2. This is discussed further in Chapter VI.

Usually the decline with frequency response is greater for class B than for class A amplifiers, because the effect of internal plate impedance is greater. As an extreme, the plate resistance may be so great that the current in the load cannot increase as the load impedance decreases. Hence the characteristic curve falls off proportionately with the load impedance indicated in Figs. 100 and 101.

The change in mode of operation in a class B amplifier as the output passes from one tube to the other in the region of cut-off has been pointed out in Section 55. This change-over may cause transient voltages in the amplifier which distort the output voltage wave form. If the two halves of the transformer primary winding are not tightly coupled, primary-to-primary leakage inductance causes nicks in the output voltage wave, in somewhat the same way as leakage inductance in a rectifier plate transformer. In a class B amplifier, the change from one tube to the other is less abrupt than in a rectifier, so higher values of leakage inductance are permissible. Perceptible nicks in the voltage wave occur if the ratio of primary-to-primary leakage reactance to average triode plate resistance is 4 or more.[4]

Balanced operation in a push-pull amplifier, i.e., equal plate current and voltage swing on both sides, is possible only if the tubes are alike and if transformer winding turns and resistances per side are equal. Shell-type concentric windings do not fulfill this condition because the half of the primary nearer to the core tongue has lower resistance than the other half. Balance is easier to achieve in the core type of arrangement shown in Fig. 102. In class A amplifiers close primary-primary coupling is not essential, and balance may be attained by arranging part coils as in Fig. 103.

Because only half of the primary winding of a class B amplifier carries current during a half-cycle, the leakage flux and therefore the primary-to-secondary leakage inductance have approximately half the

[4] See "Quasi Transients in Class B Audio-Frequency Push-Pull Amplifiers," by A. Pen-Tung Sah, *Proc. I.R.E.*, Vol. 24, Nov. 1936, p. 1522.

values with both windings active all the time. With capacitive currents, both windings *are* active, at least partially. Designing so that $B > 2.0$ results in low capacitive currents, low leakage inductance, high resonance frequency, and extended frequency range, in addition to the load impedance advantages given in Section 60.

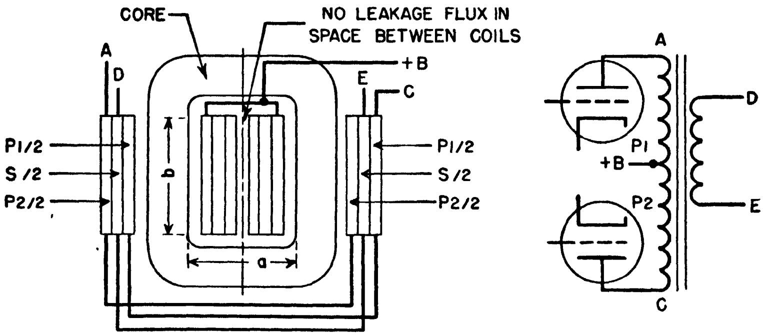


FIG. 102. Core-type push-pull balanced windings.

Capacitive currents also cause unbalance at high frequencies, even with winding arrangements like Figs. 102 and 103. This is evident if the secondary winding in these figures is grounded at one end; the effective capacitances to the two primary windings are then unequal. This problem may be solved by keeping the capacitances small with liberal spacing, but this practice increases leakage inductance and cannot be carried very far. Coil mean turn length should be kept as small as possible, by the use of the most suitable core steel. Core-type designs have smaller mean turns than shell-type. Also, the two outer coil sections have low capacitance to each other and to the case if liberal spacing is used, *without* an increase in leakage inductance. Flux in the space between the outer sections links all the windings on one leg, and hence is not leakage flux. Consequently, this space is subtracted from the term a in equation 21 (p. 59). In push-pull amplifiers the winding arrangement of Fig. 102 is advantageous because of the low capacitance between the points of greatest potential difference, A and C.

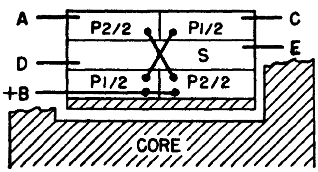


FIG. 103. Shell-type push-pull balanced windings.

62. Plate Current Increase. In a lightly loaded amplifier the frequency characteristic stays flat at high frequencies, even with a droop in load impedance, but the plate current rises in inverse proportion to

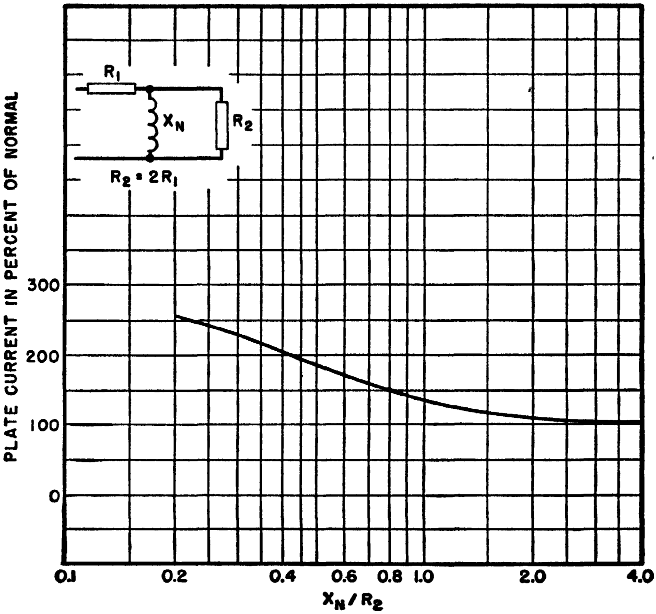


FIG 104. Rise in plate current due to transformer impedance change at low frequencies.

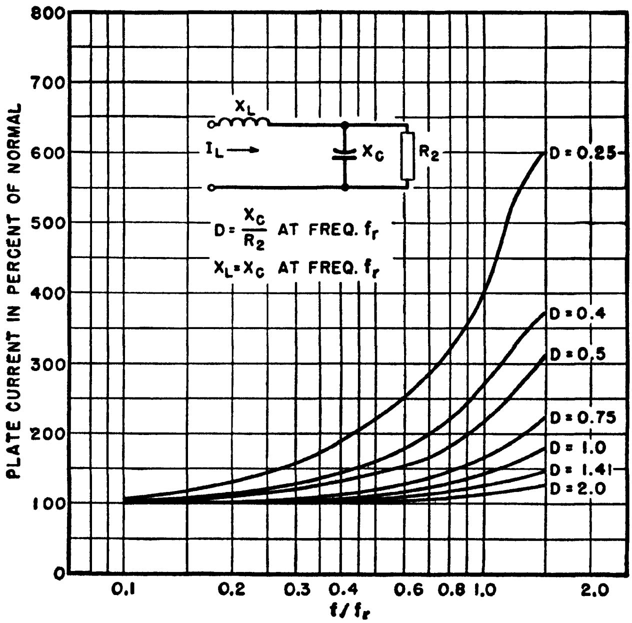


FIG. 105. Rise in plate current due to transformer impedance change at high frequencies.

the impedance. If the plate current can rise enough to maintain constant output voltage, this plate current rise may be objectionable from the standpoint of tube heating or plate supply regulation. Values of plate current rise calculated on the basis of constant output for low and high frequencies are shown in Figs. 104 and 105. Many satisfactory audio amplifiers have plate currents which would be excessive at the extremes of the range if high or low notes were amplified continuously. They are not damaged because these tones are of short duration.

63. Pentode Amplifiers. Tetrode tubes have an additional grid between anode and control grid to reduce the grid-to-anode capacitance. This additional grid is known as the screen grid and is operated at a positive potential with a-c bypass to reduce the grid-to-anode capacitance. The chief drawback to this type of tube is that the anode voltage swing is limited to the difference between the anode voltage and screen voltage. This disadvantage is overcome by the addition of a third grid known as the suppressor, which removes this limitation and allows large anode voltage swings down to the diode line of the tube. Sometimes the third electrode is connected internally to the cathode. Similar characteristics are obtained with the so-called beam tubes, which are tetrodes with special screen grid spacings. Figure 106 shows 6L6 beam tube plate characteristics, with a typical load line of 2500 ohms. As a single side amplifier, such a tube is likely to have large distortion because of the uneven spacing of constant-grid-voltage lines. Distortion is reduced in a push-pull amplifier, especially for high power output. Plate resistance r_p is very high in pentodes and beam tubes, of the order of ten times the load resistance.

Pentodes are essentially constant current devices. The value of load impedance is thus an indication of the output voltage, at least for low frequencies. Response of a low frequency transformer-coupled pentode amplifier can be taken from Fig. 100.

At high frequencies, leakage inductance of the transformer intervenes between the pentode and its load, so that the primary voltage and secondary or load voltage are not identical. In Fig. 107 the change of output voltage for a constant grid voltage at high frequencies is shown. In this figure, the equivalent circuit is a pi-filter, which is more accurate for pentode transformers than the circuit of Fig. 89 (*e*). This is true because the transformer usually connects the pentode to a load impedance lower than it could work into directly without loss of power. For this, step-down transformers are needed. Harmonic con-

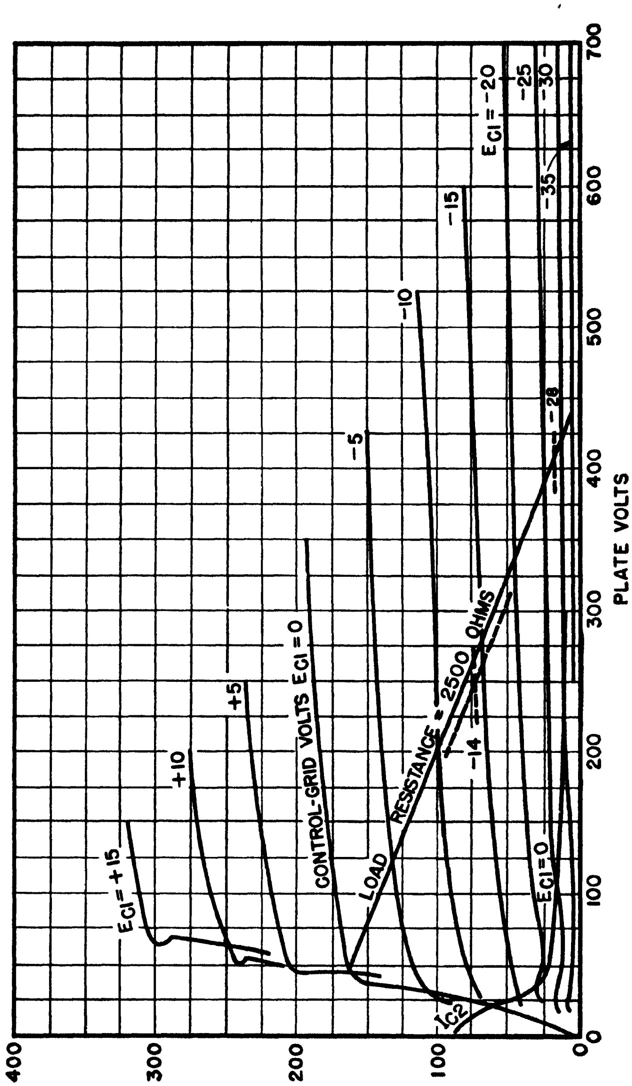


Fig. 106. Plate characteristics of type 6L6 beam tube.

tent of pentodes is high, especially in single-side amplifiers. Large phase angle and low load impedance produce undesirable distortion. It is best to use values of X_N/R_2 greater than 2 in Fig. 100 at the lowest frequency to avoid distortion.

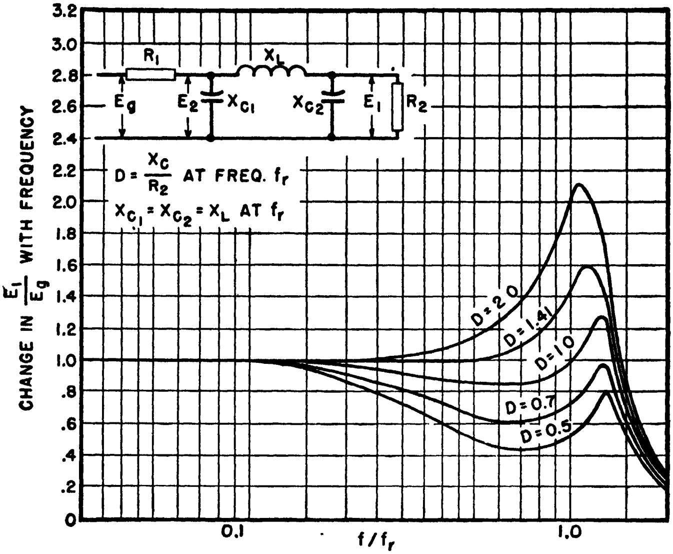


FIG. 107. Pentode frequency response with pi-filter output circuit.

64. Calculation of Inductance and Capacitance. Transformer-coupled amplifier performance is dependent at low frequencies upon transformer *OCL*, and at high frequencies upon leakage inductance and winding capacitance. Calculation of these quantities is essential in design and useful in tests for proper operation. Inductance formulas are repeated here for convenience, along with capacitance calculations.

$$OCL = \frac{3.2N^2A_c}{10^8\left(l_g + \frac{l_c}{\mu}\right)} \quad \text{(henrys)} \tag{26}$$

where N = turns in winding
A_c = core area in square inches
l_g = total length of air gap in inches
l_c = core length in inches
μ = permeability of core (if there is unbalanced direct current in the winding, this is the incremental permeability).

For concentric shell- or core-type windings the total leakage inductance referred to any winding is

$$L = \frac{10.6N^2l(2nc + a)}{n^2b \times 10^9} \quad \text{(henrys)} \tag{21}$$

where N = turns in that winding
l = mean length of turn for whole coil
a = window opening height
b = winding width
c = insulation space
n = number of insulation spaces
= number of primary-secondary interleavings (see Fig. 44, p. 58).

Winding capacitance is not expressible in terms of a single formula. The effective value of winding capacitance is almost never measurable, because it depends upon the voltages at the various points of the winding. The capacitance current at any point is equal to the voltage across the capacitance divided by the capacitive reactance. Since many capacitances occur at different voltages, in even the simplest transformer, no one general formula can suffice. The major components of capacitance are from

1. Turn to turn.
2. Layer to layer.
3. Winding to winding.
4. Windings to core.
5. Stray (including terminals, leads, and case).
6. External capacitors.
7. Vacuum tube electrode capacitance.

These components have different relative values in different types of windings. Turn-to-turn capacitance is seldom preponderant because the capacitances are in series when referred to the whole winding. Layer-to-layer capacitance may be the major portion in high voltage single-section windings, where thick winding insulation keeps the winding-to-winding and winding-to-core components small. Items 5, 6, and 7 need to be watched carefully lest they spoil otherwise low capacitance transformers and circuits.

If a capacitance C with E_1 volts across it is to be referred to some other voltage E_2, the effective value at reference voltage E_2 is

$$C_e = C\frac{E_1^2}{E_2^2} \tag{53}$$

By use of equation 53 all capacitances in the transformer may be referred to the primary or secondary winding; the sum of these capacitances is then the transformer capacitance which is used in the various formulas and curves of preceding sections.

In an element of winding across which voltage is substantially uniform throughout, capacitance to a surface beneath is

$$C = \frac{0.225A\epsilon}{t} \quad (\mu\mu\text{f}) \tag{54}$$

where A = area of winding element in square inches
ϵ = dielectric constant of insulation under winding = 3 to 4 for organic materials
t = thickness in inches of insulation under winding. This includes wire insulation and space factor.

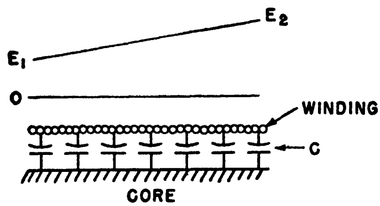


FIG. 108. Transformer winding with uniform voltage distribution.

If the winding element has uniformly varying voltage across it, as in Fig. 108, the effective capacitance is the sum of all the incremental effective capacitances. This summation is

$$C_e = \frac{C(E_1^2 + E_2^2 + E_1E_2)}{3E^2} \tag{55}$$

where C = capacitance of winding element as found by equation 54
E_1 = minimum voltage across C
E_2 = maximum voltage across C
E = reference voltage for C_e.

If E_1 is zero and $E_2 = E$, equation 55 becomes

$$C_e = \frac{C}{3} \tag{56}$$

or the capacitance, say, to ground of a single-layer winding with its low voltage end grounded is one third of the measured capacitance of the winding to ground. Measurement should be made with the wind-

ing ungrounded and both ends short-circuited together, to form one electrode, and ground to form the other.

In a multi-layer winding, E_1 is zero at one end of each layer and $E_2 = 2E/N_L$ at the other, where E is the winding voltage and N_L is the number of layers. The effective layer-to-layer capacitance of the whole winding is

$$C_e = \frac{4C_L}{3N_L}\left(1 - \frac{1}{N_L}\right) \tag{57}$$

where C_L is the measurable capacitance of one layer to another. The first and last layers have capacitance to other layers on one side only, and this is accounted for by the term in parentheses in equation 57.

Because the turns per layer and volts per layer are greater in windings with many turns of small wire, such windings have higher effective capacitance than windings with few turns. In a transformer with large turns ratio, whether step-up or step-down, this effective capacitance is often the barrier to further increase of turns ratio. With a given load impedance across the low impedance winding, there is a maximum effective capacitance C_m which can be tolerated for a given frequency response. If layer and winding capacitances have been reduced to the lowest practicable figure C_l, the maximum turns ratio is $\sqrt{C_m/C_l}$. Appreciable amounts of capacitance across which large voltages exist must be eliminated by careful design.

Since effective capacitance is greater at higher voltages, in step-down transformers the capacitance may be regarded as existing mainly across the primary winding; in step-up transformers, across the secondary winding. The effect of this on frequency response has been discussed in Section 59.

The input capacitance of a triode amplifier is given by [5]

$$C_{\text{input}} = C_{G-F} + (\alpha + 1)C_{G-P} \tag{58}$$

where C_{G-F} = grid-to-cathode capacitance
C_{G-P} = grid-to-anode capacitance
α = voltage gain of the stage.

C_{G-F} and C_{G-P} are given for many tubes in the tube handbooks. They can be measured in any tube by means of a capacitance bridge.

65. Amplifier Transformer Design. In amplifiers which operate at a single frequency, transformers are similar in design to rectifier plate transformers. Size of core is determined by the required value of *OCL.*

[5] See J. H. Morecroft, "Principles of Radio Communications," John Wiley and Sons, 2nd ed., 1927, p. 511.

See Section 58. If the winding carries unbalanced direct current, an air gap must be provided to keep B_m within the limits discussed in Section 35 (Chapter III). Winding resistances are limited by permissible loss in output, or in larger units by heating.

If the amplifier operates over a frequency range, the start of the design is with *OCL* to insure proper low frequency performance. After ample core area and turns have been chosen, attention must be given to the winding configuration. Leakage inductance and winding capacitance are calculated and, from them, f_r and B. If the high frequency response does not meet the requirements, measures must be taken to increase f_r or change B to a value nearer optimum. Sometimes these considerations increase size appreciably.

Below frequency f_r, the leakage inductance per turn is constant and equal to the total coil inductance divided by the number of turns. Capacitance per turn is constant and may be large because of the close turn-to-turn spacing. But the *LC* product per turn is smaller than the *LC* product per layer, because the layer *effective* capacitance is greater. Therefore the frequency at which the turns become resonant is higher than that at which the layers become resonant. Likewise, if there is appreciable coil-to-coil capacitance, the layer resonant frequency is higher than the coil resonant frequency f_r. If the coil design is such that resonance of part of a coil occurs at a lower frequency than f_r, the transformer frequency response is limited by the partial resonance. This condition is especially undesirable in wide range designs, but with reasonable care it can be avoided.

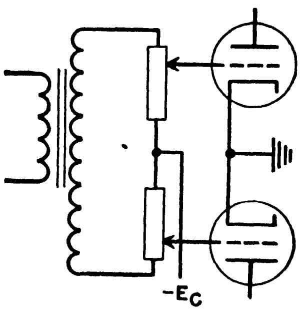


FIG. 109. Input transformer driving push-pull grids.

Two examples of audio transformer design are given here to illustrate low and high frequency response calculations.

Example (a). Input Transformer. To terminate a 500-ohm line and apply input to push-pull class A grids as in Fig. 109. Primary voltage is 2.0 volts. Frequency range 100 to 5000 cycles. Step-up ratio 1:20. No direct current flows in either primary or secondary winding. Nickel-iron laminations are used. Refer to Fig. 44 for dimension symbols and winding arrangement.

A_c = 0.5 sq in.
l_c = 4.5 in.
Permeability (initial value) = 5000
Coil mean turn 4.5 in.
Window 0.578 in.

Primary 400 turns No. 30 single enamel wire
Secondary 8000 turns No. 40 single enamel wire (total)
Primary layers 7; layer paper 0.0015 in.
Secondary layers 44; layer paper 0.0007 in.
Vertical space factor 0.9
$b = 0.75$ in.
$c = 0.008$ in.

$$\text{Secondary } OCL = \frac{3.2 \times (8000)^2 \times 0.5 \times 10^{-8}}{0.0005 + \frac{4.5}{5000}} = 730 \text{ henrys with smallest possible air gap (per Table VIII, p. 76)}$$

$$= 540 \text{ henrys with average gap} = 0.001 \text{ in.}$$

Secondary leakage inductance

$$= \frac{10.6 \times (8000)^2 \times 4.5 \times (4 \times 0.008 + 0.578)}{4 \times 0.75 \times 10^9} = 0.62 \text{ henry}$$

Capacitances:

$$\text{Secondary layer-to-layer} = \frac{0.225 \times 4.5 \times 0.75 \times 3 \times 0.9}{0.0007 + 0.0005} = 1700$$

Ditto referred to whole secondary $= \frac{1700}{44} \times 1.33 \times 0.977$	=	51 $\mu\mu$f
Primary layer-to-layer, referred to secondary	<	1
Tube input capacitance	=	25
Winding-to-core capacitance	=	40
Stray capacitance	=	10
Total secondary capacitance	=	127 $\mu\mu$f

With the primary winding located at audio ground on the secondary, there is virtually zero winding-to-winding capacitance. Secondary-to-core capacitance is based on a coil form $\frac{1}{16}$ in. thick
Total secondary load resistance $= 500 \times (20)^2 = 200{,}000$ ohms
Based on inductance with 0.001-in. gap, $X_N = 339{,}000$ ohms and $X_N/R_1 = 1.7$
Response is 0.3 db down, in Fig. 90
Resonance frequency of 0.62 henry and 127 $\mu\mu$f is 18,000 cycles and $X_c = 70{,}000$ ohms. $B = X_c/R_1 = 0.35$. $f/f_r = 5000/18{,}000 = 0.28$. Response is 0.6 db down at 5000 cycles (from Fig. 91)

Example (*b*). *Interstage Transformer.* In interstage coupling the impedance level is high, to maintain both high load impedance and high grid excitation in the following stage. The limit on the secondary side is the highest resistance which affords grid circuit stability. There is no impedance limit on the primary side except that imposed by transformer design. Usually a 1:1 ratio is about optimum. A step-down ratio gives less voltage on the following grid.

A step-up ratio reflects the secondary load into the plate circuit as a lower impedance. This reduces the voltage gain, especially with pentodes which have high plate resistance compared to load resistance. Under this condition, equation 42 becomes

$$\frac{e_p}{e_g} = \frac{\mu Z_L}{r_p} = g_m Z_L \tag{59}$$

or the voltage gain is proportional to load impedance. Interstage transformers commonly have many turns and high OCL.

Suppose that a transformer is required to connect a 6SK7 tube to a 6L6 operating class A with 10 volts rms on the grid over a frequency range of 300 to 3000 cycles. 6L6 grid resistance is 90,000 ohms, which is to be reflected into the 6SK7 plate circuit as a 90,000-ohm load. Hence a 1:1 turns ratio is used. 6SK7 plate current is 10 ma. The same core as in Example (*a*) is used, except that here it is made of silicon steel, and the stacking is reduced so that A_c is 0.32 sq in. Primary and secondary windings are single sections; with the primary start lead connected to 6SK7 plate and secondary finish lead connected to 6L6 grid. This leaves adjacent turns in both these windings at zero audio potential, and effective primary-secondary capacitance is zero.

Primary turns = secondary turns = 6600 turns No. 40 enamel wire
Primary and secondary layers = 37
Primary mean turn = 3.3 in.
Secondary mean turn = 4.2 in., l_g = 0.005 in.

$$B_{dc} = \frac{0.6 \times 6600 \times 0.010}{0.005} = 7750 \text{ gauss}$$

$$B_{ac} = \frac{3.49 \times 10 \times 10^6}{300 \times 0.32 \times 6600} = \underline{55 \text{ gauss}}$$

$$B_m = 7805 \text{ gauss}$$

From Fig. 54, μ_Δ = 1100

$$OCL = \frac{3.2 \times (6600)^2 \times 0.36 \times 10^{-8}}{0.005 + \dfrac{4.5}{1100}} = 55 \text{ henrys}$$

$$\text{Leakage } L = \frac{10.6 \times (6600)^2 \times 3.8(2 \times 0.008 + 0.578)}{0.75 \times 10^3} = 1.4 \text{ henrys}$$

Capacitances:

$$\text{Primary layer-to-layer} = \frac{0.225 \times 3.3 \times 0.75 \times 3}{0.0015} = 1120\ \mu\mu\text{f}$$

$$\text{Secondary layer-to-layer} = \frac{0.225 \times 4.2 \times 0.75 \times 3}{0.0015} = 1415\ \mu\mu\text{f}$$

Referred to the whole winding, the capacitances are

$$\frac{1120 \times 4 \times 0.973}{37 \times 3} \quad 40\ \mu\mu\text{f}$$

and

$$\frac{1415 \times 4 \times 0.973}{37 \times 3} = 50$$

Primary—core C	= 44
Secondary—core C	= 14
Tube capacitances	= 17
Stray capacitances	= 10
Total capacitance	= 175 $\mu\mu$f

$f_r = 10$ kc. $X = 90{,}000$, $D = 1$, and response is 1 db down at 3000 cycles (from Fig. 107). $X_N/R_2 = 6.28 \times 300 \times (55/90{,}000) = 1.04$. Figure 100 shows a Z/R_2 of 0.72; therefore the response is 3 db down at 300 cycles.

PROBLEMS

1. Plot load ellipses for 1550-, 2200-, and 3100-ohm loads with 30° phase angle between grid voltage and plate current in Fig. 98. From these draw plate voltage waves like that of Fig. 99.

2. Plot load ellipse for a 2500-ohm load with 30° phase angle on anode characteristics of Fig. 106. Repeat for 1250-ohm load.

3. Derive equations 55 and 57.

4. In Example (*a*), Section 65, reduce secondary turns to 6400 and arrange secondary windings in vertically split part coils like the primary windings in Fig. 103. Recalculate response at high audio frequencies.

Ans. 0 db down at 5000 cycles.

5. What is the response of the transformer in Example (*b*), Section 65, at 100 and 5000 cycles. Assume that flux density is so low that magnetizing current distortion is negligible. *Ans.* −9.5 db at 100 cycles, −1.4 db at 5000 cycles.

6. Assume the same A_c in Example (*b*) that was used in Example (*a*). What improvement in low frequency response is obtained? What change in high frequency response (remember that the mean turn of winding increases)?

Ans. Response becomes −1.44 db at 300 cycles, −1.1 db at 3000 cycles.

7. Using the same circuit and core as in Example (*b*) of Section 65, find frequency response with a 1:2 step-up ratio. Maintain the same winding resistances in primary and secondary windings. What is the change in gain at 1000 cycles?

Ans. Response −1.1 db at 300 cycles, zero at 3000 cycles. Gain reduced 5.3 db.

8. Plot a curve of harmonic voltage in output due to magnetizing current at 60 cycles of a transformer which has 30 per cent of third harmonic current when the transformer is connected to a 60-cycle supply line. Assume $R_2 = 2R_1$. Use X_N/R_1 for abscissas.

9. What turn-to-turn resonant frequency occurs in the secondary of the transformer in Example (*a*)? What layer-to-layer frequency?

Ans. 4.28×10^6 cycles; 28,000 cycles.

VI. AMPLIFIER CIRCUITS

The application of amplifiers may require control of hum, distortion, or frequency response beyond the limits imposed by ordinary design. Sometimes the additional performance is obtained by designing extra large transformers; this is usually an expensive procedure. In some cases, extra features can be incorporated into the transformers without marked increase in size. In still other cases, additional circuits are used either preceding or in conjunction with the amplifier.

In this chapter several devices for obtaining special performance are considered.

66. Inverse Feedback. If part of the output of an amplifier is fed back to the input in such a way as to oppose it, the ripple, distortion,

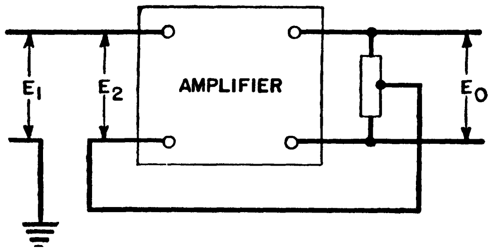


FIG. 110. Voltage feedback.

and frequency response deviations in output are reduced. The amplifier gain is reduced also, but with the availability of high gain tubes an extra stage or two compensates for the reduction in gain caused by inverse feedback, and the improvement in performance usually justifies it. In the amplifier of Fig. 110, a tapped resistor is shown across the output which has voltage E_0 and a part of this output is fed back so that the input to the amplifier is

$$E_2 = E_1 - \beta E_0 \tag{60}$$

Here β is the portion of E_0 which is fed back. If α is the voltage amplification of the amplifier and E_R and E_H are the ripple and harmonic distortion in the output without feedback, and α', E'_R, and E'_H

are the same properties with feedback, the following equations hold, if α, E_R, and E_H are assumed to be independent:

Without feedback,

$$E_0 = \alpha E_2 + E_R + E_H \tag{61}$$

With feedback,

$$E_0 = \alpha' E_1 + E'_R + E'_H \tag{62}$$

From these equations it can be shown that

$$\alpha' = \frac{\alpha}{1 + \alpha\beta} \approx \frac{1}{\beta} \tag{63}$$

$$E'_R = \frac{E_R}{1 + \alpha\beta} \approx \frac{E_R}{\alpha\beta} \tag{64}$$

$$E'_H = \frac{E_H}{1 + \alpha\beta} \approx \frac{E_H}{\alpha\beta} \tag{65}$$

With high gain amplifiers and large amounts of feedback, the output ripple and harmonic distortion can be made very small. Likewise the frequency response can be made flat, even with mediocre transformers. Inverse feedback is not used in class C amplifiers, because the output and input are not linearly related.

Incidental effects in the amplifier, like distributed capacitance and leakage inductance, have to be carefully matched in the inverse feedback circuit so that the phase shift around the loop does not become 180°. If it reaches this value, the inverse feedback turns into regenerative feedback and the amplifier becomes an oscillator with a frequency determined by the circuit constants. If it is desired to correct for distortion or hum over a frequency range of 30 to 10,000 cycles, the amplifier should have low phase shift over a much wider range, say 10 to 30,000 cycles. In the frequency intervals of 10 to 30 cycles and 10,000 to 30,000 cycles, both the amplification and the feedback should taper off gradually to prevent oscillations.

Phase shift at low and high frequencies is shown in Figs. 111 and 112 for transformer-coupled stages. At high frequencies, 180° phase shift is possible whereas at low frequencies but 90° is possible. In a resistance-coupled amplifier, only 90° phase shift occurs at either low or high frequencies. Partly for this reason, partly because less capacitance is incidental to resistors than to transformers and good response is maintained up to higher frequencies, it is in resistance-coupled amplifiers that inverse feedback is generally employed. But if the distortion of a final stage is to be reduced, transformer coupling is in-

volved. It is preferable to derive the feedback voltage from the primary side of the output transformer. This is equivalent to tapping between R_1 and X_L in Fig. 112, where the phase shift is much less.

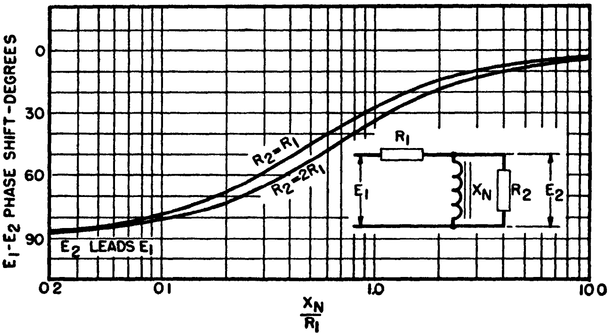


FIG. 111. Transformer-coupled amplifier low frequency phase shift.

The transformer must still present a fairly high impedance load to the output tube throughout the marginal frequency intervals to permit gradual decrease of both amplification and feedback.

Current feedback is effected in the circuit of Fig. 113 by removing capacitor C. This introduces degeneration in the cathode resistor circuit, which accomplishes the same thing as the bucking action of voltage feedback. It is less affected by phase shift and consequently is used with transformer-coupled amplifiers.

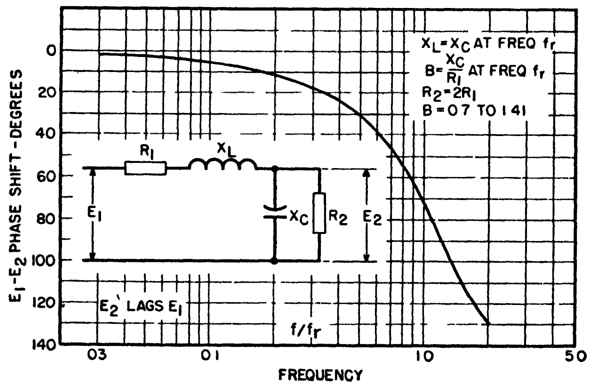


FIG. 112. Transformer-coupled amplifier high frequency phase shift.

67. Cathode Follower. The circuit of Fig. 114 is known as a cathode follower. Here the anode is connected to the high voltage supply E_B without any intervening impedance, so that for alternating currents it

is essentially grounded. Grid voltage e_g must be great enough to include the output E_0 in addition to the normal grid-to-cathode voltage at $E_{\min}$. However, the grid power is still the same as it would be were the cathode grounded. This circuit is used when the output impedance Z_L is variable or of low power factor so that normally it would be difficult to produce in it full output from the tube. The circuit has a low internal effective impedance as far as the output is concerned. It is approximately equal to the normal plate resistance r_p divided by the amplification factor μ of the tube. This is equivalent

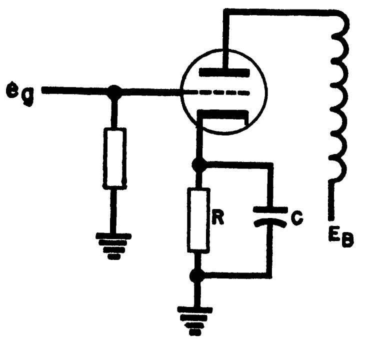


FIG. 113. Cathode bias.

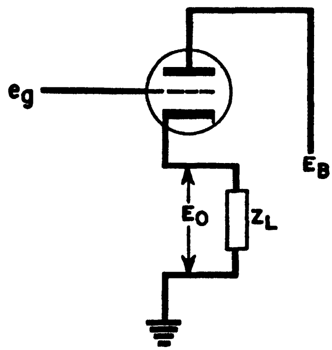


FIG. 114. Cathode follower.

to saying that the effective internal impedance is approximately the reciprocal of the mutual conductance g_m[1] for class A or B amplifiers. Cathode followers have been used to drive grids of class B modulator tubes, which are highly variable loads. The circuit produces nearly constant output voltage, but at the expense of increased grid swing.

68. Wave Filter Principles. It is sometimes desirable to allow certain frequencies which are present in a circuit to pass through at full amplitude, but to suppress as nearly as possible certain other frequencies. The means usually employed to accomplish this result is a wave filter. In any such filter, the band of frequencies which it is desired to transmit is known as the transmission band and that which it is desired to suppress is known as the attenuation band. At some frequency, known as the cut-off frequency, the filter starts to attenuate. Transition between attenuation and transmission bands may be gradual or sharp; the filter is said to have gradual or sharp cut-off accordingly.

[1] See "Feedback," by E. K. Sandeman, *Wireless Engineer*, Vol. XVII, August 1940, p. 350.

To avoid introducing losses and attenuation in the transmission bands, reactances as nearly pure as practicable are used in the elements of a wave filter. For example, in the "low pass" filter T-section of Fig. 115, the inductance arms shown as $L/2$ and the capacitance C are made with losses as low as possible. Capacitors ordinarily used in filters have low losses, but it is a problem to make inductors which have low losses. Values of inductor Q ranging from 10 to 200 are common, depending upon the value of inductance and the frequency of transmission. Therefore in wave filters the loss is mostly in the

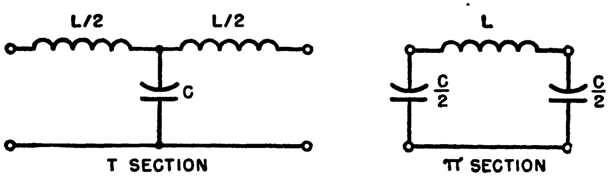


FIG. 115. Low pass filter sections.

inductances. It can be shown [2] that for pure reactance arms the values of reactance are such that in the transmission band

$$0 > \frac{Z_1}{4Z_2} > -1 \tag{66}$$

where Z_1 is the reactance of the series arm and Z_2 is the reactance of the shunt arm. In the T-section of Fig. 115, Z_1 is $2\pi f[(L/2) + (L/2)] = 2\pi fL$ and Z_2 is the reactance of C. The attenuation for sections of filter like Fig. 115 is shown in Fig. 116, for a pure reactance network starting at the cut-off frequency. The attenuation is shown in decibels, and the abscissas are one fourth of the ratio of series to shunt reactance in a full section.

It is important, in the transmission band, to terminate the sections of filter in the proper impedance. Like a transmission line, a wave filter will deliver its full energy only into an impedance which is equal to its characteristic impedance. Many wave filters are composed of several sections which simulate transmission lines. A properly terminated filter will exhibit the same impedance at either end, when terminated at the opposite end with an impedance equal to its characteristic impedance. The impedance seen at any one point in the filter is called its image impedance; it will be the same in either direction

[2] See *Transmission Networks and Wave Filters*, by T. E. Shea, D. Van Nostrand Co., 1929, p. 187.

provided the source and terminating impedances are equal. In general, however, the image impedance will not be the same for all points in the filter. For example, the impedance looking into the left or T-section of Fig. 115 (if it is assumed to be terminated properly) will not be the same as that seen across the capacitor C. For that reason, another half series arm is added between C and the termination to keep equal input and output impedances. The terminating sections at both the sending and receiving ends of a filter network are half sections, whereas

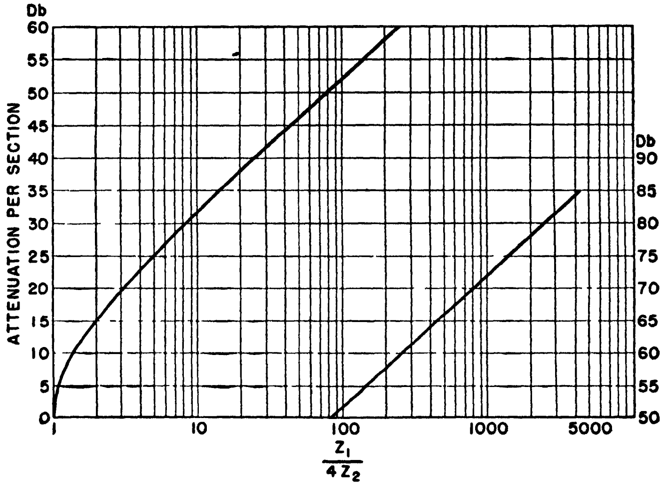


FIG 116. Attenuation per section with pure reactance arms.

the intermediate sections are full sections. A full T-section of the type shown in Fig. 115 includes an inductance L equal to $L/2 + L/2$. The image impedance seen at the input terminals of the T-section of Fig. 115 is known as the mid-series impedance, and that seen across capacitor C is known as the mid-shunt impedance.

Likewise, in the pi-section shown at the right in Fig. 115, the mid-shunt image impedance is seen at the input or output terminals. The mid-series impedance is seen at a point in the middle of coil L. This section terminates properly in its characteristic impedance at either end. Note that adjacent sections have $C/2$ for the shunt arm, so that a full section would again be composed of a capacitor C and an inductance L. The choice of T- or pi-sections is determined by convenience in termination, or by the kind of image impedance variation with frequency that is desired.

If these precautions are not observed, wave reflections are likely to cause a loss of power transfer in the transmission band.

Phase shift between input and output voltage per section for a filter having pure reactance arms is shown in Fig. 117. It is zero only at zero frequency, and it increases up to 180° at cut-off. At frequencies higher than cut-off it remains 180°.

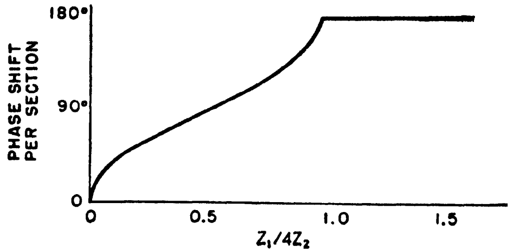


FIG. 117. Phase shift from input to output terminals of filter section as related to filter impedances.

Image impedances of a filter are not constant at all frequencies, but change as shown in Fig. 118, depending upon whether the mid-series or mid-shunt values are used. Mid-series impedance decreases slowly from its nominal or characteristic value to zero as the cut-off frequency f_c is approached, after which it changes to a pure reactance of increas-

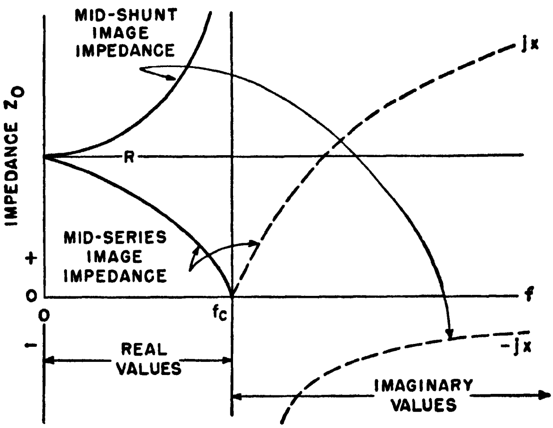


FIG. 118. Low pass filter impedance changes above and below cut-off frequency.

ing value with frequency. Mid-shunt impedance increases to infinity at cut-off, beyond which it is a pure reactance of decreasing value.

69. Filter Design Charts. Filter design information and qualitative performance are shown in Fig. 119 for the more common types of filters. The T-configurations shown are a half section followed by a whole sec-

tion and terminated by a half section. The cut-off frequency is designated as f_c, and the frequencies at which attenuation peaks are theoretically infinite are designated as f_∞.

70. Constant-*K* Filters. The constant-K filter is one in which the product of the series and shunt impedances is a constant independent of frequency; that is,

$$Z_1 Z_2 = K^2 \tag{67}$$

where K is a constant. For pure reactance arms, equation 67 becomes

$$K = \sqrt{\frac{L}{C}} = R \tag{68}$$

Equation 68 is the identical expression for the characteristic impedance for a lossless transmission line with unit inductance L and unit capacitance C. Therefore the value of R in this equation may be considered as the characteristic impedance of the filter. Constant-K filters are simple and are used widely. Filters 1, 4, 9, and 12 in Fig. 119 are constant-K filters.

For constant-K filters, equation 66 can be written:

$$\frac{-Z_1}{4Z_2} \quad \begin{matrix} \left(\dfrac{f}{f_c}\right)^2 & \text{for low pass filters} \\ \left(\dfrac{f_c}{f}\right)^2 & \text{for high pass filters} \end{matrix} \tag{69}$$

Equations 69 show that the abscissas of Figs. 116 and 117 for constant-K filters are f^2/f_c^2 or f_c^2/f^2 for low or high pass filters.

Because the image impedance changes so much with frequency, it is difficult to terminate a constant-K filter for sharp cut-off. Large reflection losses occur at or near cut-off. For this reason, the use of constant-K filters may not be feasible if the attenuation must change from a very small to a very large amount in a narrow range of frequencies.

71. *m*-Derived Filters. Filters 2, 3, 5, 6, 7, 8, 10, and 11 in Fig. 119 are m-derived filters. They are characterized by the same nominal image impedance as those of the constant-K prototype, but their attenuation, phase, and impedance vary differently with frequency. The configurations of these filters are in general different from constant-K filters. The relationships between the various elements of m-derived sections and their constant-K prototypes are shown in Fig. 119. The value of m is found from

$$m = \sqrt{1 - a^2} \tag{70}$$

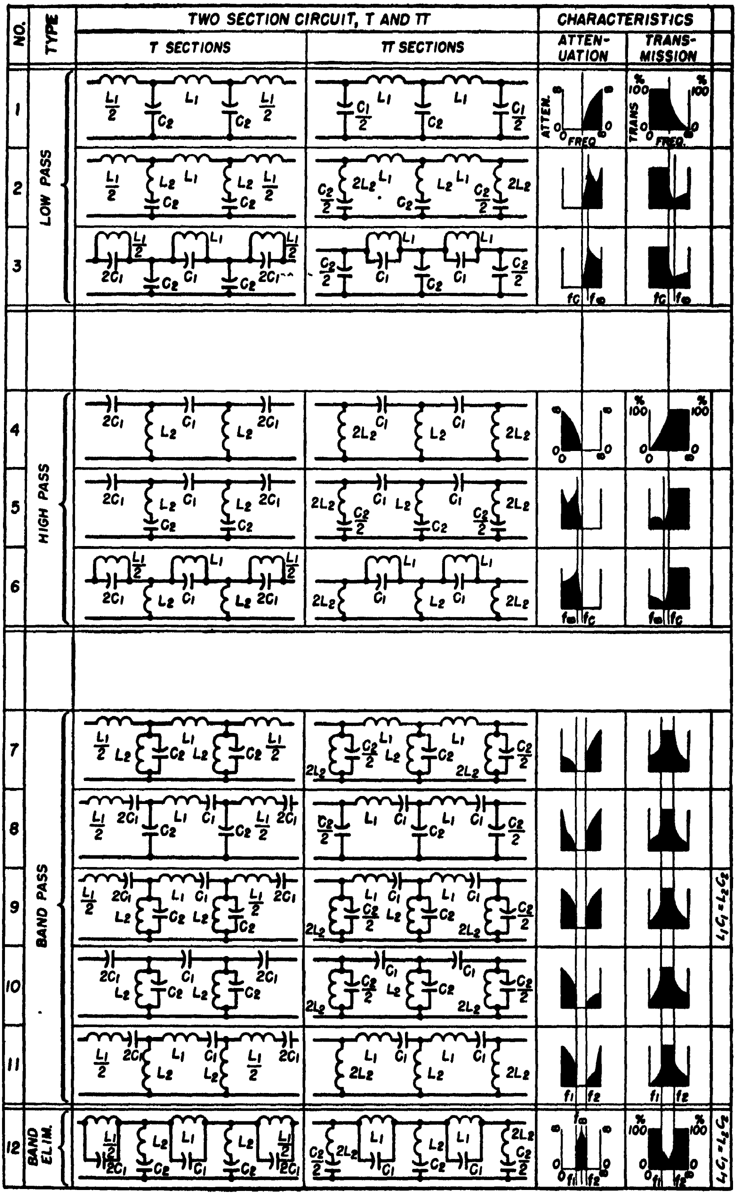


FIG. 119. Wave filter

CUT OFF POINTS FROM FILTER CONSTANTS				CONSTANTS FROM CUT OFF			
$a=\frac{f_\infty}{f_c}$	f_c	f_∞	CHAR IMPEDANCE AT ZERO $f=Z_0$	L_1	L_2	C_1	C_2
∞	$\frac{1}{\pi\sqrt{L_1C_2}}$	∞	$\sqrt{\frac{L_1}{C_2}}$	$L=\frac{Z_0}{\pi f_c}$			$C=\frac{1}{\pi f_c Z_0}$
$\sqrt{1+\frac{L_1}{4L_2}}$	$\frac{1}{\pi\sqrt{C_2(L_1+4L_2)}}$	$\frac{1}{2\pi\sqrt{L_2C_2}}$	$\sqrt{\frac{L_1}{C_2}}$	$L_1=mL$	$L_2=\frac{1-m^2}{4m}L$		$C_2=mC$
$\sqrt{1+\frac{C_2}{4C_1}}$	$\frac{1}{\pi\sqrt{L_1(C_2+4C_1)}}$	$\frac{1}{2\pi\sqrt{L_1C_1}}$	$\sqrt{\frac{L_1}{C_2}}$	$L_1=mL$		$C_1=\frac{1-m^2}{4m}C$	$C_2=mC$
$a=\frac{f_c}{f_\infty}$			CHAR IMPED AT INFINITE FREQ $=Z_0$				
∞	$\frac{1}{4\pi\sqrt{L_2C_1}}$	0	$\sqrt{\frac{L_2}{C_1}}$		$L=\frac{Z_0}{4\pi f_c}$	$C=\frac{1}{4\pi f_c Z_0}$	
$\sqrt{1+\frac{C_2}{4C_1}}$	$\frac{1}{4\pi}\sqrt{\frac{1}{L_2C_1}+\frac{4}{L_2C_2}}$	$\frac{1}{2\pi\sqrt{L_2C_2}}$	$\sqrt{\frac{L_2}{C_1}}$		$L_2=\frac{L}{m}$	$C_1=\frac{C}{m}$	$C_2=\frac{4m}{1-m^2}C$
$\sqrt{1+\frac{L_1}{4L_2}}$	$\frac{1}{4\pi}\sqrt{\frac{1}{L_2C_2}+\frac{4}{L_1C_1}}$	$\frac{1}{2\pi\sqrt{L_1C_1}}$	$\sqrt{\frac{L_2}{C_1}}$	$L_1=\frac{4m}{1-m^2}L$	$L_2=\frac{L}{m}$	$C_1=\frac{C}{m}$	
LOWER FREQUENCY f_1	HIGHER FREQUENCY f_2	f_∞	CHAR IMPED. Z_0 T SECTION AT $f_m=\sqrt{f_1f_2}$	L_1	L_2	C_1	C_2
$\frac{1}{2\pi\sqrt{L_2C_2}}$	$\frac{1}{2\pi}\sqrt{\frac{L_1+4L_2}{L_1L_2C_2}}$	∞	$\sqrt{\frac{L_1(L_1+4L_2)}{4L_2C_2}}$	$\frac{Z_0}{\pi(f_1+f_2)}$	$\frac{(f_2-f_1)Z_0}{4\pi f_1^2}$		$\frac{1}{\pi(f_2-f_1)Z_0}$
$\frac{1}{2\pi\sqrt{L_1C_1}}$	$\frac{1}{2\pi}\sqrt{\frac{C_2+4C_1}{L_1C_1C_2}}$	∞	$\sqrt{\frac{L_1(C_2+4C_1)}{4C_1C_2}}-\sqrt{\frac{L_1}{4C_1}}$	$L_1=\frac{Z_0}{\pi(f_2-f_1)}$		$C_1=\frac{f_2-f_1}{4\pi f_1^2Z_0}$	$C_2=\frac{1}{\pi(f_1+f_2)Z_0}$
$\frac{1}{2\pi}\left[\sqrt{\frac{1}{L_1C_2}+\frac{1}{L_1C_1}}-\frac{1}{\sqrt{L_1C_2}}\right]$	$\frac{1}{2\pi}\left[\sqrt{\frac{1}{L_1C_2}+\frac{1}{L_1C_1}}+\frac{1}{\sqrt{L_1C_2}}\right]$	$0\ \infty$	$\sqrt{\frac{L_2}{C_1}}=\sqrt{\frac{L_1}{C_2}}$	$L_1=\frac{Z_0}{\pi(f_2-f_1)}$	$L_2=\frac{f_2-f_1}{4\pi f_1f_2}Z_0$	$C_1=\frac{f_2-f_1}{4\pi f_2f_1Z_0}$	$C_2=\frac{1}{\pi(f_2-f_1)Z_0}$
$\frac{1}{2\pi\sqrt{L_2(C_2+4C_1)}}$	$\frac{1}{2\pi\sqrt{L_2C_2}}$	0	$\sqrt{\frac{L_2}{C_1}\left[1-\frac{C_2}{4C_1}\right]}$		$L_2=\frac{f_2-f_1}{4\pi f_1f_2}Z_0$	$C_1=\frac{f_1+f_2}{4\pi f_1f_2Z_0}$	$C_2=\frac{f_1}{\pi f_2(f_2-f_1)Z_0}$
$\frac{1}{2\pi\sqrt{C_1(L_1+4L_2)}}$	$\frac{1}{2\pi\sqrt{L_1C_1}}$	0	$\frac{\sqrt{L_1+4L_2}-\sqrt{L_1}}{2\sqrt{C_1}}$	$L_1=\frac{f_1Z_0}{\pi f_2(f_2-f_1)}$	$L_2=\frac{(f_1+f_2)Z_0}{4\pi f_1f_2}$	$C_1=\frac{f_2-f_1}{4\pi f_1f_2Z_0}$	
$\frac{\sqrt{L_1C_2+16L_1C_1}-\sqrt{L_1C_1}}{8\pi L_1C_1}$	$\frac{\sqrt{L_1C_2+16L_1C_1}+L_1C_1}{8\pi L_1C_1}$	$\frac{1}{2\pi\sqrt{L_1C_1}}$	$\sqrt{\frac{L_1}{C_2}}=\sqrt{\frac{L_2}{C_1}}$	$L_1=\frac{f_2-f_1}{\pi f_1f_2}Z_0$	$L_2=\frac{Z_0}{4\pi(f_2-f_1)}$	$C_1=\frac{1}{4\pi(f_2-f_1)Z_0}$	$C_2=\frac{f_2-f_1}{\pi f_1f_2Z_0}$

characteristics.

where a is the ratio of cut-off frequency f_c to frequency of maximum attenuation f_∞ for low pass filters and the reciprocal of this ratio for high pass filters. Filters 2 and 5 are *series-derived;* filters 3 and 6,

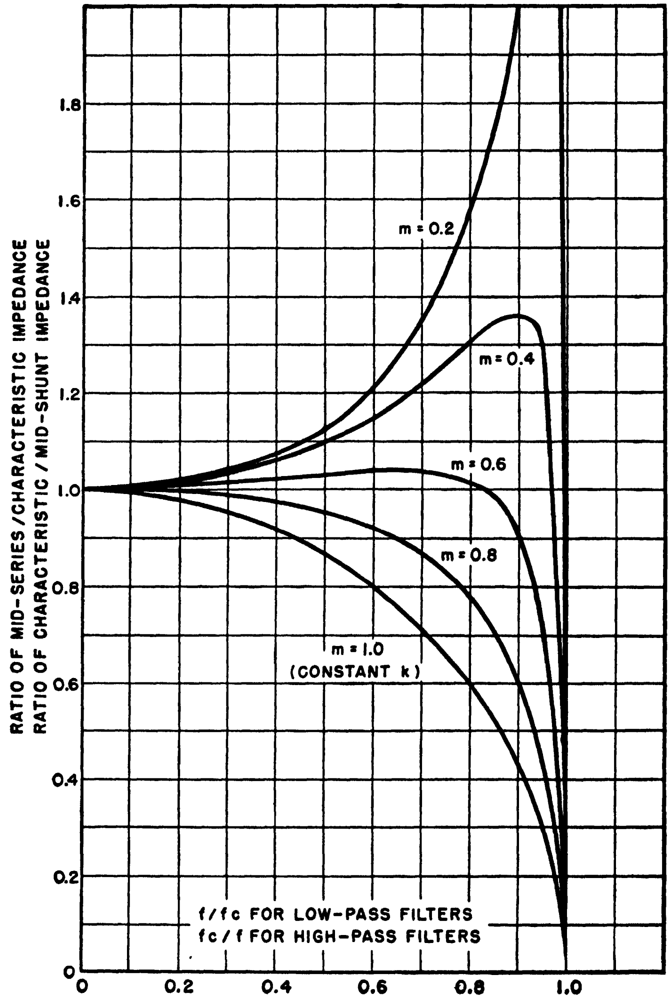


FIG. 120. Variation of image impedance in transmission band for m-derived filters.

shunt-derived. The variation of impedance is shown in Fig. 120 for several different values of m. These curves apply only to the mid-shunt image impedance of series-derived filters and to the mid-series impedance of shunt-derived filters. For such filters, with $m = 0.6$, the impedance is nearly constant over most of the transmission band.

For other m-derived filters the impedance varies as for constant-K filters. In the attenuation band, attenuation becomes infinite for pure reactance elements at frequency f_∞, after which it drops below that of its constant-K prototype. The attenuation at f_∞ is limited mainly by the Q of the chokes.

72. Limitations of Wave Filters. Several factors modify the performance of wave filters, shown in Figs. 116 and 117, especially in the cut-off region. One is the reflection due to mismatch of the characteristic impedance.[3] The load resistor is usually of constant value, whereas the image impedance drops to zero at cut-off for the usual type of filter. The resulting reflections cause a rounding of the attenuation curve in the cut-off region similar to that shown on curve V of Fig. 121. This effect is more pronounced in constant-K filters than in the favorable m-derived filters mentioned above, because the impedance deviates more from the proper value. It should not be supposed, however, that the addition of an m-derived terminating half-section to a chain of constant-K sections eliminates reflection loss at cut-off. The image impedance of each constant-K section varies like the $m = 1.0$ curve of Fig. 120, and the terminating m-derived section causes reflections in the same way as the constant resistance R. If the whole filter is composed of m-derived sections, sharp cut-off is maintained. This does not preclude the compensating effect of varying impedance in certain m-derived half-sections at frequencies near cut-off.

Another cause of rounding is the Q of the filter chokes. Curve IV of Fig. 121 shows typical rounding caused by choke loss. Figures 122 and 123 give the attenuation at cut-off in terms of Q.

Still another cause of the gradual slope of cut-off is the practice of inserting a resistor to simulate the source impedance in attenuation tests. Typical cases in which the source and terminating resistances are equal have the effect shown by curves I and IA in Fig. 121. The correct prediction of filter response near cut-off requires a good deal of care. It cannot be taken from the usual attenuation charts.

Phase shift is nearly linear with frequency for constant-K filters in the transmission band, but is non-linear for m-derived filters, especially those with smaller values of m. This fact is important in connection with networks used for the transmission of steep wave fronts, as in video amplifiers.

[3] See "An Analysis of Constant-K Low- and High-pass Filters," by O. S. Meixell, *RCA Review*, Vol. V, Jan. 1941, p. 337. Also "Single-Section m-Derived Filters," by C. W. Miller, *Wireless Engineer*, Vol. 21, Jan. 1944, p. 4.

In band-pass filters, the effects just noticed are present, with the additional complication of band width. The filter designer must choose a band width of transmission such that high attenuation is

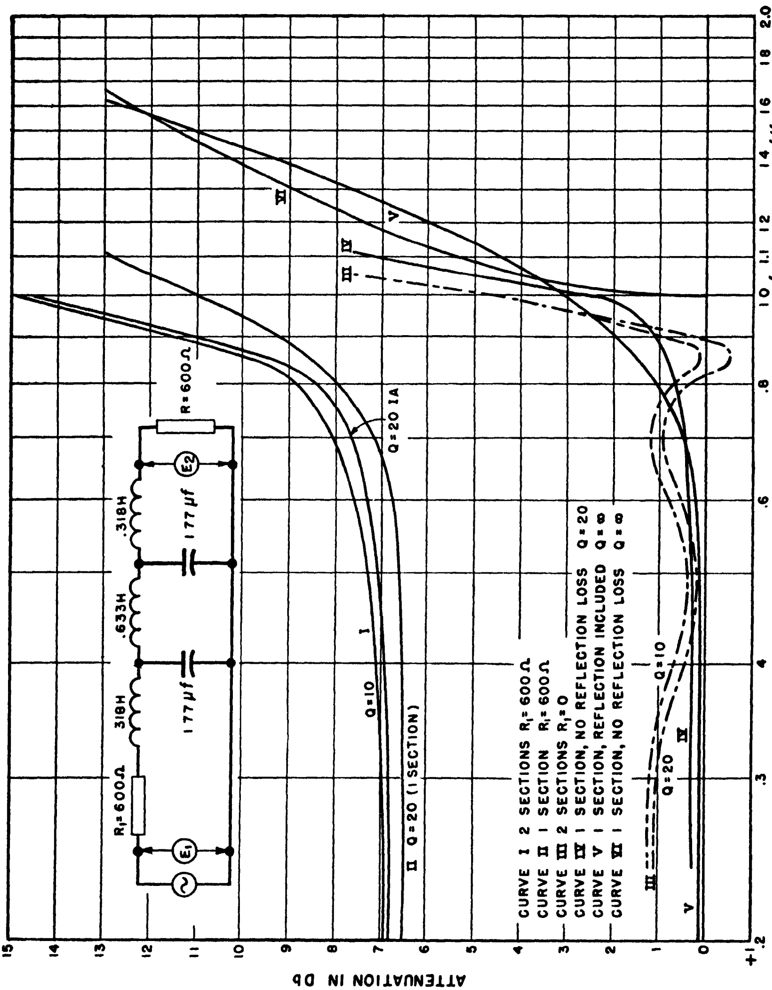


Fig. 121. Influence of reactor Q and reflections on sharpness of cut-off.

afforded at unwanted frequencies, and low attenuation at desired frequencies. This is often not a simple choice, as can be seen from Fig. 124. In this figure (*a*) shows how, for a given frequency separation from the mid-frequency, attenuation decreases as the filter band width is made wider. In (*b*) of this figure the impedance variation is much less with a wider band width. Therefore choosing a narrow band width attenuates frequencies in the transmission band because

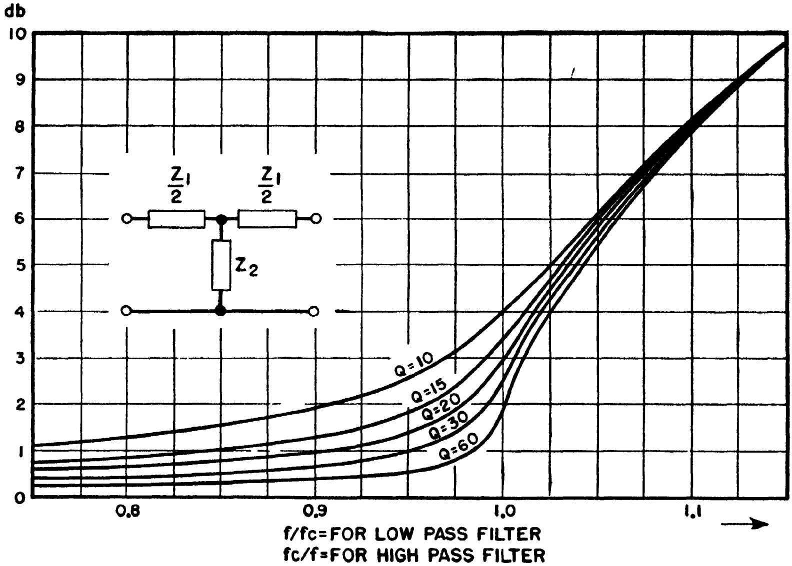


FIG. 122. Insertion loss near cut-off of a constant-K filter section.

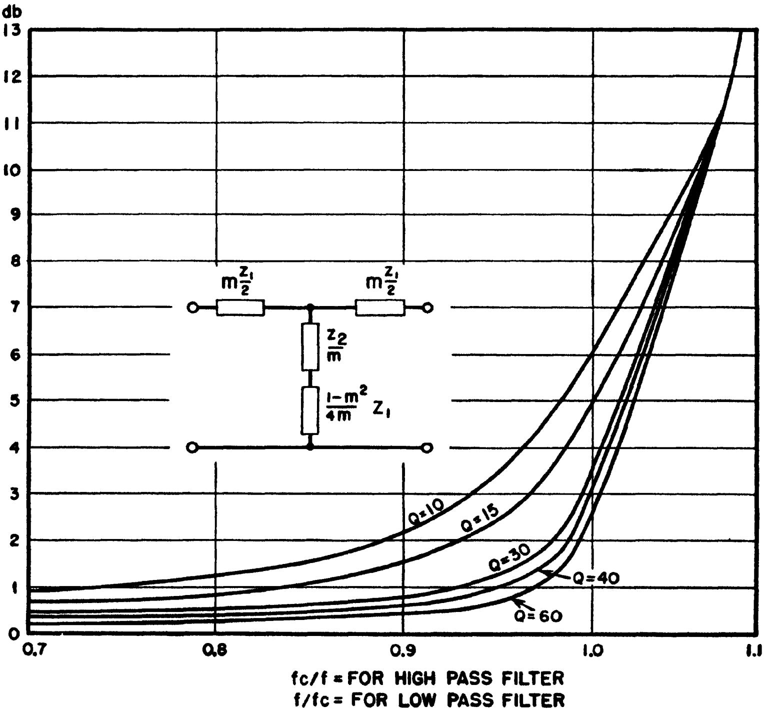


FIG. 123. Insertion loss near cut-off of an m-derived filter ($m = 0.6$).

of reflections. The use of m-derived sections is of benefit, as is evident from Fig. 124 (*b*), but here again at attenuation frequencies beyond f_∞ the attenuation may not be sufficient. m-Derived terminating half-sections for band-pass filter 9, Fig. 119, are complicated, difficult to adjust, and seldom justified.

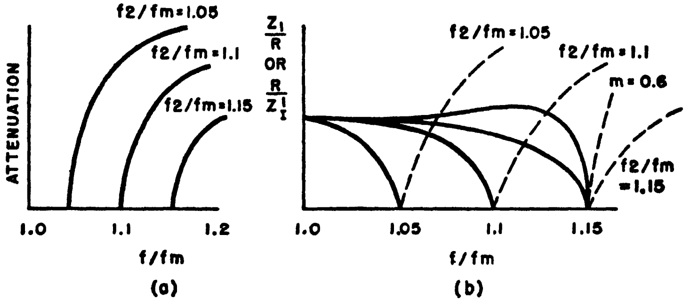


FIG. 124. (*a*) Attenuation and (*b*) impedance of band-pass filters.

73. Artificial Lines. Sometimes a certain amount of time delay must be interposed between one circuit and another. Or if the length of a transmission line is not an exact multiple of 90 degrees, some means must be found to increase its length to the next higher multiple of 90 degrees. For either of these purposes, artificial lines are used. They may operate at a single frequency or over a range of frequencies.

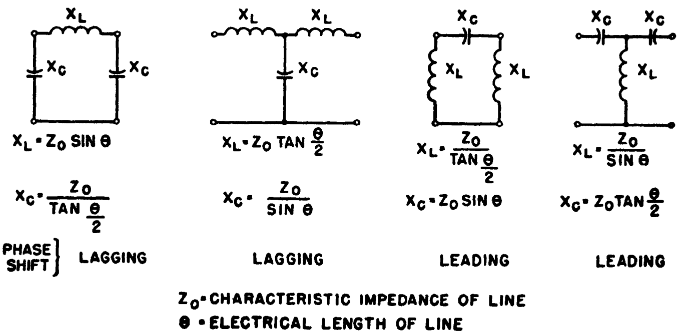


FIG. 125. Artificial line relations.

They may be tapped for adjustment to suit any frequency in a given range, so that impedance and line length are correct. The configuration may be either T or π, high- or low-pass. Figure 125 shows these four combinations for any electrical length θ of line section in degrees. It is assumed in this figure that the line operates at a single frequency

and is terminated in a pure resistance equal in value to the line characteristic impedance Z_0. Figure 126 is the vector diagram for a leading phase shift pi-section line of 90 degrees electrical length. Proportions of L and C are somewhat different in these line sections than in wave filters designed according to Fig. 119.

To obtain approximately constant time delay over a range of frequencies, several constant-K low pass filter sections may be used, each

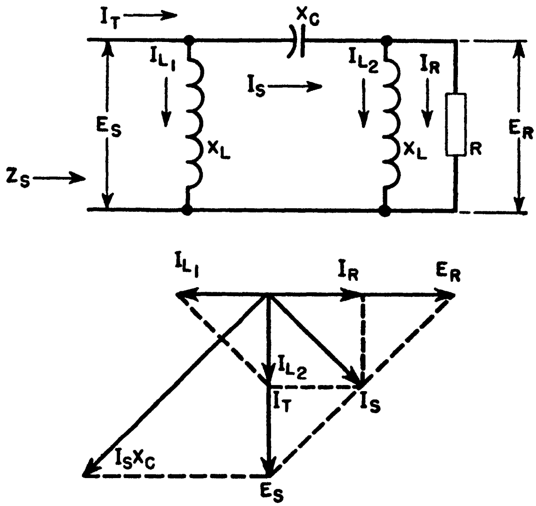


FIG. 126. Vector diagram for 90-degree line length.

having a cut-off frequency high enough so that the phase shift is proportional to frequency. The time delay per section is then $\theta/2\pi f$ at any frequency in the range, and $\theta = 2\pi f\sqrt{LC}$ where θ is the phase shift in radians, L is the inductance per section, and C is the capacitance per section.

74. Filter Inductor Design. Inductors for wave filters must have Q great enough to provide low attenuation at f_c and to provide high attenuation at f_∞. In design, attention must be given as much to Q as to inductance.

Low loss core material is essential for high Q. Nickel-iron alloys are widely used; the lamination thickness depends on frequency in approximately the same manner as Fig. 17 (p. 21) shows for Hipersil. Usually filters operate at low voltage levels, and low flux densities are used. Figure 127 shows approximate core loss for core steels 0.014 and 0.005 in. thick. Powdered iron cores have advantages at audio and higher frequencies.

Core gaps are used in filter reactors to obtain better Q. For any core, inductance per turn, and frequency, there is a maximum value of Q. The reason for this is that the a-c resistance is composed of at least two elements: the winding resistance and the equivalent core loss. In previous chapters the core loss has been regarded as an equivalent

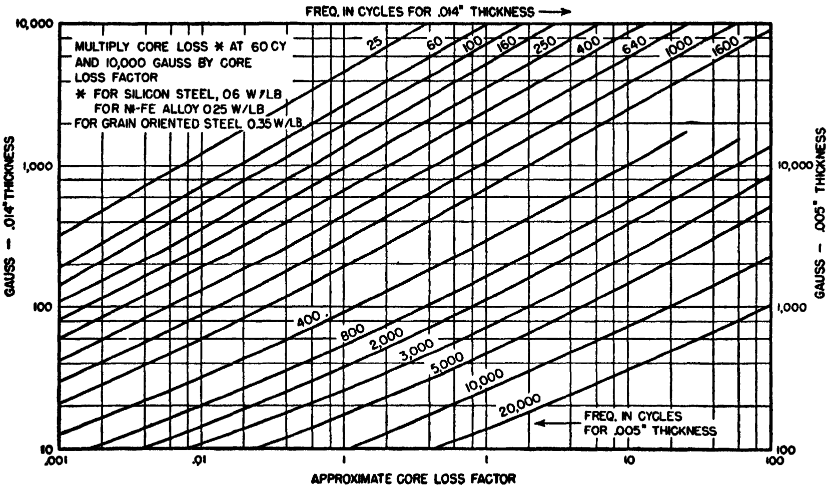


FIG. 127. Core loss in laminations 0.014 and 0.005 in. thick.

resistance *across* a winding. But it can also be regarded as an equivalent resistance *in series* with the winding. Figure 128 shows this equivalence, which may be stated:

$$R_{ser} + jX = \frac{jR_{sh}X}{R_{sh} + jX}$$

For values of $Q > 5$,

$$R_{sh} \approx \frac{X^2}{R_{ser}} \tag{71}$$

where R_{sh} = equivalent shunt resistance
R_{ser} = equivalent series resistance
X = winding reactance = $2\pi fL$.

The equivalence depends upon frequency. The formula for Q may then be changed to

$$Q = \frac{R_{sh}}{X} = \frac{R_{sh}}{2\pi fL} \tag{72}$$

or Q is proportional to shunt resistance, the winding resistance being neglected. Thus Q can be increased by lowering L. Usually the value

of L is determined from circuit considerations but, in a given reactor, the Q can be improved by increasing the core gap, up to a certain amount. As the air gap increases the flux across it fringes more and more, like that shown in Fig. 129, and L ceases to be inversely proportional to the gap. Some of the fringing flux strikes the core perpendicular to the laminations and sets up eddy currents which cause

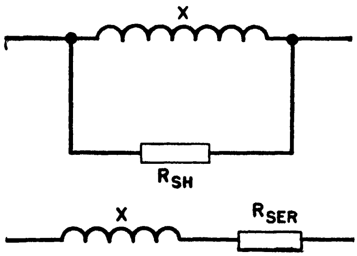


FIG. 128. Shunt and series equivalent core loss resistance.

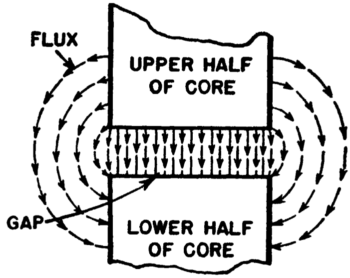


FIG. 129. Magnetic flux fringe at core gap.

additional loss. Accurate prediction of gap loss is not always possible. In general, it can be estimated from

$$W_g = Gl_g dfB_m^2 \text{ watts} \tag{73}$$

where G = a constant depending on the grade of core steel (about 10^{-8} for 0.014-in. silicon steel)
d = lamination tongue width in inches
f = frequency in cycles
B_m = peak induction in gauss
l_g = gap length in inches.

Optimum gap is that at which the gap loss and core loss are equal. These losses can be compared to the winding resistance by finding the equivalent core and gap loss resistances, and converting them to series resistances by equation 71. Optimum overall design for a given rating is that at which winding, core, and gap losses are equal. If this triple equality does not give the required Q, size must be increased. In powdered iron cores there is no gap loss.

75. Modulation Transformers. In plate-modulated power amplifiers, the modulator power required to produce 100 per cent modulation is half of the power amplifier input. Increased quality and reduction in size of components are achieved through the use of what may be called the pi-filter method.

For low modulation frequencies, this method may be illustrated by means of the circuit diagram of Fig. 130 (*a*). The modulator usually is a class B amplifier. Output transformer *OCL*, coupling capacitor,

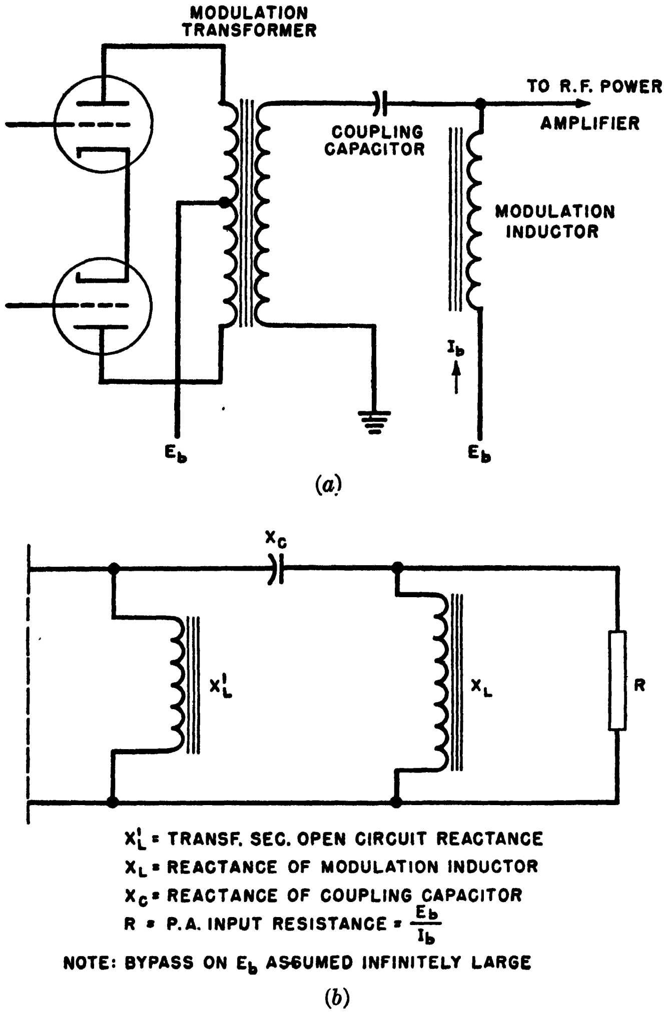


FIG. 130. (*a*) Circuit diagram of anode modulation system; (*b*) equivalent pi-filter modulator tube load.

and modulation reactor combine to form a pi-section high pass filter (Fig. 130 *b*). The elements are proportioned for characteristic impedance equal to the equivalent plate input resistance E_B/I_B of the modulated power amplifier.

Formerly these components were made as large as was considered practical. Transformer secondary and reactor reactances were each three to four times the power amplifier plate input resistance, and the coupling capacitor reactance was a fraction of this resistance, at the lowest modulation frequency. Advantages of the pi-filter are a reduction of the two inductive reactances to 1.41 times the terminating load resistance, and increase in capacitive reactance to the same value, at a low frequency f_1, which is 1.41 times cut-off frequency of the filter.

Down to f_1 the filter maintains a tube load of almost 100 per cent power factor, although the ohmic value rises to 190 per cent of the terminating load resistance at f_1. The voltage required for constant output rises to 138 per cent of normal. Partly compensating for this defect is the tendency of class B amplifier tubes to deliver higher voltages with higher tube load impedances. Thus in a certain radio transmitter, the type 805 modulator tubes deliver 1035 peak volts per side into a normal tube load of 1860 ohms. At 30 cycles, the lowest audio frequency, the load impedance rises to 3600 ohms, and the voltage required for full output is 1440 volts. Plotting a 3600-ohm load line on the tube curves shows that 1275 volts will be delivered at 30 cycles, which is 1 db down from normal.

To obtain the same frequency response with the older "brute force" method, at least twice the values of transformer and reactor inductance and much more coupling capacitance would have been necessary. The voltage across the capacitor increases as the capacitance decreases, but surge voltages often exist across the coupling capacitor in service. The voltage rating was formerly determined more by these surges than by normal voltage. With the pi-filter method, the normal voltage at low frequencies cannot be greatly exceeded in service, owing to the limitation in voltage output of the tubes. Hence, the reduction in coupling capacitance is a real one, and is offset very little by increases in voltage rating.

These points are made clearer by reference to Fig. 131. Phase shift between transformer and load voltages is 90° at f_1. At cut-off (= 70 per cent of f_1) the tube voltage for constant output is 224 per cent of the load voltage. A tube would not deliver so much voltage with this type of load, especially when the power factor is so low. The corresponding capacitor voltage at cut-off frequency is 284 per cent of E_R; it would not be delivered either, for the same reason. The useful frequency range is higher than f_1.

Another advantage of the pi-filter over the older method is the high power-factor load down to frequency f_1. The maximum tube load phase angle above f_1 is 8°, and at frequencies above $3f_1$ the phase angle

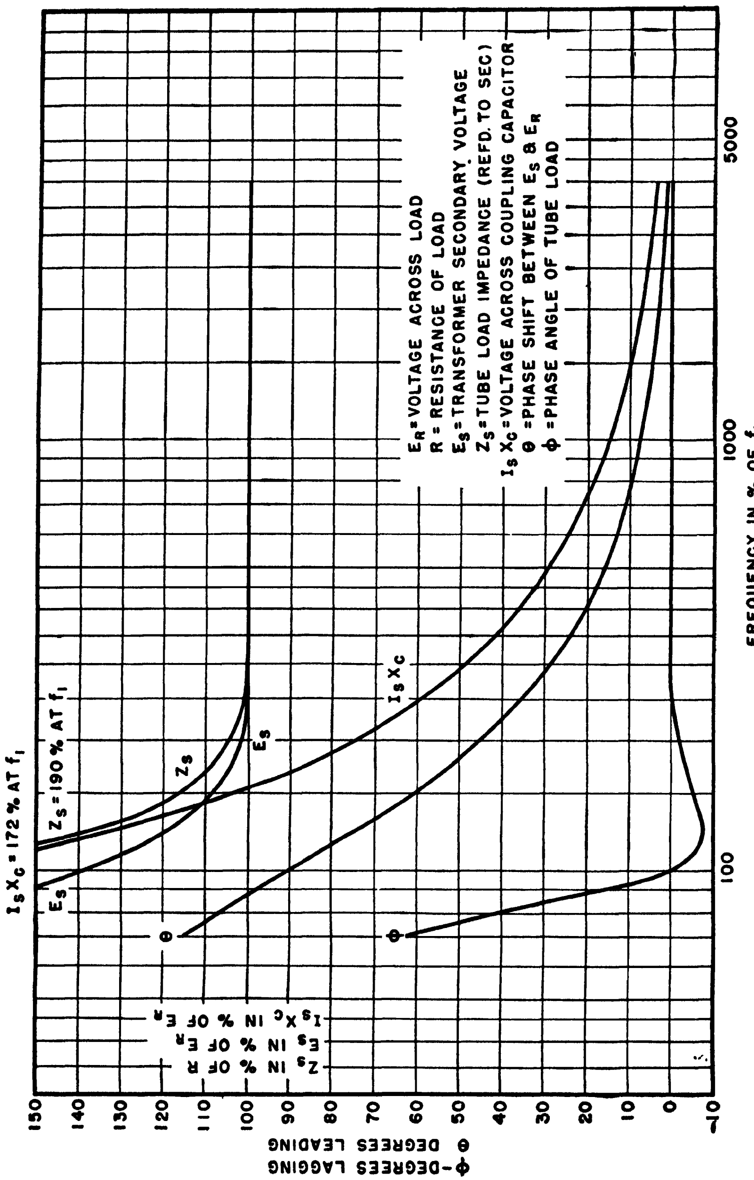


Fig. 131. Pi-filter basis of transformer design.

is zero. At $3f_1$ the tube load impedance is equal to R. Hence the tube works into a unity power-factor load of constant value at frequencies above $3f_1$. For the same size of inductive components, the "brute force" system would have been very much poorer. If these elements had been made twice as large in order to give the same frequency response at 30 cycles, the load phase angle at 30 cycles would be 35° and

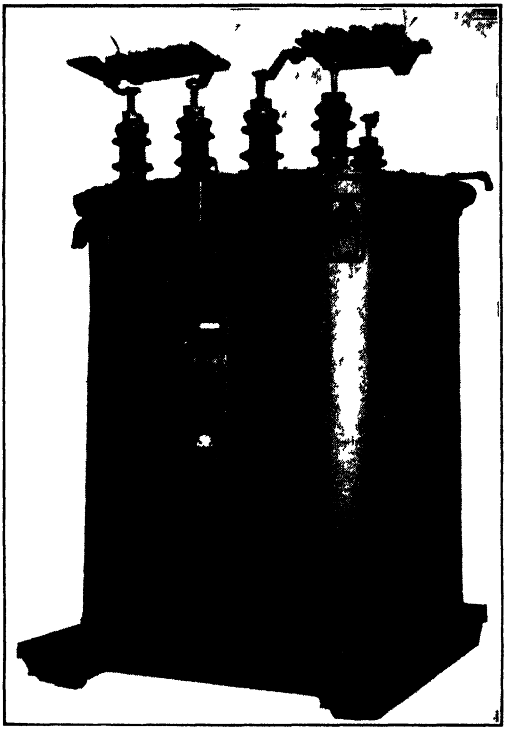


FIG 132 Oil-filled modulation transformer for a large broadcast station.

the load impedance 80 per cent of R. Hence the possibilities of low frequency distortion and low efficiency are reduced by the pi-filter method.

In very high fidelity modulators the lowest frequency is two to three times f_1 to reduce phase shift in connection with the inverse feedback.

The pi-filter just discussed conforms with the usual high pass filter in regard to values of the elements These elements can be proportioned on another basis such that the ohmic values of all the reactive elements are made equal to the load resistance at frequency f_1. These values are those of a 90° artificial line at frequency f_1. They give a unity power factor tube load equal to the modulated power amplifier

plate input resistance at frequency f_1, and thus represent 41 per cent increase in capacitor and 41 per cent reduction in transformer and reactor size. The disadvantages of the artificial line are that the tube load impedance drops to a minimum of 74 per cent R at the frequency $1.5f_1$, the maximum tube load phase angle is 16°, and it persists up to frequencies much higher than $3f_1$, which was the frequency where the response of the pi-filter became virtually perfect. This appreciable phase angle spread over a portion of the audio frequency range, together with the lower load impedance, causes distortion. The artificial line basis of design is used where larger amounts of distortion are not objectionable, or in fixed frequency modulators.

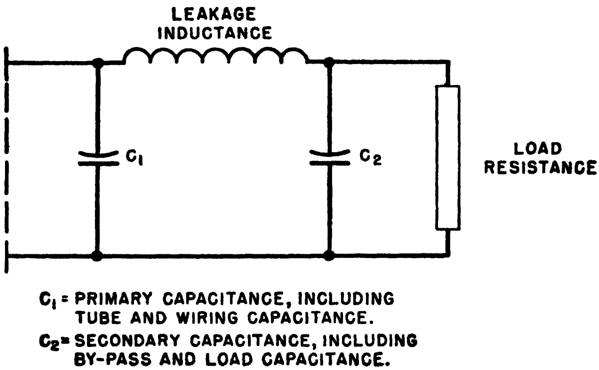


FIG. 133. Equivalent transformer diagram at high audio frequencies.

No matter what method of design is used, it is important that the modulator be loaded properly. If the power amplifier input should be interrupted while the modulator is fully excited, the voltages on the various elements are likely to rise to dangerous values, because the load line becomes flat and extends to indefinitely high voltages in the positive e_B direction. The transformer and reactor in high voltage modulators are equipped with spark gaps like those in Fig. 132 to prevent insulation breakdown due to accidental removal of load.

The pi-filter or artificial line method of design can be applied also at the higher modulation frequencies. Figure 133 shows how the usual properties of winding capacitance and leakage inductance are arranged to give a low pass filter circuit. Preferably the internal capacitance and inductance should be so low that external L and C can be added to the transformer terminals to produce the required proportions at the highest modulator frequency.[4] Figure 134 shows how both high

[4] See "An Analysis of Distortion in Class B Audio Amplifiers," by True McLean, *Proc. I.R.E.*, Vol. 24, March 1936, p. 487.

and low pass pi-filter performance can be combined in a modulator to obtain wide range, high fidelity performance. Although these methods apply chiefly to modulation transformers, they may be used in the design of loaded interstage transformers.

In high power modulation transformers, it is necessary to make the core larger in order to reduce the number of turns and obtain good high frequency response. But as the core becomes larger, so do the leakage inductance and winding capacitance. The core must grow

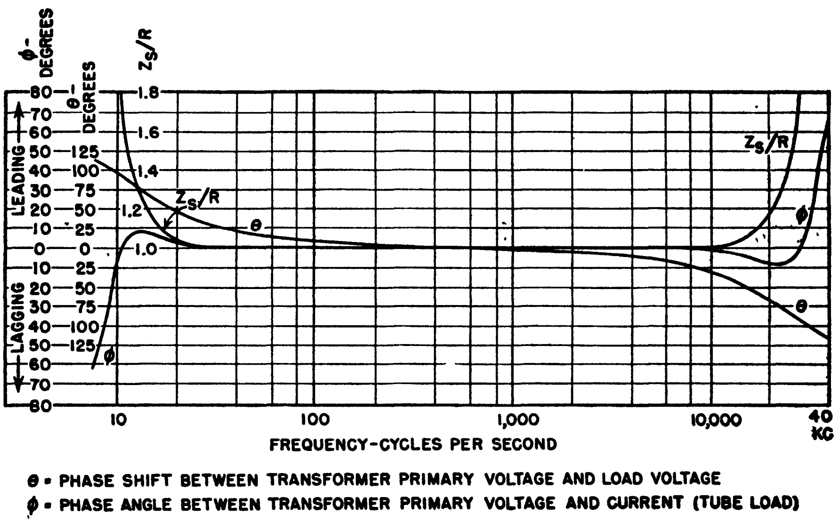


FIG. 134. High and low pass pi-filter performance in a modulator.

very large to overcome this difficulty, and high power audio transformers are much larger than commercial power transformers of the same rating. One advantage of grain-oriented steel is that it permits this process to be reversed. High induction at low frequencies means a smaller core area, smaller mean turn, and better high frequency performance, or, for the same high frequency performance, more turns and a still smaller transformer. Weight savings of 60 to 75 per cent have been made in this way.

Advantages of the pi-filter and artificial line methods are realized in transformers for 30 to 10,000 cycles; some advantage can be gained in the 100 to 5000 cycle range, but not below 100 watts. Below this size or for a narrower frequency range, the transformer becomes so small that combination with the modulation reactor into one unit is feasible and economical. The secondary current wave shape is like the first wave of Table I (p. 10). In such a transformer the core gap must be

large enough to prevent saturation by the power amplifier plate current.

76. Driver Transformers. Requirements for class B modulator driver transformers are unusually difficult to satisfy. The transformer load is non-linear, for grid current is far from sinusoidal. Although the average load is low, the driver tube must deliver instantaneous

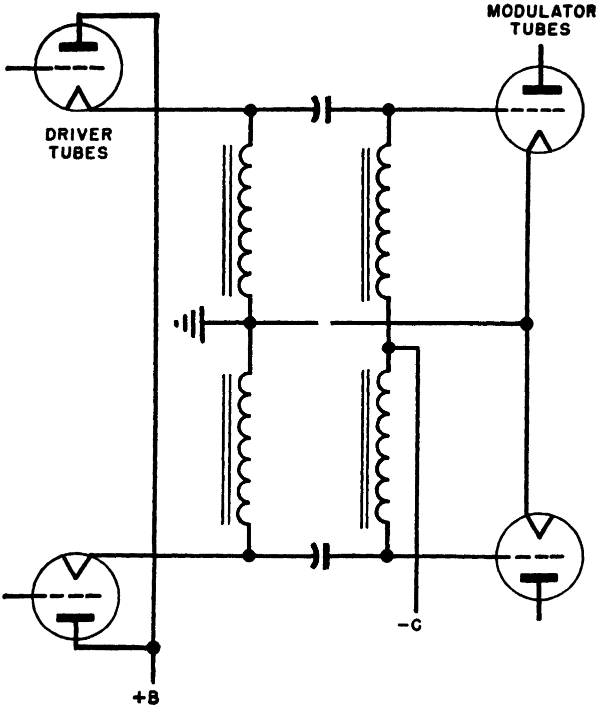


FIG. 135. Cathode follower driver circuit. (Courtesy of *Proceedings of the I.R.E.*, April 1945.)

current peaks; otherwise distortion will appear in the modulator audio output and therefore in the r-f envelope. The grid current peaks contain harmonic currents of higher order, and to insure their appearance in the modulator grid current an extension of the driver transformer frequency range at both ends is required: on the high frequency end because of the decreased leakage inductance necessary to allow the higher currents to flow, and on the low frequency end to prevent transformer magnetizing current, itself non-linear, from loading the driver tube so that it does not deliver the peak grid power. If the driver tube is a pentode or beam tube, it is usually loaded with resist-

FIG. 136. Rear view of exciter cubicle for 50-kw broadcast transmitter.

ance to minimize current variations. Driver transformers are usually step-down because the grid potentials are relatively low.

These conditions require transformers of exceptionally large size. For low (1 to 2 per cent) overall harmonic distortion, driver transformer design becomes impractical, and it is advantageous to dispense with driver transformers entirely. This is accomplished by the cathode follower circuit (Fig. 135), which for a push-pull amplifier takes the form of a symmetrical pi-filter. The two input chokes connect the driver tube cathodes to ground and carry their plate current. Coupling capacitors connect these chokes to the modulator tube grid chokes, which carry modulator grid current. Sizes of chokes and coupling capacitors are chosen to give approximately constant impedance from the lowest modulation frequency up to the higher harmonics of the highest frequency, and choke capacitance is reduced to preclude pronounced resonance effects throughout the frequency range. In Fig. 136, the filter components are mounted in the exciter cubicle; a transformer for this purpose would be too large to locate internally.

The cathode follower circuit is advantageous in another way. Leakage inductance in a driver transformer causes high frequency phase shift between driver and grid voltage, which does not exist in the coupling capacitor scheme. Since inverse feedback is often applied to audio amplifiers to reduce distortion, the absence of phase shift is a great advantage. The *low* frequency at which phase shift appears must be kept below the audio band, but this can be done without excessively large components.

77. Transformer-Coupled Oscillators. These have circuits similar to that of Fig. 137. Transformer *OCL* and capacitor C_1 form a tank circuit, to which are coupled sufficient turns to drive the grid in the lower left-hand winding. The output circuit is coupled by a separate winding. For good wave shape in such an oscillator, triodes and class B operation are preferable. The ratio of turns between anode and grid circuits is determined by the voltage required for class B operation of the tube as if it were driven by a separate amplifier stage. Single tubes may be used, because the tank circuit maintains sinusoidal wave shape over the half cycle during which the tube is not operating. Grid bias is obtained from the RC_2 circuit connected to the grid.

In such an oscillator, tube load equals transformer loss plus grid load plus output. In small oscillators, transformer loss may be an appreciable part of the total output. This loss consists of core, gap, and copper loss. Copper loss is large because of the relatively large tank current, and the wire size in the anode winding is larger than would be normally used for an ordinary amplifier. The gap is neces-

sary to keep the inductance down to a value determined by the tank circuit Q or volt-amperes. This in turn is dictated by the required harmonic content. The use and selection of core materials are approximately the same as those indicated in Section 74.

Class C oscillators are less desirable for very low harmonic requirements, because of the difficulty of designing tank circuits with sufficiently high Q. Where large harmonic values can be tolerated, the transformer can be designed for low Q, but the wave form becomes nonsinusoidal. Transformer grid circuit turns are large, approximately

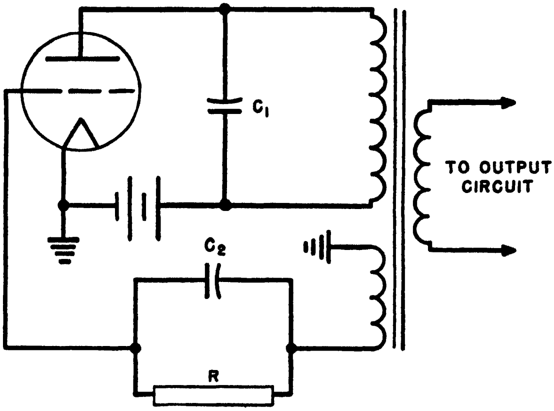


FIG. 137. Transformer-coupled audio oscillator.

the same as plate turns, and the grid voltage would be high if grid current did not limit the positive voltage swing. During the half cycle when the tube is operating, the voltage wave has a roughly rectangular shape, and during the rest of the cycle it peaks sharply to a high amplitude in the opposite direction. Core losses are difficult to predict because loss data are not normally available for such wave forms. Consequently, designs of this type are usually cut-and-try. The frequency of oscillation varies with changes in load; hence low Q class C oscillators are to be avoided if good frequency stability is required.

78. Shielding. A gain of 80 to 100 db is often reached in high gain amplifiers. It is important in these amplifiers that only the signal be amplified. Small amounts of extraneous voltage introduced at the amplifier input may spoil the quality or even make the received signal unintelligible. One source of extraneous voltage or hum is in stray magnetic fields emanating from power transformers in or near the amplifier. The stray fields enter the magnetic cores of input transformers and induce small voltages in the windings, which may be

amplified to objectionably high values by the amplifier. Several devices are used to reduce this hum pick-up:

1. The input transformer is located away from the power transformer.
2. The coil is oriented for minimum pick-up.
3. Magnetic shielding is employed.
4. Core-type construction is used.

The first expedient is limited by the space available for the amplifier but, since the field varies as the inverse cube of the distance from the source, it is obviously helpful to locate the input transformer as far

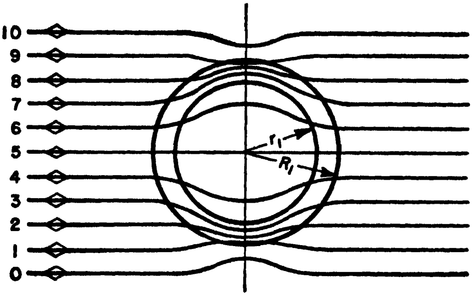


FIG. 138. Refraction of magnetic field by iron shield.

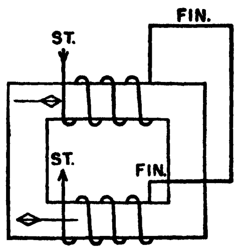


FIG. 139. Flux directions in a core-type transformer.

away from the power transformer as possible. The second method is to orient the coil so that its axis is perpendicular to the field. It requires extra care in testing. Magnetic shielding is the "brute force" method of keeping out stray fields; core-type construction is effective and does not materially increase the size. Of course, any of these methods increases manufacturing difficulties to a certain extent.

Magnetic shielding is ordinarily accomplished by a thick wall of ferrous metal or a series of thin, nesting boxes of high permeability material encasing the windings and core of the input transformer. Neither type of shield is applied to the power transformer because the flux lines originate at the power transformer and fan out in all directions from it. A large percentage of the flux would strike the shield at right angles and pass through it. On the other hand, the stray field near the input transformer is relatively uniform, and very few flux lines strike the shield at right angles. Thus more flux is bypassed by it. The action of a thick shield in keeping stray flux out of its interior is roughly illustrated in Fig. 138.

Multiple shields increase the action just mentioned because eddy currents induced in the shields set up fluxes opposing the stray field.

Sometimes alternate layers of copper and magnetic material are used for this purpose, when hum pick-up 50 or 60 db below the no-shield value is required.[5]

In core-type transformers the flux normally is in opposite directions in the two core legs, as shown in Fig. 139. A uniform external field, however, travels in the same direction in both legs, and induced voltages caused by it cancel each other in the two coils.

The relative effectiveness of these expedients is shown in Fig. 140. Hum pick-up is given in decibels with zero decibel equal to 1.7 volts across 500 ohms, and distance from a typical small power transformer as abscissas. All curves are for 500-ohm windings working into their proper impedances, and with no orientation for minimizing hum. Using impedances much less than 500 ohms reduces the hum picked up. Orientation of coil position also reduces hum. For all types of units, there is a position of minimum hum. With the unshielded shell type the angle between the transformer coil and the field is almost 90°, and is extremely critical. With shielding, this angle is less critical, but the minimum amount of hum picked up in this position is not noticeably reduced. The core type is less critical, especially with a shield. The minimum amount of hum picked up is from 10 to 20 db less than the shielded shell type in its minimum position. Removing the shields from the core type may change its position of minimum pick-up. This is because the shields reduce hum by a process different from that of the two bucking coils.

It is advantageous to have power transformers of the core type. Leakage fluxes from like coils on the two legs approximately cancel at a distant point. The U-and-I shape of lamination is preferable because of its symmetry.

Static shielding does not prevent normal voltage on a primary winding from being transferred inductively into a secondary winding. It is effective only against voltage transfer by interwinding capacitance. High frequency currents from vacuum tube circuits are thus prevented from flowing back into the 60-cycle power circuits via filament and plate transformers. Without shielding, such currents may interfere with operation of nearby receivers. Likewise, voltages to ground on telephone lines are kept from interfering with normal voice frequency voltages between lines. The extent of static shielding depends upon the amount of discrimination required. Usually a single, thin, grounded strip of metal between windings is sufficient, with ends insulated to

[5] See S. L. Gokhale, *Jour. A.I.E.E.*, Vol. 48, Oct. 1929, p. 770. Also, "Magnetic Shielding of Transformers at Audio Frequencies," by W. G. Gustafson, *Bell System Tech. Jour.*, Vol. 17, July 1938, p. 416.

prevent a short circuit. Magnetic flux in the interwinding space causes eddy currents to flow in such shields, and even shields with insulated ends indicate a partial short circuit on test. This effect

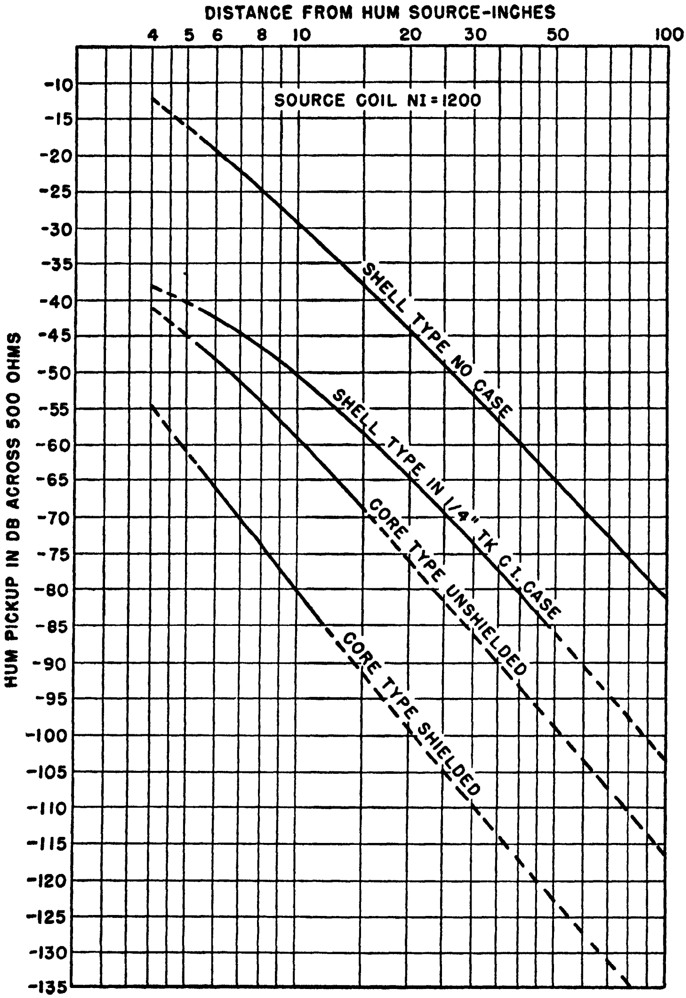


FIG. 140. Hum pick-up in input transformers.

reduces the *OCL* of the transformer. If volts per layer are small compared to total winding voltage, a layer of wire is an effective shield. The start turn is grounded and the finish left free, or vice versa. A wire shield has none of the short-circuit effect of a wide strip shield.

Usually a transformer that requires static shielding has a low voltage

winding; the shield can be placed close to this winding, needs little additional insulation, and occupies but a small fraction of the total coil space. If shields are placed between high voltage windings, as in modulation transformers, the shields must be insulated from each winding with thick insulation. This materially increases the coil mean turn length, transformer size, and difficulty in obtaining good high frequency response. Shields have questionable value in such transformers, and usually are omitted.

79. Hybrid Coils. Hybrid coils are used to isolate an unwanted signal from certain parts of a circuit, and allow the signal to be used in

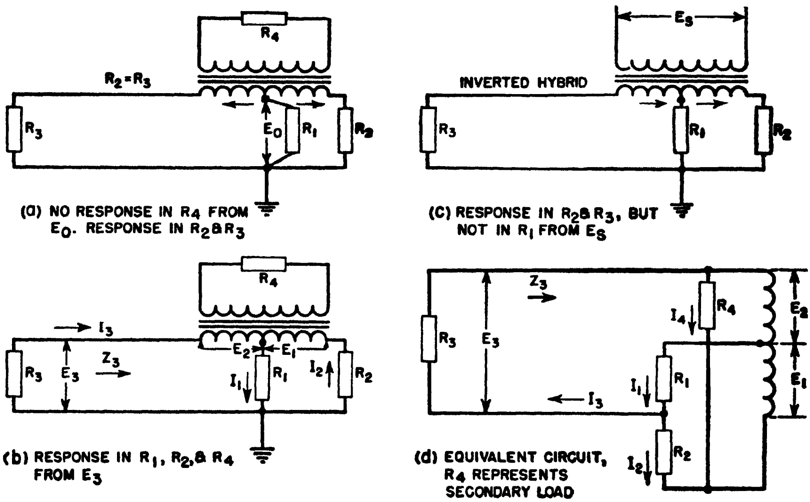


FIG. 141. Hybrid coil operation.

other parts of the circuit. In the hybrid coil shown in Fig. 141 (*a*) the lower windings or primary sections are balanced with respect to each other, and the two resistors R_2 and R_3 are equal. Voltage E_0 applied between the primary center tap and ground causes equal currents to flow in opposite directions through the two halves of the primary winding, and therefore produces zero voltage in the secondary winding. By this means, signal E_0 arrives at resistors R_2 and R_3 undiminished, but there is no voltage in R_4 connected across the secondary coil. Figure 141 (*b*) shows what happens in this circuit if the voltage is applied across R_3 instead of across R_1. In this case, the voltage E_3 appears across resistors R_1, R_2, and R_4; that is, in all parts of the circuit.

An inverted hybrid coil is shown in Fig. 141 (*c*). Here voltage E_s is applied across the upper coil, which is now the primary. The second-

ary sections are assumed to be balanced. Therefore, there is zero voltage between the center point of the secondary winding and ground, and though a signal appears at R_2 or R_3, there is no signal across R_1. Thus a hybrid coil works in both directions.

It has been assumed that R_2 and R_3 are equal and that the two primary half-windings are of equal number of turns. This is not necessarily the case, for if the resistance of R_2 is twice that of R_3, then the number of turns connected to R_2 should be twice those connected to R_3. However, it is important that through the range of frequency in which the hybrid coil is desired to function, the balance between the two halves be maintained closely. The most exact balance is achieved for $R_2 = R_3$ by winding the two halves simultaneously with two different wires. This method gives good isolation of the undesired signal. Other methods introduce some ratio error which reduces the isolation. For the same reason, it is necessary to balance the circuit with regard to capacitance and leakage inductance. That is, if a capacitance exists across R_3, such as line capacitance for example, then an additional equivalent amount should be added across R_2 in order to achieve the balance desired. Likewise, any inductive apparatus, adding either series or parallel inductance in one circuit, should be compensated for by inductance of like character in the other circuit. Adding series inductance, for example, in series with R_3 will not compensate for shunt inductance across R_2 or vice versa, as the two have opposite effects with regard to frequency and therefore balance is attained only at one frequency.

Assume a perfect transformer having no exciting current and no leakage inductance between the two halves, and a transformer with equal turns in the two halves of the primary winding. Assume currents in the directions shown in Fig. 141 (*d*). Then

$$I_1 = I_2 + I_3 \tag{74}$$

$$E_1 = I_2R_2 + I_1R_1 \tag{75}$$

$$E_3 = I_1R_1 + E_2 \tag{76}$$

On the assumption of equal turns in the two half-windings, $E_1 = E_2$. If the magnetizing current is assumed to be zero, the ampere-turns and hence the volt-amperes in the two primary halves are equal. The secondary load can be considered as reflected into the primary winding as resistor R_4.

$$I_4 = \frac{E_1 + E_2}{R_4} \tag{77}$$

$$(I_3 - I_4)E_2 = (I_2 + I_4)E_1 \tag{78}$$

If equations 74 to 78 are combined, an expression for Z_3 can be found:

$$Z_3 = \frac{E_3}{I_3} = \frac{4R_1R_2 + 4R_1R_4 + R_2R_4}{4R_1 + 4R_2 + R_4} \tag{79}$$

If the secondary circuit is open, $R_4 = \infty$, and equation 79 becomes

$$Z_3 = 4R_1 + R_2 \tag{80}$$

80. Amplifier Tests. Tests for hum, distortion, linearity, and frequency response can be made with meters in the output circuit when voltage of a known frequency and wave form is applied to the input. Hum and distortion are conveniently measured by instruments specially made for the purpose. Linearity is measured by varying the input voltage and measuring corresponding output voltage. Frequency response is measured similarly, but frequency is varied. Normal production testing of amplifiers requires no more than such overall tests. But, in the development of the amplifier, excessive hum, distortion, or other defects may be indicated, and tests must be applied stage by stage to locate the trouble. Voltage is usually measured by a tube voltmeter, one terminal of which is grounded. In a push-pull amplifier, it is therefore necessary to block the direct voltage and measure the alternating voltage on each side. A cathode-ray oscilloscope is helpful in checking wave form at various points.

Before being assembled in the amplifier, transformers are tested for *OCL*, winding resistance, core loss, and insulation strength. Although with new designs it is desirable to check leakage inductance and winding capacitance, these properties vary less in a given design than the others.

In the diagram of Fig. 142, the first two stages have current feedback, so initial tests were made with the circuit shown. But overall voltage feedback from the modulator plates back to the 6J7 grids was not applied until the amplifier was first tested without it. Then resistors from which feedback is derived were adjusted to produce the feedback voltage necessary to give the required performance. Also, the carrier power amplifier was completely adjusted before modulation was applied. Percentage of modulation was measured by the increase of carrier output current when modulation was applied. Inductance *RFC* and capacitor C_1 were added to maintain the modulator load constant at high frequencies. C_2 in this circuit is the audio coupling capacitor. Separate meters are provided to measure the plate current of each driver and modulator tube, so that bias may be adjusted for the same plate current on each side.

Proper operation is predicated on amplifier stability, which often is not obtained when power is first turned on. Local or parasitic oscilla-

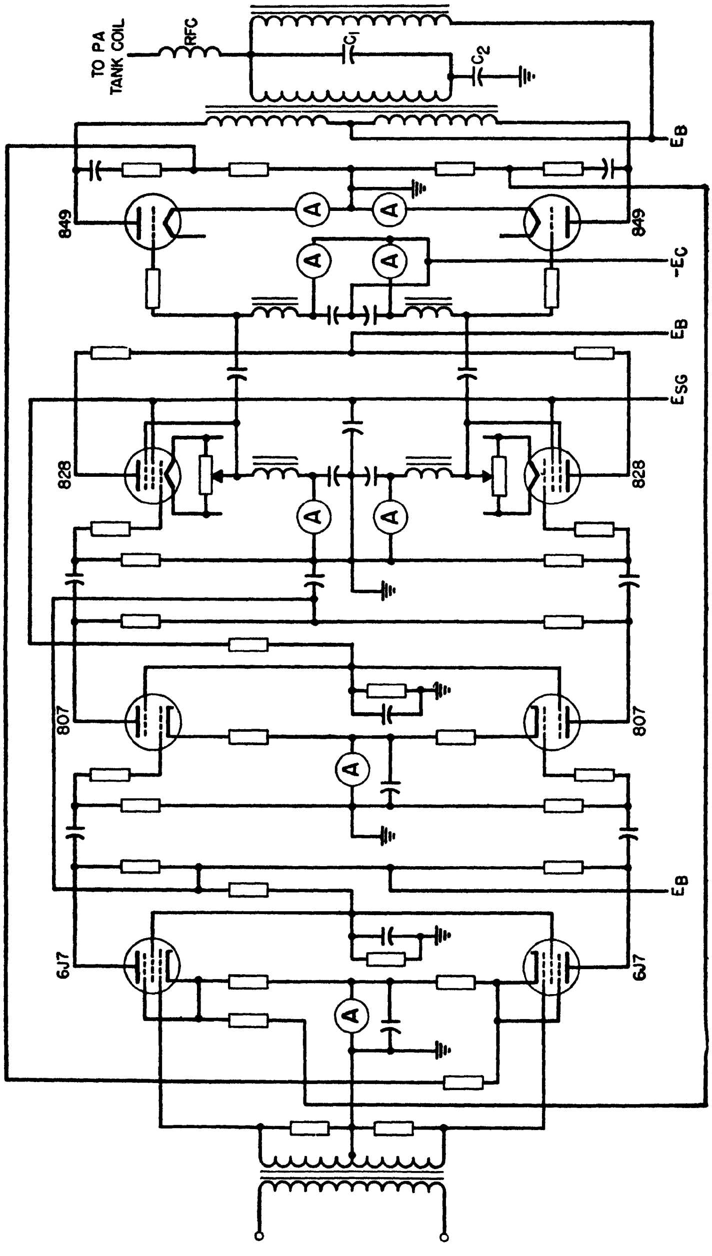


FIG. 142. Amplifier schematic diagram.

tions may easily occur as a result of natural resonance of circuit elements or even in connections and tube electrodes. These must be detected and eliminated by corrective measures which apply to the trouble. Some of these troubles may be caused by long leads, especially in the grid circuit. Tubes may require resistors in the plate and grid leads to damp out parasitic oscillations. Resistors are used in this manner in the amplifier shown in Fig. 142. Coils in circuits with widely different voltages should not be coupled closely, because regeneration may result. In circuits with high voltage, and therefore large capacitive currents, it may be necessary to add shielding to prevent stray pick-up from one stage to another. In push-pull amplifiers, if some circuit element is unbalanced, it may give rise to a push-push oscillation which can be eliminated by better balance, or by decoupling the tube plates at the unwanted frequency. If insufficient bypass capacity is used on plate or bias supplies, interstage coupling may occur at low frequencies. The frequency may be less than 1 cycle per second. This kind of instability is known as "motor-boating." Operating tubes so that some electrode becomes a negative resistance during a portion of the cycle may give rise to oscillations which cannot be prevented except by avoidance of the cause, or by some power-absorbing circuit which does not affect normal operation. The elimination of such trouble requires much testing time and skill, but it must be done before performance tests are made.

81. Design Examples.

Example (*a*). *Transformer for Pi-Filter Modulator.*
Frequency range 100 to 5000 cycles
Audio output 400 watts
Power amplifier E_B/I_B = 10,000 ohms
Voltage ratio primary/secondary (1180 + 1180)/2000
f_1 = 60 cycles
Core: 4-in. stack of silicon steel lamination B, Fig. 31 (p. 39)
Turns primary/secondary (800 + 800)/1380 No. 26 wire

A_c = 7.2 sq in. net; 8.0 sq in. gross
l_c = 12.75 in.
l_g = 0.012 in.

Possible tube current unbalance = 0.032 amp

$$B_{dc} = \frac{0.6 \times 800 \times 0.032}{0.012} = 1260 \text{ gauss}$$

$$B_{ac} = \frac{3.49 \times 2000 \times 10^6}{100 \times 7.2 \times 1380} = 7000 \text{ gauss}$$

$$B_m = 8260 \text{ gauss.}$$ From Fig. 54 (p. 76), μ_Δ = 9000+

$$\text{Secondary } OCL = \frac{3.2 \times (1380)^2 \times 8 \times 10^{-8}}{0.012 + \dfrac{12.75}{9000}} = 36.5 \text{ henrys}$$

X_L at $f_1 = 6.28 \times 60 \times 36.5 = 13{,}800$ ohms. $Z_s = 115$ per cent R at 100 cycles from Fig. 131. Winding arrangement as in Fig. 143, to reduce layer voltage and capacitance.

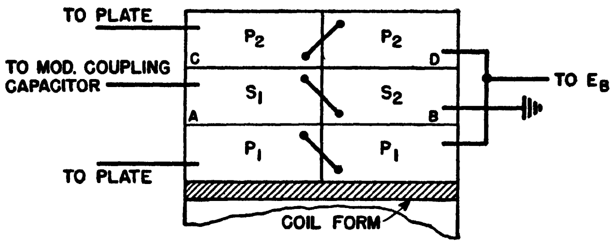


FIG. 143. Section of transformer coil wound for low layer voltage.

Winding resistances: Total primary 90 ohms
Secondary 80 ohms
Secondary leakage inductance = 53 millihenrys
Capacitances (referred to secondary):

$P_1 - S_1$(at A)	=	292 $\mu\mu$f
$P_1 - S_2$(at B)	=	0
$P_2 - S_1$(at C)	=	112
$P_2 - S_2$(at D)	=	58
Secondary layer to layer	=	140
Primary layer to layer	=	170
Stray and tube	=	50
Power amplifier r-f bypass	=	500
Total	=	1322 $\mu\mu$f

At high audio frequencies $f_r = 19{,}000$, $D = 0.6$, and Z/R_2 from Fig. 101 (p. 130) is 87 per cent at 5000 cycles.

Example (b). Audio Oscillator.
Circuit of Fig. 137, with 6C5 tube, $E_B = 150$ volts, $E_C = -10$ volts
Frequency 800 cycles
Plate load impedance = 20,000 ohms
Class B operation; grid swings to +2 volts

$$\left.\begin{aligned} \Delta e_g &= -12 \text{ volts} \\ \Delta e_p &= 150 - 35 = 115 \text{ volts} \\ \Delta i_p &= 5.6 \text{ ma} \end{aligned}\right\} \text{during positive half-cycle}$$

Average power output = $(115 \times 5.6)/4 = 160$ mw
Transformer voltage ratio $P/G = 115/12 = 81/8.5$ rms
For low harmonic distortion, volt-amperes = 10 times tube output = 1.6 watts

$$X_c = \frac{(115 \times 0.707)^2}{1.6} = 4140 \text{ ohms}$$

$$C = \frac{1}{6.28 \times 800 \times 4140} = 0.048\ \mu\text{f}$$

$$\text{Plate } OCL = \frac{4140}{6.28 \times 800} = 0.823 \text{ henry}$$

Current in plate winding = $\frac{81}{4140} = 0.02$ amp
Core is the same as in Example (*b*), Section 65.
Primary 2100 turns No. 32 enamel winding resistance = 125 ohms
Grid 250 turns No. 42 enamel winding resistance = 180 ohms
$l_g = 0.060$ in., $l_c = 4.5$ in., $A_c = 0.32$ sq in., core weight = 0.4 lb

$$B_{dc} = \frac{0.6 \times 2100 \times 0.0056}{0.060 \times \pi} = 35 \text{ gauss}$$

$$B_{ac} = \frac{3.49 \times 81 \times 10^6}{800 \times 0.32 \times 2100} = 525 \text{ gauss}$$

$B_m = 560$ gauss. (From Fig. 54, $\mu_\Delta = 2000$)

$$\text{Primary } OCL = \frac{3.2 \times (2100)^2 \times 0.36 \times 10^{-2}}{0.060 + \dfrac{4.5}{2000}} = 0.815 \text{ henry}$$

From Fig. 127 core loss = $0.2 \times 0.6 \times 0.4 = 0.048$ watt
Gap loss = 0.030 watt
Primary copper loss = $(0.02)^2 \times 125$ = 0.05 watt
This leaves 32 mw available for secondary output.

Example (*c*). *Cathode Follower*. Assume that tubes 828 and 849 in Fig. 142 operate at $E_B = 1700$ volts. With $E_c = -75$ and 700 watts output, 849 plate swing is 1200, $E_{\min} = 500$, peak grid voltage = +105, and peak grid current is 0.090 amp. This is peak load for the 828 tube which can operate class AB1. The 828 tube plate swing is 1200 + 180 = 1380, or $E_{\min} = 1700 - 1380 = 320$ volts, and the 828 peak load is 105/0.090 = 1170 ohms. The cathode choke of 828 and the grid chokes and capacitors of 849 should have $1.41 \times 1170 = 1650$ ohms reactance for 100 per cent modulation at frequency f_1. At 10 cycles this would be 27 henrys. Peak voltage across these chokes is the 828 plate swing.

PROBLEMS

1. In a modulation transformer which carries the power amplifier plate current, what rms current flows with 100 per cent modulation? When there is a modulation reactor, what currents flow in the transformer and reactor?

***Ans.* $1.225I_B$; $0.707I_B$; I_B.**

2. Draw vector diagrams like Fig. 126 for a pi-filter modulator design, with $X_L = X_C = 1.41R$ at f_1. Draw for $f = f_1$, $1.2f_1$, $1.5f_1$, $2f_1$, and $3f_1$. Check with Fig. 131.

3. Draw diagrams like those in Prob. 2 for the high frequency pi-filter in Fig. 133.

4. What effect does −25 per cent tolerance in transformer and reactor inductance have on pi-filter $3f_1$ phase angle and impedance? What effect does +25 per cent tolerance have? What effect does +25 per cent in transformer inductance and −25 per cent in reactor inductance have? *Ans.* 7°, $0.95R$; 6°, $1.04R$; 0°, $0.96R$.

5. Design a modulation choke to use with the transformer of Example (*a*), Section 81. Try the same quantity and kind of laminations.

6. If a core-type input transformer with a cast iron shield is spaced 15 in. from the amplifier power transformer, what is the inherent hum level in the amplifier output, with perfect filtering of the power supply? *Ans.* −91 db.

7. A pair of type 6L6 beam tubes operating class AB2 is rated at 47 watts output with $E_B = 360$, $E_c = -22.5$, $I_B = 205$ ma for the pair. Using the curves of Figs. 87, 88, and 106 (pp. 113, 114 and 137 respectively), see whether a pair of 6L6 tubes are suitable for driving a pair of 851 tubes, operating class B at $E_B = 2000$, $E_c = -85$, with 500 volts grid-to-grid driving voltage and 20 watts grid driving power. Assume transformer coupling and sinusoidal 6L6 grid voltage.
Ans. Not suitable.

8. Complete winding calculations for Example (*a*), Section 81. Include turns per layer, layer and winding insulation, mean turn, leakage inductance, layer and winding capacitance.
Ans. 50; 0.002 in., 0.060 in.; 15.8 in.; 53 mh (secondary); 3000 $\mu\mu$f, 140 (secondary), 120 (primary).

9. Investigate the possibilities of improved uniformity of output impedance by addition of external inductance and capacitance in Example (*a*), Section 81.

10. If the performance of the driver of Prob. 7 is improved by the use of four 6L6's in push-pull parallel and the cathode follower circuit, how much grid voltage on the 6L6 tubes is required? *Ans.* 570 volts grid-to-grid.

11. In the hybrid coil of Fig. 141 (*b*), suppose that 1 watt is delivered into both R_1 and R_4, for $R_1 = R_2 = R_3 = R_4 = 1500$ ohms. What value of E_3 is required, how much total power, and what turns ratio?
Ans. 77.4 volts; 6 watts; $N_p:N_s = 2:1$.

12. Design a modulation transformer for the frequency range of 30 to 10,000 cycles having secondary impedance of 1500 ohms constant within −0 + 25 per cent, with step-down ratio of 2:1 and output = 2 kw. Use the pi-filter method.

13. What is the equivalent primary core loss resistance of Example (*b*), Section 81? How many turns are necessary for 500-ohm output load?
Ans. 138,000 ohms; 104 turns.

14. Design pi-filter chokes for the 828 cathode follower of Example (*c*), Section 81, with a frequency f_1 of 10 cycles.

15. Design a driver transformer to replace the 828 cathode follower chokes in Prob. 14 for a frequency range of 30 to 10,000 cycles. Load secondary with 1200 ohms per side; at 30 cycles use $X_N = 3R_2$; secondary leakage inductance 0.006 henry or less.

16. An artificial delay line consists of 10 sections of 0.054 henry and 0.15 μf each. At 1000 cycles, what are the time delay, phase shift, and characteristic impedance of the line? *Ans.* 0.9 milliseconds, 320°, 600 ohms.

VII. HIGHER FREQUENCY TRANSFORMERS

In Chapter VI the influence of low frequency performance on size was mentioned. If high transformer *OCL* is required, to maintain good low frequency response many turns or a large core are necessary, either of which limits the high frequency response. But if the amplifier frequency range is wholly composed of high frequencies, this limitation is in large part removed. For example, in a power-line carrier amplifier, the frequency range is 50 to 150 kc. It is then only necessary that *OCL* be high enough to effect good response at 50 kc. This is a great help in designing for proper response at 150 kc, and makes possible the use of laminated iron-core transformers for these and higher frequencies.

82. Iron-Core Transformers. At power-line carrier frequencies, the principles discussed in preceding sections for lower frequency transformers apply. In terms of the mean frequency, the band is narrow, but at 50 kc, the curves for low frequency operation portray amplifier performance just as they do at 30 cycles. Likewise at 150 kc care must be used that the limiting factors of leakage inductance and winding capacitance do not interfere with proper operation.

In carrier frequency transmitters, transformers are normally used for coupling between stages and for coupling the output stage to the line. They sometimes transform a large amount of carrier power. Coils are usually wound in single layers, spaced well apart to reduce capacitance, and have but few turns. If the necessary turns cannot be wound in a single layer, a bank winding like that shown in Fig. 144

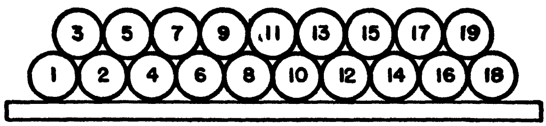


FIG. 144. Two-layer bank winding.

may be used. This winding has more capacitance than a single layer, but much less than two layers wound in the ordinary way. Since intrawinding layer-to-layer capacitance is zero in these transformers,

the resonance frequency f_r is usually determined by winding-to-winding capacitance.

In high impedance circuits, the winding-to-winding capacitance may be reduced by winding "pies" or self-supporting vertical sections side by side. Pies are wound with a throw of one turn per layer, but may be several turns wide. They have the general appearance of Fig. 145. High, narrow core windows or several pies are desirable to reduce leakage inductance. Transformer loss is mostly core loss. Two-mil grain-oriented steel can be used advantageously in such transformers, because of its low losses and high permeability. In transmitter operation, class AB or class B amplifiers are commonly used, with or without modulation, which may be as high as 100 per cent. In a receiver, input and interstage transformers also are employed, mainly to obtain voltage gain or for isolation purposes. Simi-

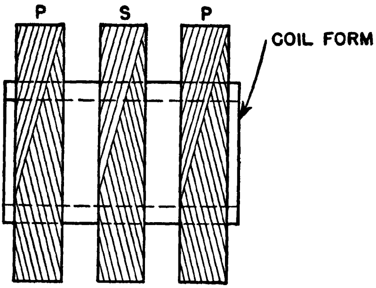


FIG. 145. Pie-section windings.

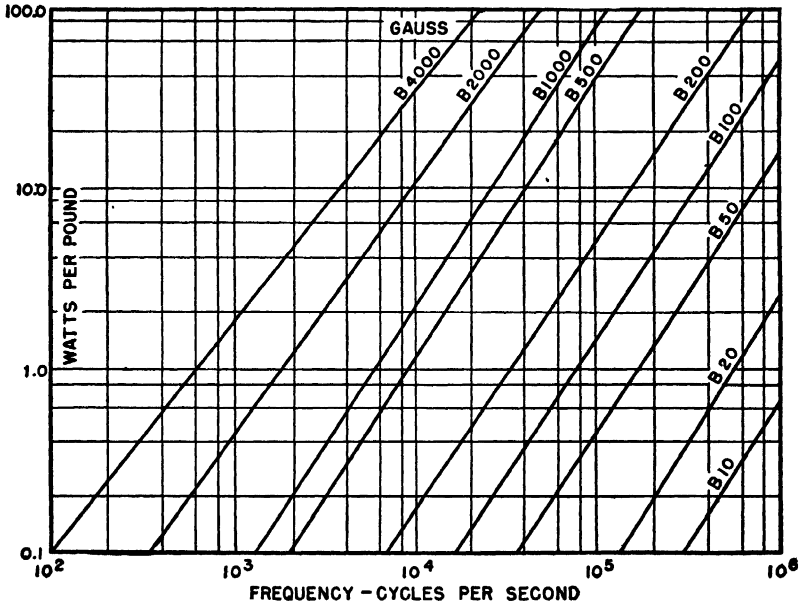


FIG. 146. Approximate loss of 2-mil steel at higher frequencies.

lar transformers are used for line matching, especially where overhead lines are connected to underground cables. Line impedance changes abruptly, and transformers may be necessary for good power transfer.

Core data at these frequencies are usually not available except for a limited choice of materials and gages. Approximate losses for two-mil

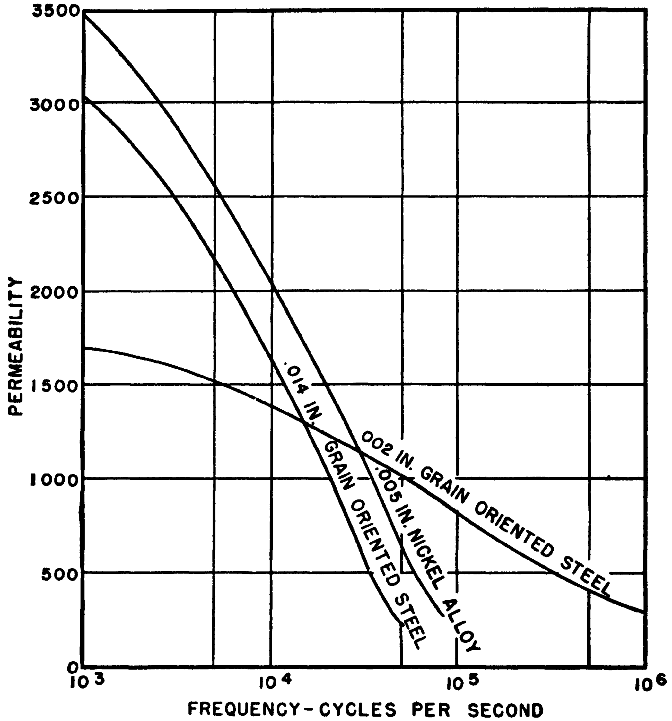


FIG. 147. Approximate permeabilities of core steels at higher frequencies.

oriented steel are given in Fig. 146. Interpolation or extrapolation from known data may be necessary to estimate core losses. In spite of this limitation, carrier frequency transformers are widely used. Some of the transformers in Fig. 13 operate in the carrier band. Core steel permeability decreases at high frequencies, depending on the lamination thickness. Oriented steel and nickel alloys have high permeability at low frequencies but, unless thin laminations are used, this advantage disappears at frequencies of 20 to 50 kc. The approximate decline for low induction is shown in Fig. 147. Decrease of permeability may be so rapid that *OCL* nearly decreases inversely as

frequency with 0.014-in. and even 0.005-in. material.[1] Grain-oriented steel 0.002 in. thick is well suited to these frequencies.

Transformers are also used up to 500 kc or higher. Capacitance limits the upper frequency at which amplifier transformers may be operated. In a tuned circuit amplifier, the tuning includes the incidental and tube capacitance as well as the tank circuit capacitance. A transformer has no tuning to compensate for such capacitances. Even with zero winding capacitance there would be a frequency limit at which any tube could operate into an untuned transformer without spoiling its efficiency or other characteristics. The most favorable condition for the use of transformers at higher frequencies is low circuit impedance. With low leakage inductance and low impedance circuits, transformer operation is possible at several megacycles.

83. Capacitance Evaluation. In high frequency transformers, the capacitances differ from those in audio transformers in that the windings are usually single layers, whose turn-to-turn capacitance is negligible compared to capacitance between windings and to the core. For example, in the transformer in Fig. 148, the primary and secondary are each wound in a single layer concentrically, in the same rotational direction, and in the same traverse direction (right to left). It will be assumed that the right ends of both windings are connected to ground (or core) through large capacitances, as shown dotted, so that the right ends are at substantially the same a-c potential. Primary capacitance C_1 is composed of many small incremental capacitances C_p, and secondary C_2 of many small incremental capacitances C_s, each of which has a different voltage across it. Likewise, many small incremental capacitances C_a exist between primary and secondary, which have different potentials across them. If the transformer is step-up,

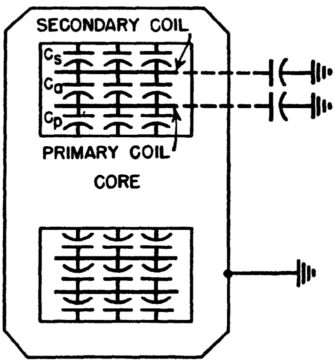


FIG. 148. Single-layer windings.

$$C_1 = \frac{1}{3}\Sigma C_p \quad \text{and} \quad C_2 = \frac{1}{3}\left[\Sigma C_s + \frac{(N_s - N_p)^2}{N_s^2}\Sigma C_a\right] \qquad (81)$$

[1] For additional core loss and permeability data at higher frequencies see "The Variation of the Magnetic Properties of Ferromagnetic Laminae with Frequency," by C. Dannatt, *Jour. I.E.E.*, Vol. 79, Dec. 1936, p. 667.

If the transformer is step-down,

$$C_1 = \frac{1}{3}\left[\Sigma C_p + \frac{(N_p - N_s)^2}{N_p^2}\Sigma C_a\right] \quad \text{and} \quad C_2 = \frac{1}{3}\Sigma C_s \tag{82}$$

If the ratio is 1:1,

$$N_s = N_p, \quad C_1 = \tfrac{1}{3}\Sigma C_p \quad \text{and} \quad C_2 = \tfrac{1}{3}\Sigma C_s \tag{83}$$

For transformers with opposite angular rotations of primary and secondary windings, or with opposite traverse directions (but not

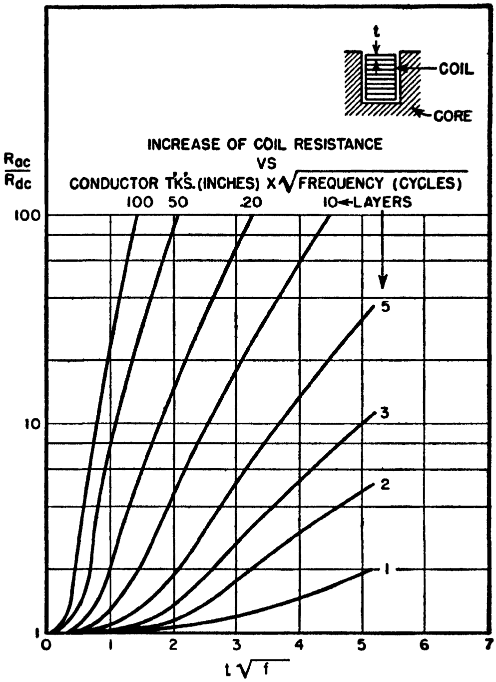


FIG. 149. Increase in coil resistance at high frequencies.

both), minus signs in the factors $(N_p - N_s)^2/N_p^2$ and $(N_s - N_p)^2/N_s^2$ in equations 81 and 82 become positive; there is no other change. For transformers with both angular rotations and traverse directions opposite there is no change at all in these equations. If there is a shield between primary and secondary, omit terms containing C_a in these equations, and make ΣC_s and ΣC_p include the capacitance of secondary and primary to shield, respectively. ΣC_s is the measurable capacitance of the short-circuited secondary to core, and ΣC_p that of the short-circuited primary to core.

In push-pull amplifier transformers, the secondary winding is interleaved between two primary halves. The rotational direction of winding and traverse are important, as they affect not only effective capacitance, but also plate-to-plate coupling. It is usually best to have all windings with the same rotational direction and traverse, and to connect the primary halves externally.

Winding resistance increases with frequency because of eddy currents in the larger wire sizes, and copper loss increases proportionately. Formulas and curves based on wires in free space do not apply to transformers because of the leakage flux. Eddy current resistance of layer-wound coils in deep open slots is plotted in Fig. 149 as a function of conductor thickness, frequency, and number of coil layers; it approximates the increase of winding resistance in a transformer.[2]

84. Example. Line matching transformer 50 to 500 ohms.

Frequency range 50 to 150 kc
Power output 100 watts
Primary voltage $= \sqrt{ZW} = \sqrt{50 \times 100} = 70.7$ volts
Secondary voltage $= \sqrt{500 \times 100} = 224$ volts
Core 2-mil oriented steel
$A_c = 0.45$ sq in., $l_c = 6$ in., $l_g = 0.002$ in. (incidental), core weight $\frac{3}{4}$ lb
Window $\frac{5}{8}$ in. $\times$ $1\frac{1}{2}$ in.
Primary 31 turns No. 22 wire. Mean turn 3.8 in.
Secondary 100 turns No. 30 wire. Mean turn 4.6 in.
Windings arranged as in Fig. 148
Insulation between primary and core, and between secondary and primary $\frac{1}{8}$ in. of organic material
Secondary effective capacitance 40 $\mu\mu$f
$B_m = 350$ gauss
Secondary $OCL = 20$ mh
Secondary leakage inductance $= 260\ \mu$h
Core loss $= 8$ watts per lb $\times$ $0.75 = 6$ watts. This is practically the only loss.
X_N/R_1 at 50 kc $= (6.28 \times 50{,}000 \times 0.020)/500 = 12.56$
$f_r = 1560$ kc. $B = 5.00$
According to Figs. 90 and 91, this transformer has nearly flat response over the entire range.

85. Air-Core Transformers. Transformers considered hitherto have had laminated iron cores. A class of transformers is widely used in radio frequency circuits without cores, or with small plugs of powdered iron. In a transformer with an iron core, the exciting current required

[2] See "Eddy-Current Resistance of Multi-layer Coils," by T. H. Long, *Trans. AIEE*, Vol. 64, Oct. 1945, p. 716.

for inducing the secondary voltage is a small percentage of the load component of current. In an air core transformer *all* the current is exciting current, and induces a secondary voltage proportional to the mutual inductance.

Consider the circuit of Fig. 150, in which Z_1 is complex and includes the self-inductance of the primary coil. Likewise, secondary impedance Z_2 is complex and includes the self-inductance of the secondary coil. With a sinusoidal voltage applied, Kirchhoff's laws give the following.

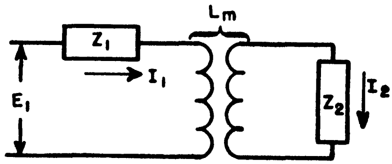


FIG. 150. General case of inductive coupling.

$$E_1 = Z_1 I_1 + j\omega L_m I_2 \qquad (84)$$

$$0 = Z_2 I_2 + j\omega L_m I_1 \qquad (85)$$

where $\omega = 2\pi$ times operating frequency, and L_m is the mutual inductance between the primary and secondary coils.

From equation 85 we see that the voltage in the secondary coil is numerically equal to $\omega L_m I_1$, the product of primary current and mutual reactance at the frequency of applied voltage E_1. The equivalent impedance of the circuit of Fig. 150 when referred to the primary side is given by

$$Z' = Z_1 + \frac{{X_M}^2}{Z_2} \qquad (86)$$

where $X_M = j\omega L_m$.

In the above formulas, the impedances Z_1, Z_2, and Z' are complex quantities whose real and imaginary terms depend upon the values of resistance, inductance, and capacitance in the circuit. One common practical case arises when the primary resistance is zero, or virtually zero, and the secondary coil is tuned to resonance so that Z_2 is a pure resistance R_2. Under these conditions, equation 86 reduces to

$$R' = \frac{{X_M}^2}{R_2} \qquad (87)$$

where R' is the equivalent resistance in the primary.

Equation 87 gives the value of mutual inductance required for coupling a resistance R_2 so that it will appear like resistance R' with a maximum power transfer between the two coils, and states that the mutual reactance X_M is the geometric mean between the two values of resistance.

The ratio of mutual inductance to the geometric mean of the primary and secondary self-inductances is the coefficient of coupling:

$$k = \frac{L_m}{\sqrt{L_1 L_2}} \tag{88}$$

The value of k is never greater than unity, even when coils are interleaved to the maximum possible extent. Values of k down to 0.01 or lower are common at high frequencies.

A tuned air-core transformer often used in receivers is shown in Fig. 151. Here a sinusoidal voltage E_1 may be impressed on the primary circuit by a vacuum tube amplifier. Resistances R_1 and R_2 are

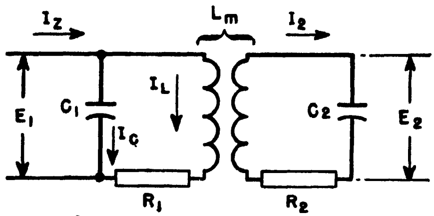


FIG. 151. Tuned air-core transformer.

usually the inevitable resistance of coils, but occasionally resistance is added to change the circuit response. The value of voltage E_2 obtained from this circuit depends on the impressed frequency; in Fig. 152 it is shown for resonance at three different values of coupling. If the value of coupling is such that

$$X_M = \sqrt{R_1 R_2}$$

we obtain a condition similar to that of equation 87, in which the maximum power or current is produced in the secondary circuit. Maximum current through condenser C_2 gives maximum voltage E_2. This value of coupling is known as the critical value. Smaller coefficient of coupling gives a smaller maximum value of E_2. Greater coefficient of coupling results in a "double hump" as shown in Fig. 152. The heights of resonant peaks and frequency distance between peaks depend upon circuit Q and coefficient of coupling k. The double hump curve of Fig. 152 is desirable because, with modulated waves, frequencies in adjacent channels are rejected; yet very little attenuation is offered to audio frequencies which effectively add or subtract from the carrier frequency normally corresponding to resonance. Close tuning control and high Q are essential to good response and selectivity.

If the primary circuit is made to resonate at a different frequency

from the secondary, audio response is much worse, and considerable distortion is likely. Moreover, the response at mean frequency is less than it would be if the circuits were properly tuned. Air-core transformers are usually made adjustable for tuning and coupling.

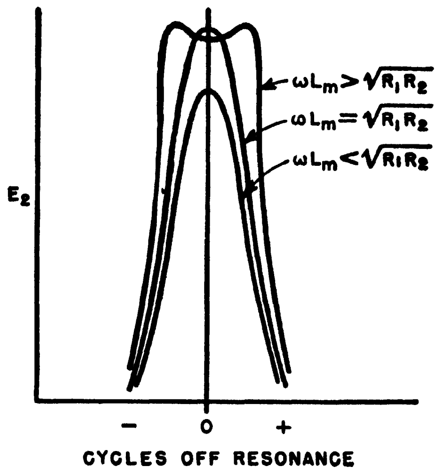


FIG. 152. Response curves for circuit of Fig. 151.

86. Mutual Inductance. It is evident from equation 85 that the secondary voltage depends upon the mutual inductance between the coils. Mutual inductance can be calculated by formulas which depend upon the geometric configuration of the coils. If the coils are arranged

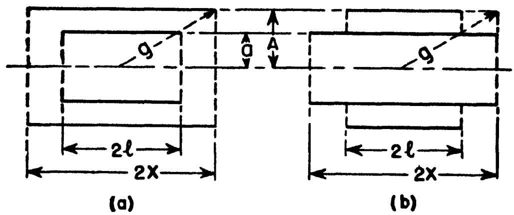


FIG. 153. Concentric co-axial coupled coils.

concentrically, as shown in either (*a*) or (*b*) of Fig. 153, the mutual inductance of the coils can be found from

$$L_m = \frac{0.05a^2N_1N_2}{g}\left[1 + \frac{A^2a^2}{8g^4}\left(3 - 4\frac{l^2}{a^2}\right)\right] \mu\text{h} \qquad (89)$$

where N_1 = primary turns and N_2 = secondary turns. All dimensions are in inches. For most purposes, the bracketed portion of this

formula is approximately unity, and it has been plotted in Fig. 154 for a single-turn secondary. To find the mutual inductance for any

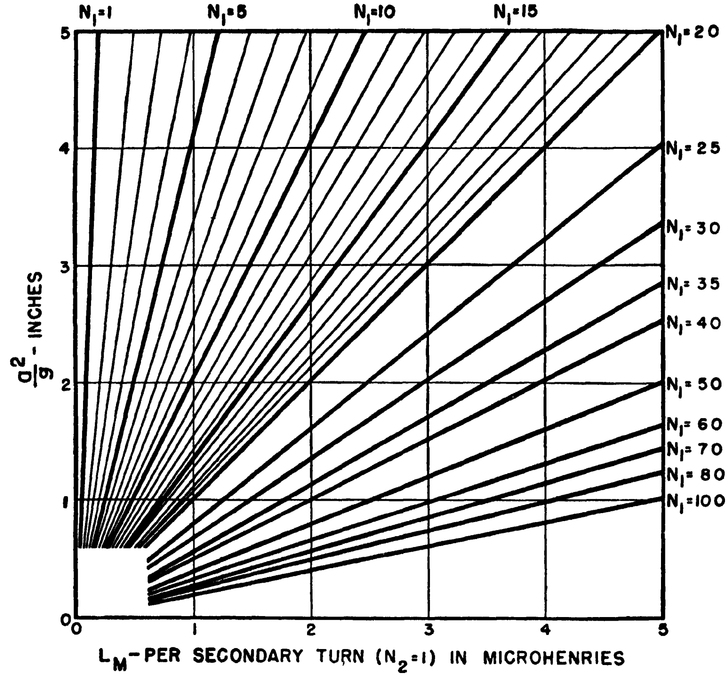


FIG. 154. Mutual inductance of coils in Fig. 153.

given number of secondary turns, multiply the mutual inductance found from this curve by the number of secondary turns. The range of ordinates and abscissas can be extended indefinitely.

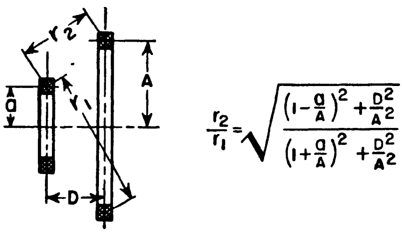


FIG. 155. Co-axial non-concentric coils with rectangular sections.

If the coils are arranged co-axially as in Fig. 155, then approximate values of mutual inductance are found as follows.

$$L_m = FN_1N_2\sqrt{Aa} \tag{90}$$

In this formula all dimensions are in inches and the mutual inductance is in microhenrys. The factor F can be conveniently found in Fig. 156.

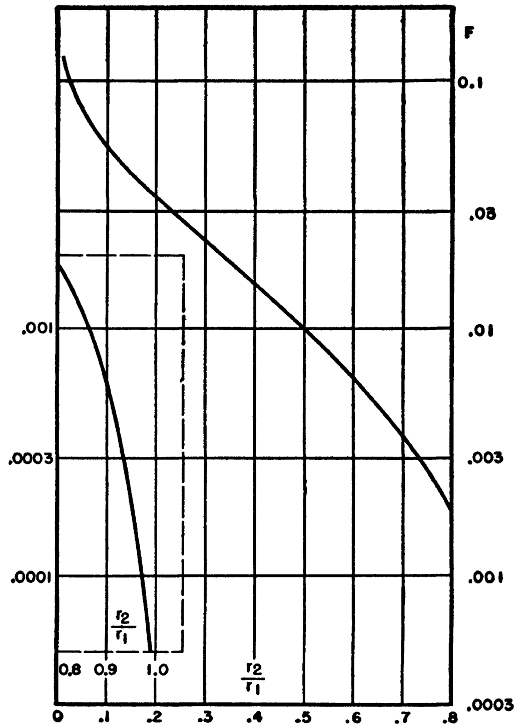


FIG. 156. Factor F in equation 90 as a function of r_2/r_1 (see Fig. 155).

Self-inductance of single-layer coils is

$$L = \frac{0.1a^2N^2K}{l} \tag{91}$$

where a = mean coil radius in inches
N = number of turns
l = length of coil in inches
L = inductance in microhenrys
K may be found from Fig. 157.

Equations 89, 90, and 91 are based on equations 192, 187, and 153 in Bureau of Standards Circular 74.

Receiver intermediate-frequency tuned transformers generally have co-axial coils. If the wire is more than 0.005 in. in diameter, it is com-

monly subdivided into several strands in the type of cable known as Litzendraht, to reduce losses and increase Q. In transmitters, the size of the coils becomes much larger, and concentric coils are employed.

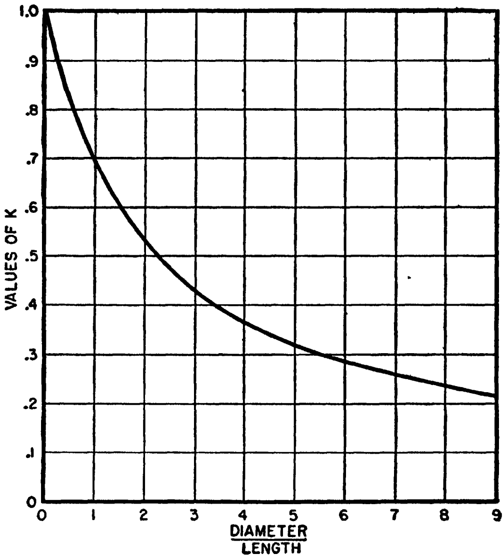


FIG. 157. Factor K in equation 91.

The wire used is Litzendraht at 600 kc or lower frequency, and may contain many strands for carrying heavy currents. At higher frequencies the wire is of solid or tubular section.

87. Powdered Iron Cores. Both self-inductances and mutual inductances of a coil may be increased by inserting a plug of powdered iron inside the coil tube. Tuning a coil to a given frequency is often

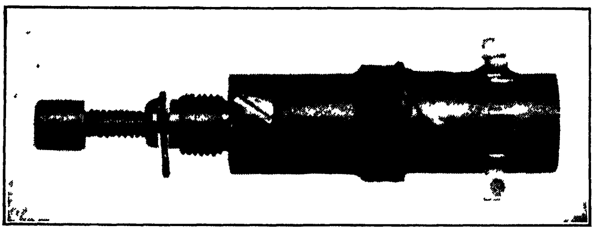


FIG. 158. Coil inductance is varied by powdered iron core.

effected in this manner with fixed capacitors instead of tuning with variable capacitors. Such a coil is shown in Fig. 158, with the powdered iron core hidden by the coil form. At the left end is the screw

and lock by which the inductance can be adjusted and maintained at a given value. The mutual inductance of a pair of coils can be changed similarly. This is preferable to attempting to vary the distance between the coils, since it requires no flexible connections. Powdered iron is available in several grades, from ordinary powdered iron to powdered nickel alloy. The particles are coated with an insulating compound which separates them, and reduces the permeability of the core to values ranging from 2 to 80, depending on the grade of iron and the frequency. In a given coil, the insertion of a powdered iron core raises the inductance from 2 to 3 times the value which it would have if no iron were present. Circuit Q increases similarly.

The question may be asked: Why are powdered iron cores not always used at higher frequencies? When high Q coils are required, as in filters or tuned circuits, powdered iron cores are best. But in untuned circuits with low Q and wide frequency range, the low magnetizing current of a laminated, closed iron core is preferable.

88. R-F Chokes.[3] When a choke is used to pass direct current and present high impedance to radio frequencies, it may have high r-f voltage across it. High choke impedance at operating frequency is necessary to avoid loss of r-f current which reduces the useful power and overheats the choke. If a single-layer choke is connected to an r-f generator at a given voltage, and if its current is measured as in Fig. 159, the choke impedance is the ratio of voltage to current measured.

By disconnecting the choke from the circuit, the tuning reaction may be noted, and from this whether the reactance of the choke is inductive or capacitive. The difference in watts input to the generator, when the coil is removed and the tank condenser is retuned for minimum plate current, is readily observable. This difference times the generator efficiency is the loss in the coil at a particular voltage and frequency.

The impedance of a typical coil, found as described above, is plotted in Fig. 159 against frequency. At low frequencies (a), the curve follows straight reactance line $X_L(= 2\pi fL)$. At a frequency somewhat below natural frequency b (determined by the choke inductance and effective capacitance), the slope starts to increase, and reaches a maximum point at a frequency c of $1.2b$ to $1.7b$. Above this frequency, the impedance decreases until a minimum value is reached at d, which is from 2.2 to 3.0 times b. At higher frequencies, the increase and decrease are repeated in a series of peaks and valleys at approximately

[3] This section is based on the author's "A Study of R-F Chokes," *Electronics*, April 1934, p. 120.

equal frequency intervals. The second, fourth, and sixth peaks are of lower value than the first, third, and fifth respectively. The seventh peak is followed by a flattened slope which suggests a submerged eighth peak. The points of minimum impedance rise in value, so that at higher frequencies the valleys appear to be partly filled in and the peaks to be

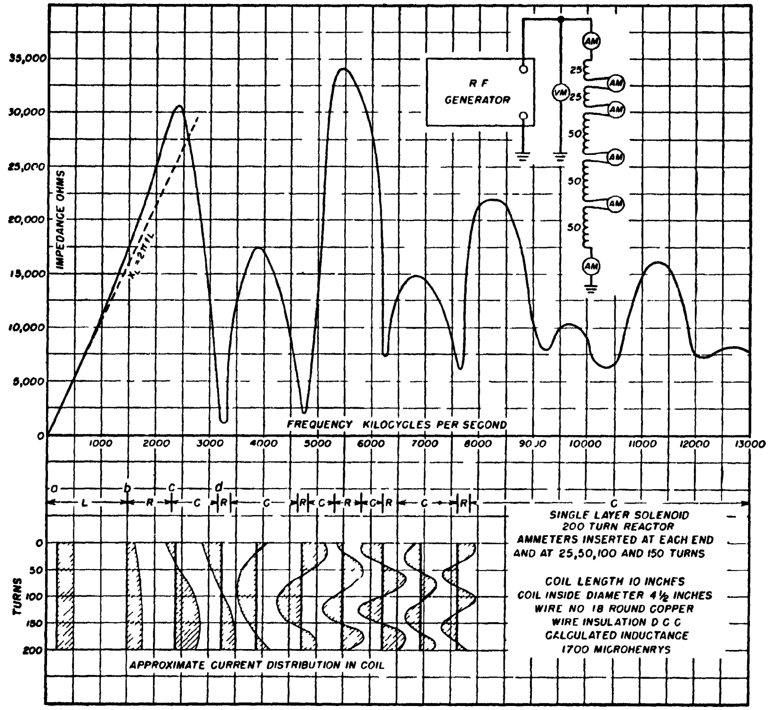


Fig. 159. R-f choke impedance.

leveled off. The watts loss are high at points of low impedance, and they rise sharply at the frequency d.

The change in reactance is shown in Fig. 159. The coil is inductive up to frequency b. From b to c it has no noticeable effect on the tuning and hence is pure resistance, or nearly so. Above c it is capacitive up to a frequency slightly below d, where it again becomes of indefinite reactance. Thereafter, it is capacitive, except for brief frequency intervals, where it is resistive, or only slightly inductive. At all frequencies higher than the fifth peak, the coil is capacitive.

Since a coil has distributed constants it is subject to standing waves at the higher frequencies. The character of these waves may be found

by tapping the coil at various points and inserting thermogalvanometers in series with the coil at these points. The current distribution is plotted in Fig. 159 against coil length. These diagrams show the kind of standing waves as the frequency increases.

Current distribution is uniform at all frequencies below *b*. Most chokes are used within the first impedance peak. The useful range for choke impedance of 20,000 ohms in Fig. 159 is 1700 to 2800 kc. This choke could be operated at 5500 kc safely also, but the frequency range is narrower. Also, the safe loss dissipation is less because it takes place over half of the coil surface. Pie-section chokes have similar impedance curves, but impedance peaks following the first are less pronounced.

PROBLEMS

1. Calculate the effective winding capacitance, winding resistances, and secondary *OCL* of the transformer in Section 84.

Ans. 40 $\mu\mu$f; 0.161 ohm, 4.06 ohms; 20 mh.

2. Derive formulas for the winding capacitances of three single-layer windings, all grounded at the left end and wound in the same direction. Assume the middle winding to have a small number of turns compared to the other two.

Ans. $\frac{1}{3}(\Sigma C_p + \Sigma C_s + 2\Sigma C_a)$.

3. Using the same core and induction as in Section 84, and with the secondary interleaved between two half-primary windings, design a transformer to deliver 30 watts output from a pair of 6L6 tubes operating class AB_2 at $E_B = 360$ volts, over the frequency range of 50 to 150 kc, into a 500-ohm line.

4. Find the mutual inductance between two single-layer concentric coils, each having 25 turns of wire and 6 in. winding length. The inner coil is 4 in. in diameter and the outer one 6 in. What is the coupling coefficient between these coils?

Ans. 30 μh; 0.66.

5. What is the mutual inductance of two co-axial pie-sections, each having 100 turns and ½ in. mean radius, if the axial distance between them is ¾ in.?

Ans. 30 μh.

6. Half the flux in a long air-core solenoid leaves through the turns. What is the maximum possible increase in inductance if a powdered iron core of equal length is inserted? How does a pie-section coil compare?

Ans. Less than 2:1; very little increase.

7. Assuming a dissipation rate of 0.008 watt per sq in. per deg C, from the external cylindrical surface of the coil in Fig. 159, how many watts can it dissipate for a temperature rise of 40 centigrade degrees at 1000 and at 5500 kc?

Ans. 46; 23.

VIII. ELECTRONIC CONTROL TRANSFORMERS

Electronic devices are used to control speed, voltage, and current, or may require control of these quantities. Most of the circuits can be grouped into a few basic types. This chapter comprises typical circuits which use transformers and reactors.

89. Electronic Control Circuits. Vacuum tube control circuits are used for amplification of the input voltage, not always at a single frequency. With thyratrons the input voltage triggers the tube, which then allows current to flow into the controlled circuit, but the output wave may not resemble the input wave.

A simple circuit for thyratrons operated with alternating anode supply and resistive load is shown in Fig. 160.[1] During that part of

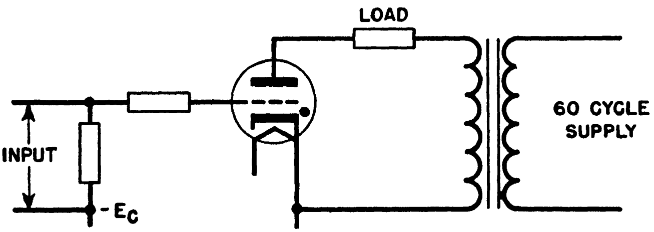


Fig. 160. Basic thyratron circuit.

each cycle when the anode is positive with respect to the cathode, the tube conducts current which passes through the load, provided the grid is at the right potential. In Fig. 161 is shown the positive anode voltage for a half-cycle, with the corresponding critical grid voltage. Any value of grid voltage higher than this critical value will permit the tube to conduct. Once tube conduction is started, change of grid voltage to a value less than critical will not stop conduction. Conduction does stop, however, at the end of the half-cycle, or when the anode voltage falls to zero. Three methods of controlling the load current

[1] See *Industrial Electronics*, by F. H. Gulliksen and E. H. Vedder, John Wiley and Sons, 1935, p. 45.

are shown in Fig. 162. Figure 162 (*a*) shows how direct voltage applied to the grid permits conduction through the tube over the shaded portion of the cycle. Minimum controllable current is half that which would flow if the tube were free to conduct over the entire positive half-cycle. This method of control is not precise, especially near the half-power point, because a small difference in d-c input control voltage produces a comparatively large change in conduction angle or may cause the tube to fail to fire altogether.

Figure 162 (*b*) shows a more satisfactory method of amplitude control. The grid is maintained at a positive d-c potential, and an alternating voltage is superposed on it which lags the anode voltage by

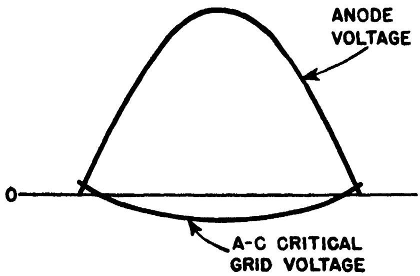


FIG. 161. Anode and critical voltages in basic thyratron circuit.

90 degrees. Varying the magnitude of the d-c grid voltage shifts the zero axis of the a-c wave up or down, and intersects the critical a-c grid voltage at different points of the cycle. Tube current can then be varied from zero to maximum. Close control of the tube current can be obtained because the grid voltage wave intersects the critical curve at a large angle.

In Fig. 162 (*c*) another method is shown. The phase of a superposed alternating voltage is shifted upon a negative d-c bias which is more negative than the critical characteristic. Changing the phase position of the a-c grid voltage varies the tube current from zero to maximum. The phase position of the grid voltage can be shifted by several methods, one of which is discussed in Section 90.

The anode supply transformer carries load direct current. Core saturation may be prevented by an air gap; heating and regulation in the primary winding due to excitation current govern the length of air gap. Ordinarily, permissible maximum induction may be higher than in a single side amplifier transformer because impedance or frequency response considerations are irrelevant with a 60-cycle supply line. Excitation current may be comparable in magnitude to load

current. However, there is this difference: with a resistive load, current flows only during the positive half-cycle, whereas magnetizing current flows during the whole cycle. Secondary current is a series of

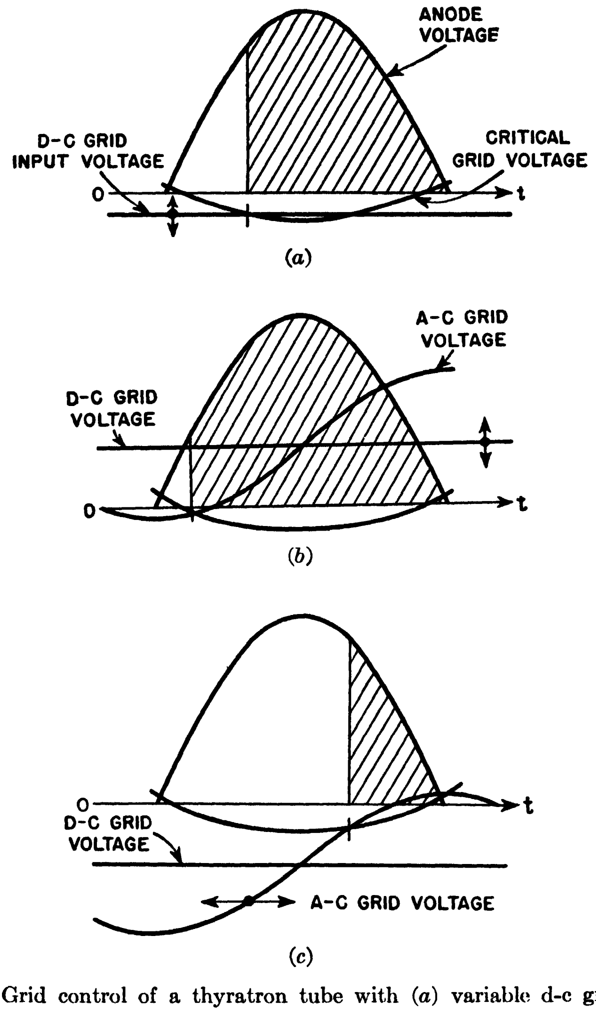


FIG. 162. Grid control of a thyratron tube with (*a*) variable d-c grid voltage, (*b*) variable d-c grid voltage with superposed a-c grid voltage, (*c*) fixed d-c grid voltage with superposed a-c grid voltage of variable phase position.

pulses, the maximum width of which is 180 degrees. The rms value of these pulses is half the peak amplitude, and this is the current which governs secondary wire size. Rms secondary voltage is 2.22 times maximum d-c load voltage, as in a single-phase half-wave rectifier.

Design of the transformer is similar to the anode transformers in Chapter III, except for the higher induction.

Full-wave circuits [2] operate with two thyratrons and a center-tapped transformer in which the net d-c flux is zero. The design is the same as for a full-wave rectifier anode transformer.

90. Grid-Controlled Rectifiers. The basic a-c grid control circuit described in the last section may be extended to more than one tube and may control large amounts of power. Any of the rectifier circuits of Table VII (p. 46) may be used with grid control of output voltage, which supplants the older practice of using induction regulators in the supply lines.

Smooth control of rectifier d-c voltage under load conditions is possible through the use of thyratrons or ignitrons with phase shift control of the grid or igniter. Stable control of filtered output is possible only with choke input filters. In Chapter IV the regulation of a rectifier is shown to be lowest if the input choke has inductance greater than critical value. With grid control, if the filter choke inductance is great enough, the tube conducts even after the anode reaches zero. The tendency of current to stop at voltage zero builds up voltage across the filter choke in such a direction that cathode potential is less than zero after the anode reaches zero. Thus conduction in the tube is maintained until the next tube fires. If the choke inductance is less than critical, tube current wave is discontinuous, regulation is poor, transient surges and oscillations in the output voltage occur, and control is unstable.

For a single-phase full-wave rectifier with grid control, the direct voltage output decreases as shown by curve I in Fig. 163. Critical value of inductance increases with firing angle and so does ripple voltage as shown by curves II and III in this figure. For a three-phase full-wave rectifier, the direct output voltage is approximately 41 per cent greater than the single-phase values shown in Fig. 163, and the critical value of choke reactance less filter capacitor reactance is approximately one tenth of the single-phase values over the range of 20° to 90° firing angle. At 90° firing angle, the d-c output in all cases is zero. Voltage across the choke reverses in sign but does not increase in magnitude even with the maximum angle of 90°. Therefore the maximum voltage from choke to ground is not changed, and the design of a reactor for this type of rectifier is the same as for a rectifier without grid control, except for the value of inductance.

Choke input filters can be used to maintain continuous current flow in single-phase half-wave rectifiers. Although the output voltage is

[2] See Gulliksen and Vedder, *op. cit.*, p. 54.

reduced, as mentioned in Chapter III, this combination is occasionally useful.[3]

Grid-controlled rectifiers have more irregular current wave forms and therefore more pronounced a-c line harmonics than ordinary rectifiers.[4]

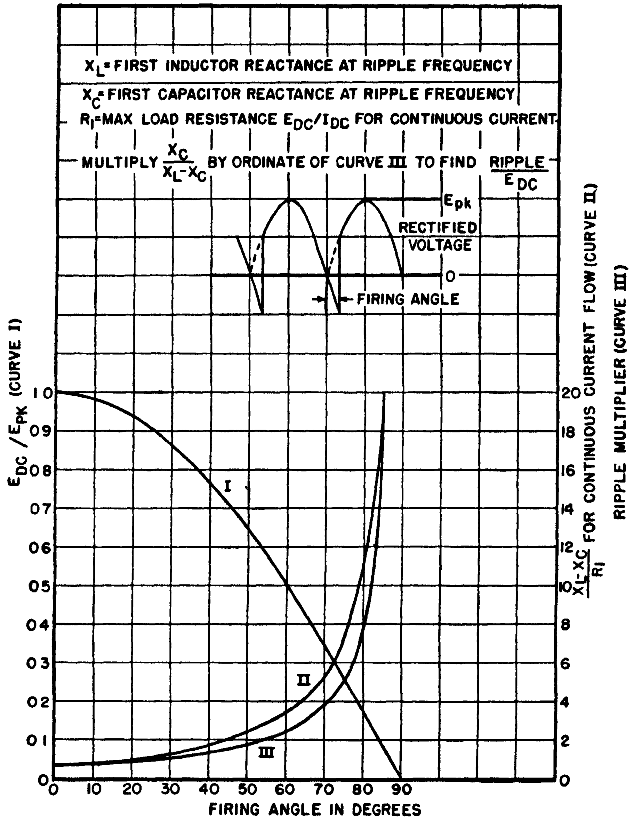


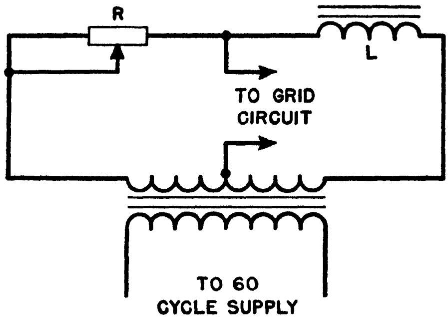
FIG. 163. Output voltage, ripple, and current continuity in single-phase full-wave grid-controlled rectifier.

Two methods of providing phase shift control of a constant amplitude a-c grid voltage for grid-controlled rectifiers are shown in Fig. 164. In

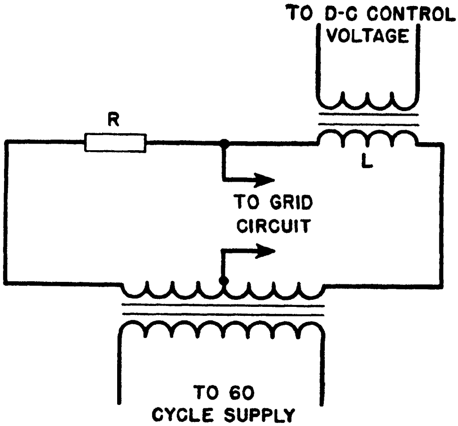
[3] For general calculation of discontinuous waves, see "Voltage and Current Relations for Controlled Rectification with Inductive and Generative Loads," by K. P. Puchlowski, *Trans. AIEE*, Vol. 64, May 1945, p. 255. For theory of controlled rectifiers, see "Critical Inductance and Control Rectifiers," by W. P. Overbeck, *Proc. I.R.E.*, Vol. 27, Oct. 1939, p. 655.

[4] See "Harmonics in A-C Circuits of Grid Controlled Rectifiers and Inverters," by R. D. Evans and H. N. Muller, Jr., *Trans. AIEE*, Vol. 58, 1939, p. 861.

(*a*) a small value of resistance R effectively connects the upper grid circuit terminal to the left-hand terminal of the supply transformer, and a large value of R shifts it nearer to the right-hand terminal of the supply transformer. If the supply transformer is center-tapped, the



(*a*)



(*b*)

FIG. 164. Resistance-inductance phase shift circuits.

vector diagram of Fig. 165 shows the phase position of the grid voltage E_g in solid lines for a small value of R, and in dotted lines for a large value of R. Varying the rheostat R thus varies the rectifier output from full voltage to a low voltage.

In Fig. 164 (*b*) resistor R is fixed and inductance L is varied by means of direct current flowing in one of its windings. The vector diagram of Fig. 165 still applies; the solid lines are for high inductance and the dotted lines for low inductance. Direct current for varying the induct-

ance may be obtained through a thyratron or a vacuum tube, especially when rectifier output voltage is automatically controlled. The reactor is usually of the saturable type described in the next section.

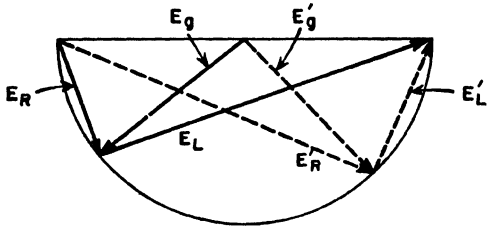


FIG. 165. Vector diagram for Fig. 164.

91. Saturable Reactors. Although any iron-core reactor is saturable, the term is reserved here for the three-legged type shown in Fig. 166. Equal turns in the a-c coils set up equal a-c magnetomotive forces which cancel in the center leg, and cause flux to flow as indicated by the solid line. No alternating voltage is induced in the d-c coil, but d-c flux flows in both outer legs as indicated by the dotted lines. A change of current in the d-c coil causes a change in total flux linking

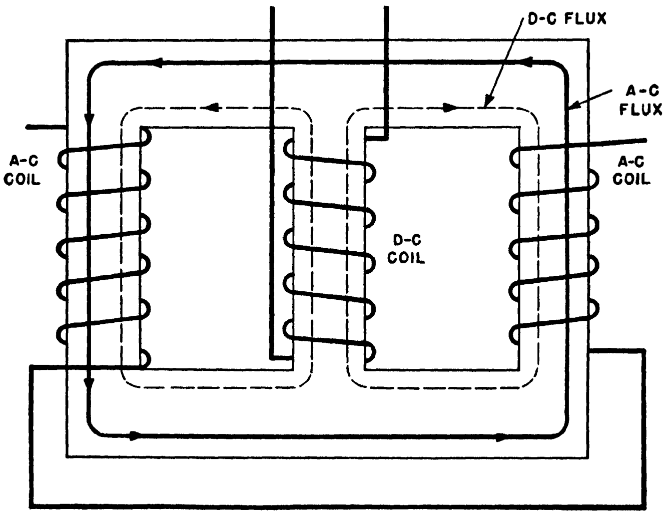


FIG. 166. Windings and core flux paths in a saturable reactor.

the a-c coils and hence a change of inductance. Since the core has no air gap, wide changes of inductance are obtained. A-c coils may be connected in parallel instead of series, provided equal turns in each

coil and the flux polarity of Fig. 166 are maintained; for the same total number of turns the inductance is halved and the alternating current doubled.

Saturation curves for a saturable reactor are plotted as shown in Fig. 167 from test data, and are suitable for designing any reactor with

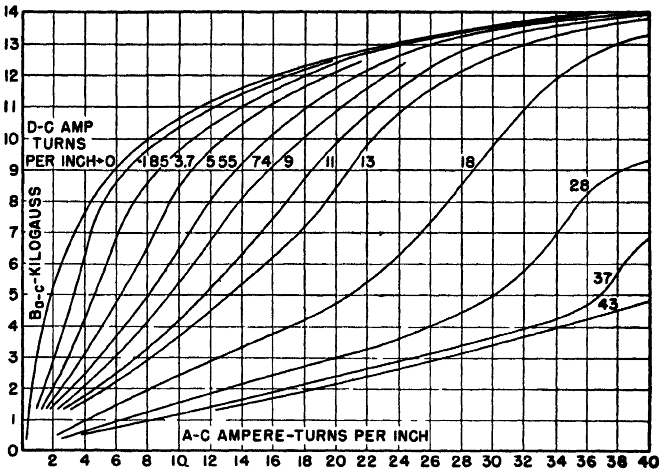


FIG. 167. Typical saturation curves for saturable reactor.

the same relative flux paths and grade of core steel. The curves of Fig. 167 are for 4% silicon steel. The use of these curves will be illustrated by an example.[5]

Example. Minimum and maximum rectifier tube firing angles in Fig. 164 (*b*) fix the extremes of inductance required for a given resistance R. Suppose that $R = 10{,}000$ ohms, that an inductance change of 50 to 6 henrys is required with a change in direct current of zero to 0.030 amp, and that supply transformer secondary voltage = 220 volts, at 60 cycles; see Figs. 164 (*b*) and 165.

Reactance at zero direct current $= 2\pi \times 60 \times 50 = 18{,}800$ ohms

Reactance at 30 ma direct current $= 2\pi \times 60 \times 6 = 2260$ ohms

$$E_L = \frac{18{,}800 \times 220}{\sqrt{(18{,}800)^2 + (10{,}000)^2}} = 194 \text{ volts}$$

$$E'_L = \frac{2260 \times 220}{\sqrt{(2260)^2 + (10{,}000)^2}} = 48.5 \text{ volts}$$

[5] This example was first worked out by Mr. D. S. Stephens.

Reactor alternating current with zero direct current

$$= \frac{194}{18,800} = 0.0103 \text{ amp}$$

Reactor alternating current with 0.030 amp direct current

$$= \frac{48.5}{2260} = 0.0215 \text{ amp}$$

A reactor designed for this purpose has a core window $^{11}\!/_{16}$ in. high by $1\frac{5}{16}$ in. wide, center leg $^{11}\!/_{16}$ in. wide, outer legs $^{11}\!/_{32}$ in. wide, a-c flux path

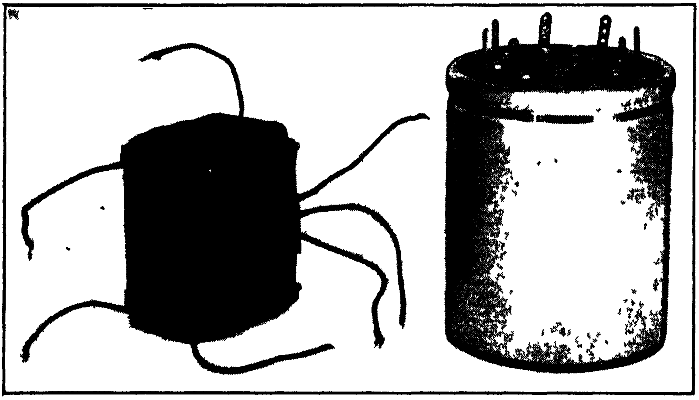


FIG. 168. Saturable reactor before and after assembly in case.

7.85 in. long and d-c flux path 5 4 in. long. Let $B_{ac} = 7$ kilogauss at zero direct current. From Fig. 167, the a-c ampere-turns per inch are 3.4, or

$$N_{ac} = \frac{3.4 \times 7.85}{0.0103} = 2600 \text{ turns}$$

Two a-c coils in series have 1300 turns each. Core area in the outer legs is, by equation 24,

$$A_c = \frac{3.49 \times 194 \times 10^6}{60 \times 7000 \times 2600} = 0.62 \text{ sq in.}$$

so that the lamination stack is $0.62/(0.344 \times 0.9) = 2$ in. of 0.014-in. laminations.

The number of turns in the d-c coil depends on the lower value of reactance, across which are 48.5 volts. $B_{ac} = 7 \times (48\ 5/194) = 1.75$ kilogauss and a-c ampere-turns per inch $= (0.0215 \times 2600)/7.85 = 7.15$. Figure 167 shows that 18 d-c ampere-turns per inch are required. The d-c coil turns are $(18 \times 5.4)/0.030 = 3240$. Figure 168 is a photograph of this reactor.

Choice of core flux density depends on the d-c power available. As the flux density at the highest inductance is raised, less d-c power is

necessary for obtaining the lowest inductance, and for a given number of turns in the a-c coils, less core area. The wider the range of inductance, the more d-c power is required.

The widest range of inductance is obtained with zero direct current at the higher inductance. In some vacuum tube circuits, the minimum

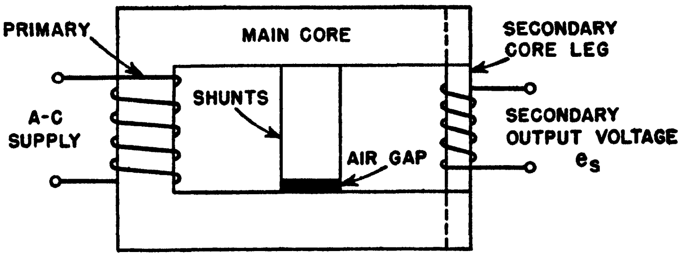


FIG. 169. Peaking transformer.

direct current is not zero, and a bias winding is added to the center leg to cancel the d-c ampere-turns with minimum current in the main d-c winding. Saturable reactors have many uses besides that described here.

92. Peaking Transformers. It is stated in Section 89 that a large angle of intersection between the grid firing voltage wave and critical grid voltage is desirable for accurate control. A grid wave form with vertical front edge would be ideal. To produce a steep peaked wave form for firing thyratron tubes, sometimes special transformers are used. Usually the design depends on the non-linearity of the magnetizing current. Figure 169 shows a peaking transformer in which the magnetic core is made of special laminations. The primary is wound on the full-width left leg, and the secondary on the right leg which is made of a few laminations of small width. In the space between primary and secondary is a laminated shunt path with an air gap. In Figure 170 are shown the core fluxes ϕ_m and ϕ_s, linking the primary and secondary coils respectively. At low inductions, the same flux links both coils. As the flux rises from zero in each cycle, at first all the flux links the secondary coil, but because of the smaller cross section of the right leg, it saturates at the value ϕ_s and the main flux ϕ_m flows through the shunt path. Thus

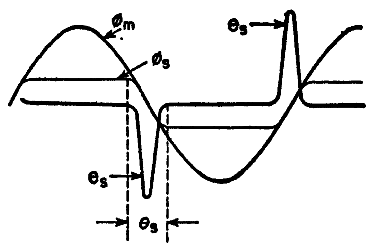


FIG. 170. Fluxes and secondary voltage in peaking transformer.

there is a long interval in each cycle during which the flux change is substantially zero, and no voltage is induced in the secondary coil. During the short period θ_s, a voltage is induced in the secondary coil which has a very peaked wave form. This happens twice in each cycle.

Because of the shorter time for the change in ϕ_s, $d\phi/dt$ would remain nearly constant over the angle θ_s if there were no leakage flux, and for 1:1 turns ratio there would be approximately equal volts in the primary and secondary coils. The secondary flux change takes place over a much shorter period of time and the flux rises to only a fraction of its maximum value ϕ_m. Therefore less core area is needed in the secondary leg to obtain the desired voltage e_s. This leads to the following approximate ratio.

$$\frac{A_s}{A_p} = \frac{\theta_s}{\pi} \sin\frac{\theta_s}{2} \tag{92}$$

where A_s = core area in the secondary leg
A_p = core area in the primary leg.

Peaked secondary voltage may be made steeper by the use of nickel-iron laminations in the secondary leg, because these alloys have sharp saturation. The air gap in the center leg prevents it from shunting all the primary flux, which would reduce the secondary voltage to zero. This air gap should be no more than 5 to 10 per cent of the window height, to keep leakage flux from threading through the secondary coil and giving a less peaked wave form. With this length of air gap and a total window length of twice the window height, the secondary turns are, for a 1:1 voltage ratio,

$$N_s \approx 2N_p \tag{93}$$

where N_p is the number of primary turns.

93. Current-Limiting Transformers. Filaments of large vacuum tubes sometimes must be protected against the high initial current they draw at rated filament voltage. This is done by reducing the starting voltage automatically through the use of a current-limiting transformer, with magnetic shunts between primary and secondary windings. The shunts carry very little flux at no load; as the load increases, the secondary ampere-turns force more of the flux into the shunts until at current I_{sc}, Fig. 171 (*B*), the output voltage is zero. This same principle is used to limit current in high impedance tube circuits and neon signs.

Cross-sectional area through each shunt path is the same as that of the upper or lower leg of the shell laminations, so flux in the shunts

does not exceed that in the core. At short-circuit current I_{sc}, half of the total flux flows through each set of shunts. The air-gap length in each shunt path can be found from equation 23:

$$l_g = \frac{0.6NI_{sc}}{B_m} \quad \text{(inches)} \tag{94}$$

where N = secondary turns
B_m = allowable induction in the shunts (in gauss).

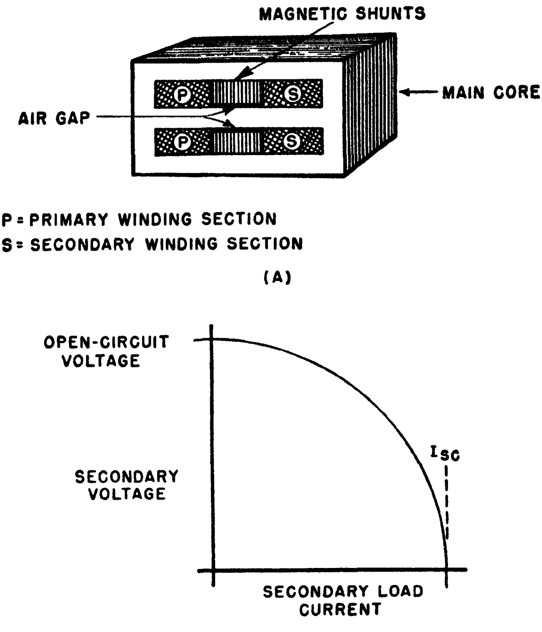


FIG. 171. (*A*) Current-limiting transformer; (*B*) output voltage versus current curve.

The constant 0.6 is generally too small because of the flux fringing around the gap. A closer value is 0.9. If the shunts are too short, the transformer does not limit the current properly. It is best to have slightly less air gap than necessary, and find by trial the right length of shunt. The fringing flux heats the coil windings somewhat more than in an ordinary transformer. If the secondary current is heavy, coils are wound pancake fashion and connected in parallel; they may have to be cross-connected for the coils to divide the load equally.

If the ordinate for open-circuit voltage and abscissa for short-circuit

current in Fig. 171 (B) are equal, the curve is a quarter-circle for a perfect transformer because the secondary current at short-circuit is all reactive. With core, shunt, and winding losses the curve for an actual transformer falls some 10 to 15 per cent less than the quarter-circle at currents 0.5 to 0.75 times I_{sc}.

94. Autotransformers. An autotransformer has a single winding which is tapped as shown in Fig. 172 to provide a fraction of the primary voltage across the secondary load. The connections may be reversed so that a step-up voltage is obtained. The regulation, leakage inductance, and size of an autotransformer for a given rating are all less than for a two-winding transformer handling the same power. Where the voltage difference is slight, the gain is large. Where the voltage difference is great, there is not much advantage in using an autotransformer, nor can it be used where isolation of the two circuits is required.

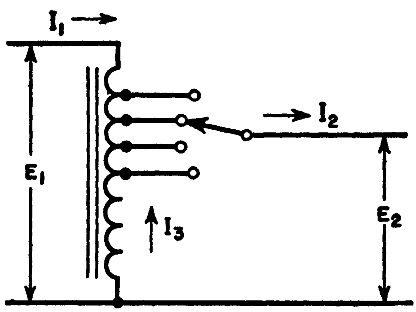


FIG. 172. Autotransformer voltages and currents.

Autotransformers are used in electronic applications chiefly for the adjustment of line voltage, either to change it or to keep it constant. Examples are the reduction of plate voltage for tuning an amplifier, and the maintenance of constant filament voltage. Taps may be chosen by means of a tap switch to adjust the load voltage. The load voltage may be adjusted to within half of the voltage increment between taps.

If the voltage is adjusted while load remains connected, bad switching arcs occur, either from breaking the circuit or from short-circuiting turns. To provide for adjustment under load conditions, a resistor may be momentarily connected in the circuit as the tap switch bridges from one tap to the next, and current is limited to full load value. In large power tap changers, a reactor replaces the resistor to avoid heating and losses.

The v-a rating of an autotransformer depends on the ratio of input to output voltage. In Fig. 172 the output current $I_2 = I_1 + I_3$. Let p = per cent tap/100 = E_2/E_1. Neglecting losses, $I_2 = I_1/p$ and $I_3 = (1/p - 1)I_1$. Then

$$\text{Volt-amperes (in the upper portion)} = (1 - p)E_1I_1 \tag{95}$$

$$\text{Volt-amperes (in the lower portion)} = pE_1I_3 = (1 - p)E_1I_1$$

which satisfies equality of volt-amperes in each section. For ratio p close to unity, the v-a rating and hence size for a given output can be made very small; for small values of p the size is not much less than that of a two-winding transformer, but the autotransformer has much

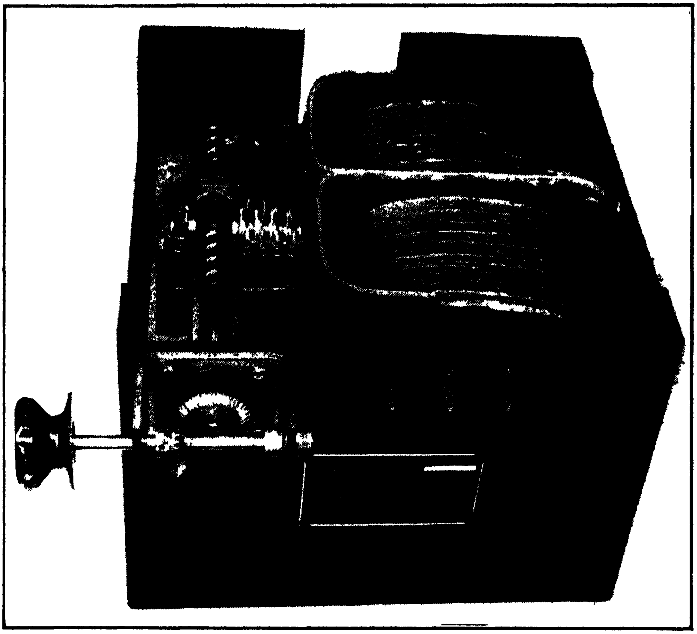


FIG 173 Adjustable primary anode transformer.

less regulation. Its effective winding reactance and resistance decrease as $(1 - p)^2$; that is, for a given unit,

$$\frac{X \text{ (or } R\text{) as autotransformer}}{X \text{ (or } R\text{) as two-winding transformer}} = (1 - p)^2 \qquad (96)$$

Appreciably less regulation is obtained in an autotransformer, even when size is not reduced much, because the right-hand term in equation 96 is squared.

When the power for electronic equipment is supplied by a 230-volt line, but auxiliary items such as relays and small motors are used at 115 volts, a convenient way of obtaining the latter voltage is to center-tap the primary of a large plate transformer, and use it as a 2:1 step-down autotransformer. The larger primary winding copper requires little extra space, and an additional transformer is thereby saved.

To improve the closeness of voltage control, a variable autotransformer has been developed in which the moving tap is a carbon brush which slides over exposed turns of the winding. Brush resistance prevents excessive transition current and permits smooth voltage control; yet it offers little additional series resistance to the load. The same idea can be applied to two-winding transformers for secondary voltage adjustment. A typical unit of this kind is shown in Fig. 173.

When autotransformers are used on three-phase supply lines, they may be connected the same as two-winding transformers in star, delta, open-delta, or Scott connections. The latter two connections are less subject to objectionably high regulation in autotransformers and, if they supply three-phase anode transformers, cause no serious primary voltage unbalance for voltage ratio p close to unity.

95. Static Voltage Regulators. Automatic regulators of various kinds have been devised for keeping comparatively small amounts of

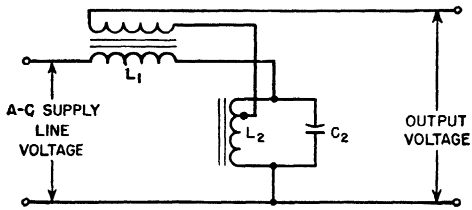


FIG. 174. Resonant-circuit voltage regulator.

power at a constant voltage. Figure 174 shows one circuit for a resonant-reactor voltage regulator. Inductance L_1 is linear. Inductance L_2 and capacitor C_2 are parallel-resonant at the supply line frequency and rated voltage. The pair draws very little current, so that the reactive voltage in L_1 is low. Output current flows through its secondary winding which is of such polarity as to maintain rated voltage. Inductance L_2 is partially saturated at this voltage. If line voltage falls below rated value, less current is drawn by L_2, and the L_2C_2 combination becomes untuned. Total current to the parallel circuit is then capacitive, and this capacitive current, drawn through L_1, raises the output voltage. Conversely, if line voltage rises above rated value, the L_2C_2 combination becomes untuned on the inductive side, and the output voltage falls below the line value. Output voltage variations of ± 1 per cent are obtained with ± 10 per cent line voltage variations in this manner, and with load changes from zero to full load.

Constant supply frequency is a condition for resonance at rated voltage; with the good frequency control of modern power systems this condition is generally fulfilled. Load power factor variations cause output voltage to change. Some regulators are provided with

taps to minimize this effect. Output wave form contains a noticeable third harmonic, because the large magnetizing current of L_2 must flow through appreciable impedance in L_1. Owing to the partial saturation of reactor L_2, it tends to operate at a high temperature and requires good ventilation. Practical regulators are in use with ratings from 25 v-a to 1500 v-a.

96. Automatic Gain Control. Vacuum tube amplification factor is not constant under all conditions of operation. For example, in Fig. 86 (p. 110), in the vicinity of the operating point P, the amplification factor is constant. When the data are expanded to take into account high current operation as in Fig. 87, the amplification factor in the region of high anode current and low anode voltage is no longer constant.

Some tubes are designed to have large variations in amplification factor. These are known as variable-mu, remote cut-off, or super-control tubes. The mutual conductance of such tubes is highly variable with grid bias. Figure 175 (*a*) is the curve of mutual conductance for a tube of this kind. Such a characteristic can be used to reduce gain at high amplitudes and thus prevent overmodulation in audio systems. In Fig. 175 (*b*) a circuit is shown which automatically reduces gain for excessive values of applied grid voltage e_g on the grid of the 6SK7 tube. This tube drives a 6L6 output tube through transformer T_1. On this transformer there is an auxiliary winding S_2 which is connected to rectifier tube 6H6-1 and produces the rectified output across resistance R_2 having a negative potential at the point shown. With large signals, the voltage rectified across R_2 is large and reduces the mutual conductance and plate voltage swing of the 6SK7 tube. Nearly constant output voltage is maintained in the 6L6 output.

If the power output of the 6L6 tube is delivered mainly into a linear a-c impedance, the slight additional load imposed by the gain control makes little difference. But if all the output is delivered to rectifier loads, as it is in Fig. 175 (*b*), the non-linearity of both tube and load causes output distortion. This is true particularly of beam or pentode output tubes. The normal class A output of a 6L6 beam tube is 6 watts but, if the output power is all rectified, only 50 mw can be drawn without excessive distortion. Half-wave rectifiers and capacitor-input filter outputs are worst in this respect, because of the current discontinuities. If the automatic gain control rectifier input is taken from a tuned amplifier, these difficulties decrease. The tuned circuit capacitor readily supplies irregular current wave forms, provided the amplifier has sufficient power output available.

Automatic volume control (*AVC*) is applied in receivers to either the r-f or audio stages, to maintain approximately constant volume in

spite of fading or other causes of input voltage variations. It is applied in audio amplifiers to maintain better output volume with differing voice levels.

If the input grid resistor R_1 in Fig. 175 (*b*) is connected to a fixed negative bias the *AVC* is inoperative below the value of bias voltage.

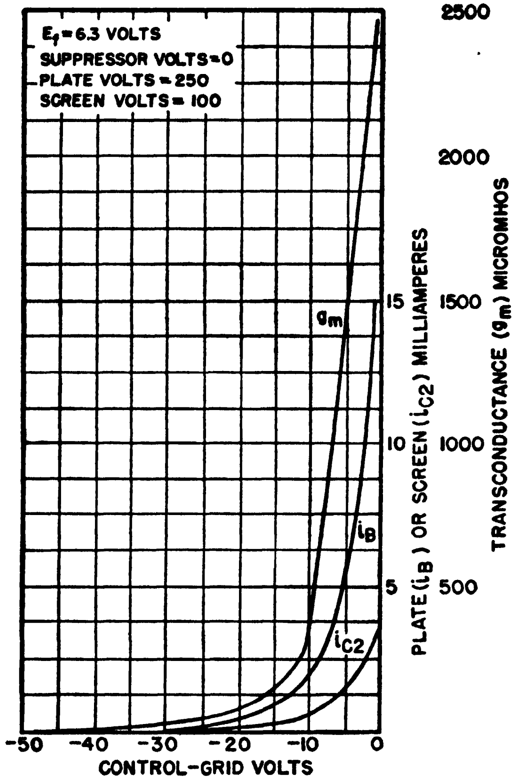


FIG. 175 (*a*). Variable-mu tube (6SK7) mutual conductance curve.

This is called delayed *AVC*; with it, no *AVC* is applied until a certain output level is reached. In some receivers, more than one stage may be controlled, and the *AVC* action is amplified.

Demodulation is accomplished in the circuit of Fig. 175 (*b*) by means of diode 6H6-2. Each half of the r-f cycle is rectified and the d-c power is absorbed in resistor R_3. Audio power is bypassed around R_3 by capacitor C_2 and the voltage is impressed upon the primary of transformer T_2. If an amplitude modulated wave is used in this amplifier, the output voltage of winding S_1 on transformer T_1 has the form shown in Fig. 176. The first few cycles are shown as full-wave rectified loops

with constant amplitude, that is, with no modulation. The audio output for this section of the wave is zero. A sine wave envelope of 100 per cent modulation is shown in the rest of the figure. Average volt-

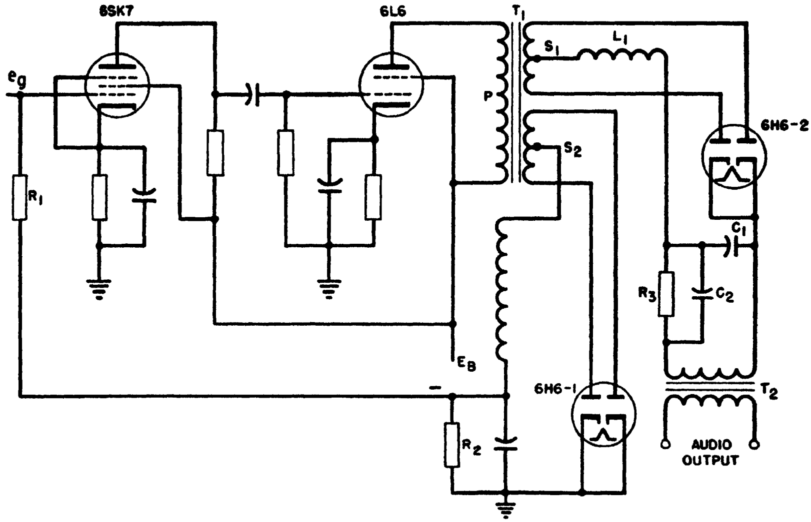


FIG. 175 (b). Automatic gain control and demodulator circuits.

age left after the carrier frequency half-loops have been absorbed by the r-f filter L_1C_1 is the audio voltage impressed on transformer T_2.

The method of demodulation just described is known as diode demodulation. It is often accomplished by means of a single diode, in which case every other lobe of the wave in Fig. 176 is omitted. Methods

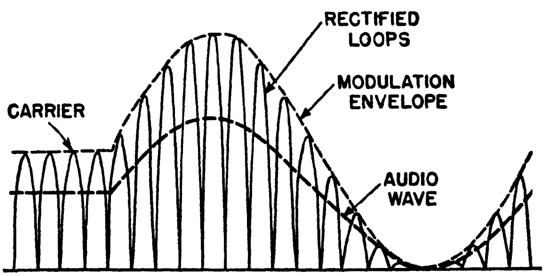


FIG. 176. Rectified amplitude-modulated wave.

are in use also for demodulation with a triode, in which some amplification of the demodulated wave is obtained.

Circuits similar to this are used in power-line carrier receivers. The carrier frequency is 50 to 150 kc, and audio frequencies are employed

for modulation. In transformer T_1 some special problems are encountered because the transformer operates over a range of 50 to 150 kc, delivers the correct amount of voltage to the automatic gain control tube 6H6-1 for proper AVC action, delivers the proper output to the audio load without distortion, and obtains these voltages from a nearly constant current source. The transformer ratio is obtained by estimating the r-f voltage swing obtained with a square primary input current wave, and dividing this by the voltage required to produce the necessary audio output after choke L_1 smooths the rectified lobes to the average value shown by the heavy dotted lines in Fig. 176. Transformer voltages and currents are calculated as in Table VII (p. 46) for a single-phase full-wave rectifier, but with peak audio-current and voltage taking the place of d-c output.

PROBLEMS

1. What minimum and maximum phase shift of E_g and E'_g with respect to the supply voltage are obtainable with the circuit elements in the example of Section 91? *Ans.* 56.2°; 154.4°.

2. With No. 34 enamel wire in all coils of the reactor in Section 91, calculate coil dimensions and winding resistances and d-c power required. *Ans.* A-c coils 170 ohms each; d-c coil 530 ohms; 0.477 watt.

3. Design a saturable reactor for zero to 60° phase shift of E_g, with zero to 20 ma d-c, other conditions remaining as in Section 91.

4. What is the rms anode current of Fig. 162 (*c*) when conduction begins at 60°? At 90°? *Ans.* $0.635I_{pk}$; $0.5I_{pk}$.

5. Design an anode transformer for the circuit of Fig. 160 to obtain 300 volts 0.5 amp in the load with a 60-cycle supply. Assume 25 volts drop in the tube and B_m = 14,000 gauss, with a silicon steel core.

6. Design an anode transformer as in Prob. 5 except that B_m = 11,000 gauss. Compare sizes.

7. With a 6L6 peak plate voltage swing of 50 volts at 100 per cent modulation, what transformer ratio P/S_1 is necessary for peak audio voltage of 50 volts on the primary of T_2 Fig. 175 (*b*)? Assume that capacitor C_2 reactance is negligible. *Ans.* 0.636.

8. By constructing a table like Table XI (p. 111) show that, with audio output of 200 mw, the voltage wave of transformer T_1, Fig. 175 (*b*) becomes badly distorted. Assume that the sinusoidal peak grid voltage swing is 5 volts, and that plate current = 70 ma for $0 < \theta < 180°$ and 30 ma for $180° < \theta < 360°$, with class A conditions in the 6L6 tube (E_B = 250 volts; E_C = −14 volts). Assuming that this current includes the AVC component, find the transformer ratio P/S_2 necessary to supply −40 volts to the 6SK7 grid, with 5 volts drop in 6H6, R_2 = 50,000, and a 2-μf filter capacitor across R_2. *Ans.* 1:1.6.

9. Design a current limiting transformer to deliver 11 volts at 60 amp full load with the primary connected to 230 volts single-phase 60 cycles. Limit the secondary short-circuit current to 120 amp. Try a 4-in. stack of 0.014-in. silicon steel laminations B (Fig. 31, p. 39), and B_m = 11,000 gauss.

IX. PULSE AND VIDEO TRANSFORMERS

97. Square Waves. Square waves or pulses differ from sine waves in that the front and back sides of the wave are very steep and the top flat. Such pulses are used in the television and allied techniques to produce sharp definition of images or signals. A square wave can be thought of as made up of sine waves of a large number of frequencies starting with, say, audio frequencies and extending into the medium r-f range. The response of a transformer to these frequencies determines the fidelity with which the square wave is reproduced by the transformer. Some pulses are not square, but have sloping sides and a round top, like a half-wave rectifier voltage. Such pulses will not be discussed here, because if a transformer or circuit is capable of reproducing a square wave, it will reproduce a rounded wave at least as well.

Square waves can be generated in several ways: sometimes from sine waves by driving a tube into the diode region each cycle. The plate circuit voltage wave form is then different from that of the grid voltage because the round top of the sine wave has been removed. By repeating this process (called clipping) in several stages, it is possible to produce very square waves, alternately plus and minus, like those of Fig. 177 (*a*). If a voltage having such a wave form is applied across a capacitor, it causes current to flow in the capacitor only at the vertical edges, as in Fig. 177 (*b*). If a voltage proportional to this current is then successively amplified and clipped at the base, it results in the wave form of Fig. 177 (*c*). Here the negative loops are assumed to be missing, as they could be after single-side amplification. The wave is again square, but of much shorter duration than in Fig. 177 (*a*), and the interval between pulses greatly exceeds the pulse duration. The pulse duration is usually referred to as the pulse width, and the fre-

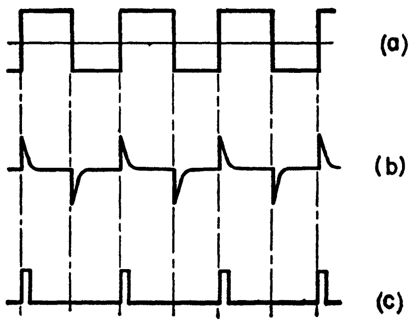


FIG. 177. Square waves differentiated and clipped.

quency at which the pulses occur is called the repetition rate and is expressed as pulses per second (pps). Common pulse widths lie between 0.5 and 10 microseconds; the intervals between pulses may be between 10 and 1000 times as long as the pulse width. These values are representative only, and in special cases may be exceeded several-fold. The general wave shape of Fig. 177 (*c*), with short pulse duration compared to the interval between pulses, is the main subject of this chapter. The ratio of peak to average voltage or current may be very high, and the rms values appreciably exceed average in such pulses.

There are two ways in which the response of any circuit to a square wave can be found. The first of these consists in resolving the pulse into a large number of sine waves of different frequencies, finding the response of the circuit to each frequency, and summing up these responses to obtain the total response. This can be formulated by a Fourier integral, but for most circuits the formulation is easier than the solution. An approximation to this method is the arbitrary omission of frequency components having negligible amplitude, and calculation of the circuit response to the relevant frequencies. This approximation has two subjective criteria: The number of frequencies to be retained and the evaluation of the frequency components for which the circuit has poor response.

The second method, which will be used here, consists in finding the transient circuit response to the discontinuities at the front and trailing edges of the square wave. It is possible to reduce the transformer to a circuit amenable to transient analysis, without making any more assumptions than would be necessary for practical design work with the Fourier method. The transient method has the advantage of giving the total response directly, and it can be plotted as a set of curves which are of great convenience to the designer. The major assumption is that one transient disappears before another begins. If good square wave response is obtained, this assumption is justified.

Analyses are given of the influence of iron-core transformer and choke characteristics on pulse wave forms. In all these analyses the transformer or choke is reduced to an equivalent circuit; this circuit changes for different wave forms, portions of a wave form, and modes of operation. Curves are included which are calculated from the formulas derived and which facilitate design procedure. Derivations of formulas are given in the Appendix.

98. Transformer-Coupled Pulse Amplifiers.[1] The analysis here given is for a square- or flat-topped pulse impressed upon the transformer by

[1] Sections 98, 99, and 100 are based on the author's "Iron-Core Components in Pulse Amplifiers," *Electronics*, Aug. 1943, p. 115.

some source such as a vacuum tube, a transmission line, or even a switch and battery. Such a pulse is shown in Fig. 178, and a generalized circuit for the amplifier is shown in Fig. 179. The equivalent circuit for such an amplifier is given in Fig. 180. At least this is the circuit which applies to the front edge a of the pulse shown in Fig. 178

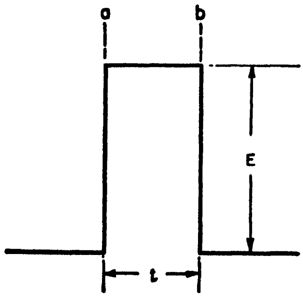


FIG. 178. Flat-topped pulse.

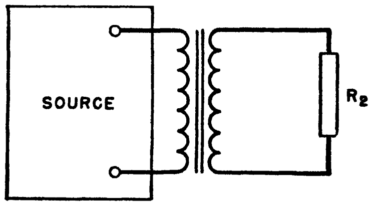


FIG. 179. Transformer coupling.

as rising abruptly from zero to some steady value E. This change is sudden, so that the transformer OCL can be considered as presenting infinite impedance to such a change, and is omitted in Fig. 180. Transformer leakage inductance, though, has an appreciable influence and is shown as inductance L in Fig. 180. Resistor R_1 of Fig. 180 represents the source impedance; transformer winding resistances are generally negligible compared to the source impedance. Winding capacitances are shown as C_1 and C_2 for the primary and secondary windings, respectively. The transformer load resistance, or the load resistance into which the amplifier works, is shown as R_2. All these values are

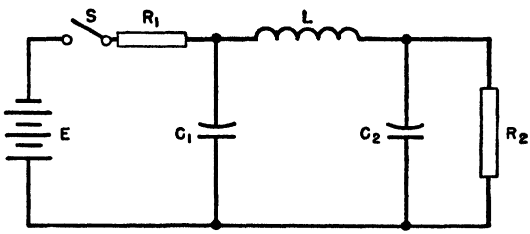


FIG. 180. Equivalent circuit.

referred to the same side of the transformer. Since there are two capacitance terms C_1 and C_2, it follows that for any deviation of the transformer turns ratio from unity, one or the other of these becomes predominant. Turns ratio and therefore voltage ratio affect these capacitances, as discussed in Chapter V; for a step-up transformer, C_1

may be neglected and, for a step-down transformer, C_2 may be neglected. The discussion here will be confined first to the step-up case.

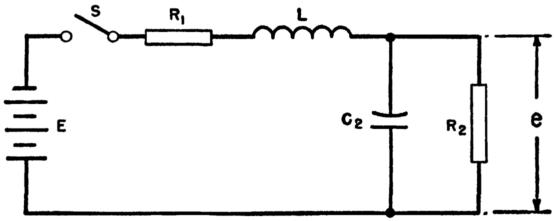


FIG. 181. Circuit for step-up transformer.

99. Front Edge Response. The step-up transformer is illustrated by Fig. 181. When the front of the wave (Fig. 178) is suddenly impressed on the transformer, it is simulated by the closing of switch S.

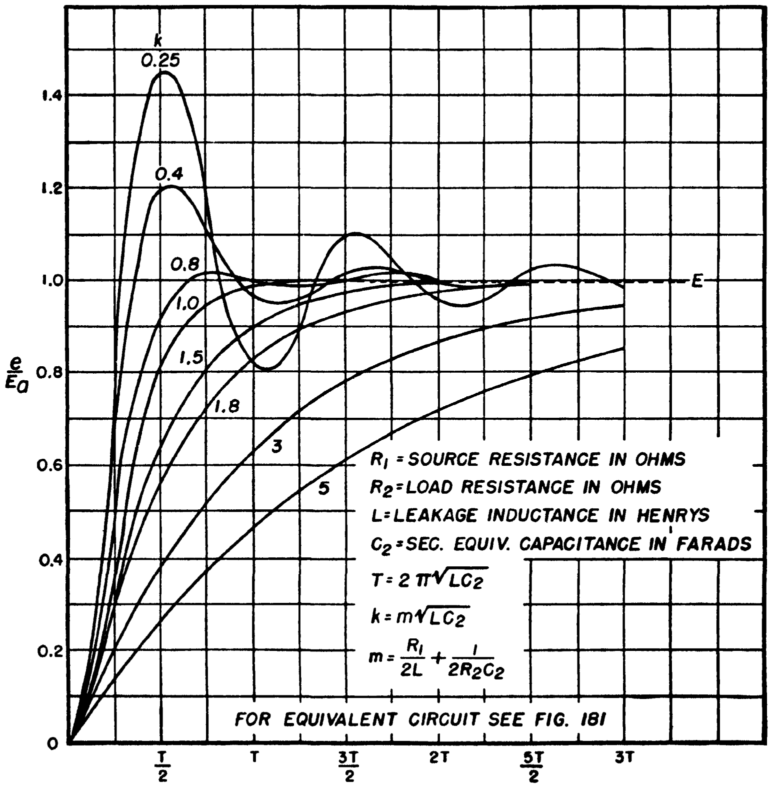


FIG. 182. Influence of transformer constants on front edge of pulse.

At this initial instant, voltage e across R_2 is zero, and the current from battery E is also zero. This furnishes two initial conditions to be used in the derivation of equation 107 which expresses the rate of rise of voltage e from its initial value of zero to its final steady value of $E_a = ER_2/(R_1 + R_2)$. Equation 107 is derived in Appendix A. Figure 182 shows the rate of rise of the transformed pulse.

The scale of abscissas for these curves is not time but percentage of the time constant T of the transformer. The equation for this time

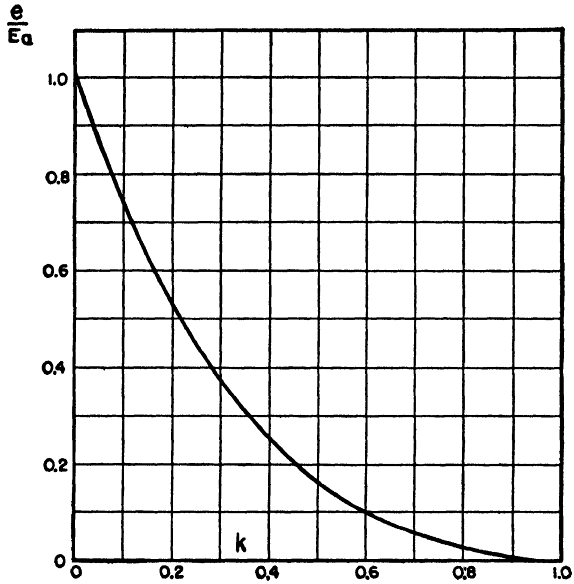


FIG. 183. Pulse transformer overshoot voltage.

constant is given on the curves, and is a function of leakage inductance and of capacitance C_2. The rate of voltage rise is governed to a marked extent by another factor k which is the ratio of the decrement to the angular frequency for an oscillatory circuit, but retains the same form even if the circuit is not oscillatory. The relation of this factor k and the various constants of the transformer is given directly in Fig. 182. The greater the transformer leakage inductance and distributed capacitance, the slower is the rate of rise, although the effect of R_1 and R_2 is important, for they affect the factor k. If a slight amount of oscillation can be tolerated, the wave rises up faster than if no oscillation is present. Yet if the circuit is damped very little, the oscillation may reach a maximum initial voltage of twice steady-state voltage E_a, and usually such high peaks are objectionable. The values for k given

in Fig. 182 are those which fall within the most practicable range. Voltage overshoot above steady value E_a is given in Fig. 183 for oscil-

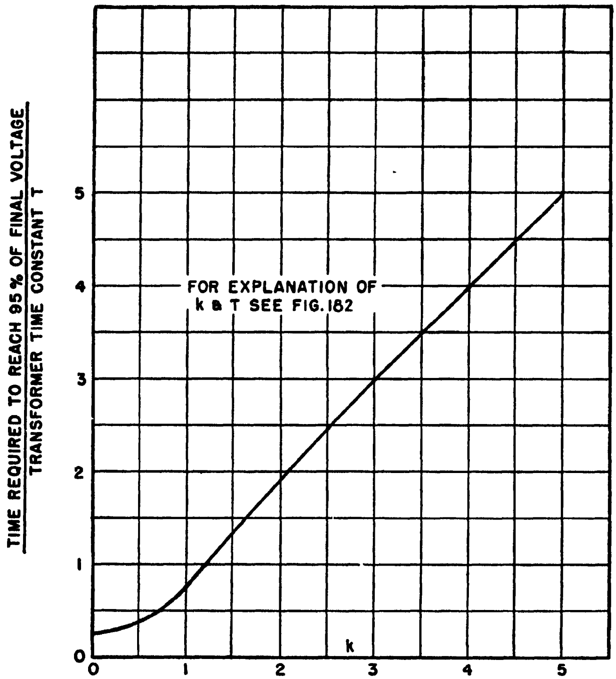


FIG. 184. Time required to reach 95 per cent of final voltage.

latory transformers. The time required to reach 95 per cent of E_a is shown by Fig. 184.

100. Response at the Top of the Pulse. Once the pulse top is reached, E_a is dependent on the transformer OCL for its maintenance at this value. If the pulse stayed on indefinitely at the value E_a, an infinite inductance would be required to maintain it so, and of course this is not practical. There is always a droop at the top of such a pulse. The equivalent circuit during this time is shown in Fig. 185. Here the inductance L is the OCL of the transformer, and R_1 and R_2 remain the same as before. Since the rate of voltage change is relatively small

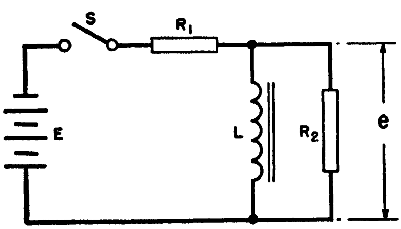


FIG. 185. Circuit for top of pulse.

during this period, capacitances C_1 and C_2 disappear. Also, since leakage inductance usually is small compared with the OCL, it is neglected. At the beginning of the pulse, the voltage e across R_2 is assumed

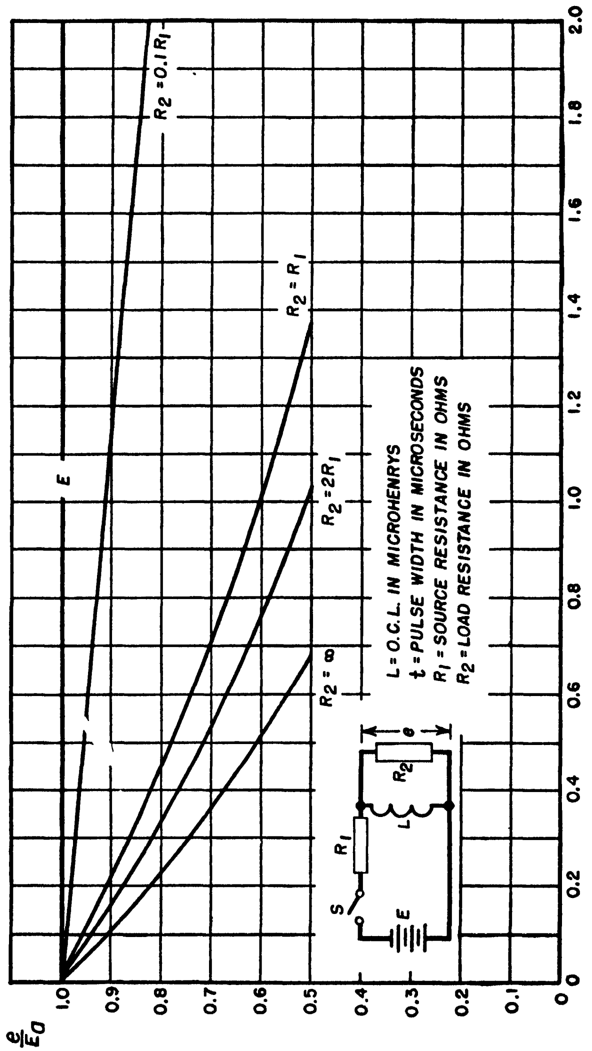


FIG. 186. Influence of transformer OCL on square-topped pulses.

to be at steady value E_a which is true if the voltage rise is rapid. Curves for the top of the wave are shown by Fig. 186. Several curves are given; they represent several types of pulse amplifiers ranging from a pentode in which R_2 is one tenth of R_1, to an amplifier in which load resistance is infinite and output power is zero. In the latter curve, the

voltage e has for its final value the voltage of the source. All the curves are exponential, having a common point at 0, 1. Abscissas are not time, but are the product of time and R_1/L, the time being the duration of the pulse between points a and b in Fig. 178. The greater the inductance L the less the deviation from constant voltage during the pulse.

101. Trailing Edge Response. At instant b in Fig. 178, it is assumed that the switch S in Fig. 185 is opened suddenly. The circuit now reverts to that of Fig. 187, in which L is the OCL, but secondary capacitance C_2 is again of consequence. Figures 188, 189, and 190 illustrate the decline of pulse voltage after instant b (Fig. 178), the derivation for which is given in Appendix A. Abscissas are the time constant determined by the OCL and capacitance C_2. The constant k on these curves is again the decrement divided by the angular frequency for the oscillatory case, but the decrement has a different meaning, as indicated in Fig. 188. The time constant T is greater in these figures than in Fig. 182; but with low capacitance, k is high and the curves with higher values of k drop rapidly. The slope of the trailing edge can be kept within tolerable limits, provided the capacitance of the transformer can be kept small enough. Accurate knowledge of this capacitance is therefore important. The measurement and evaluation of transformer capacitance are treated as in Chapters V and VII.

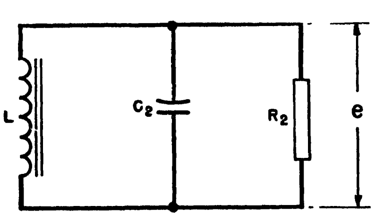


FIG. 187. Circuit for trailing edge.

If the transformer has appreciable magnetizing current, the shape of the trailing edge is changed. The greater the magnetizing current, the more pronounced the negative voltage backswing. The ordinates of Figs. 188, 189, and 190 are given in terms of the voltage E_a, at instant a, as if there were no droop at the top of the pulse. These curves apply when there is droop, but then the ordinates should be multiplied by the fraction of E_a to which the voltage has fallen at the end of the pulse. Exciting current at the end of the pulse is

$$i_L = \frac{E_a}{mL}(1 - \epsilon^{-mt}) \tag{97a}$$

where $m = R_1R_2/(R_1 + R_2)L$
t = pulse duration in seconds
L = primary OCL in henrys.

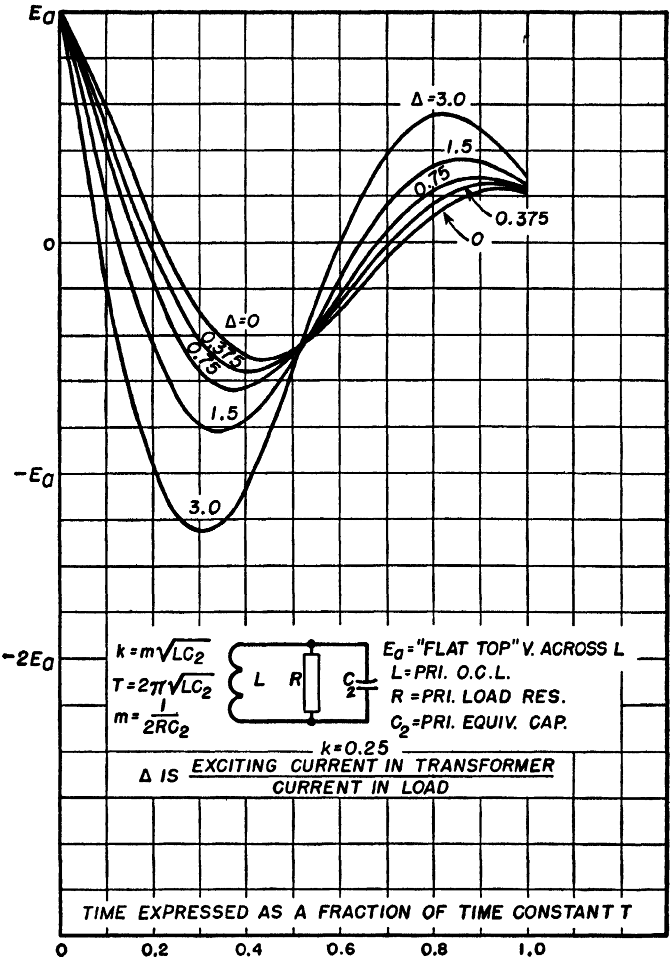


FIG. 188. Trailing edge of pulse, oscillatory transformer.

Exciting current can be expressed as a fraction Δ of the primary load current I, or $\Delta = i_L/I$. For any R_1/R_2 ratio, $\Delta = [(R_1 + R_2)/R_1] \times$ voltage droop at point b (Fig. 178), or

$$\frac{E'}{E_a} = 1 - \frac{R_1 \Delta}{R_1 + R_2} \tag{97b}$$

where E_a = voltage at point a (Fig. 178), and E' = voltage at point b.

This equation gives the multiplier for finding the actual trailing edge voltage from the backswing curve parameters in Figs. 188, 189, and 190. With increasing values of Δ the backswing is increased, especially

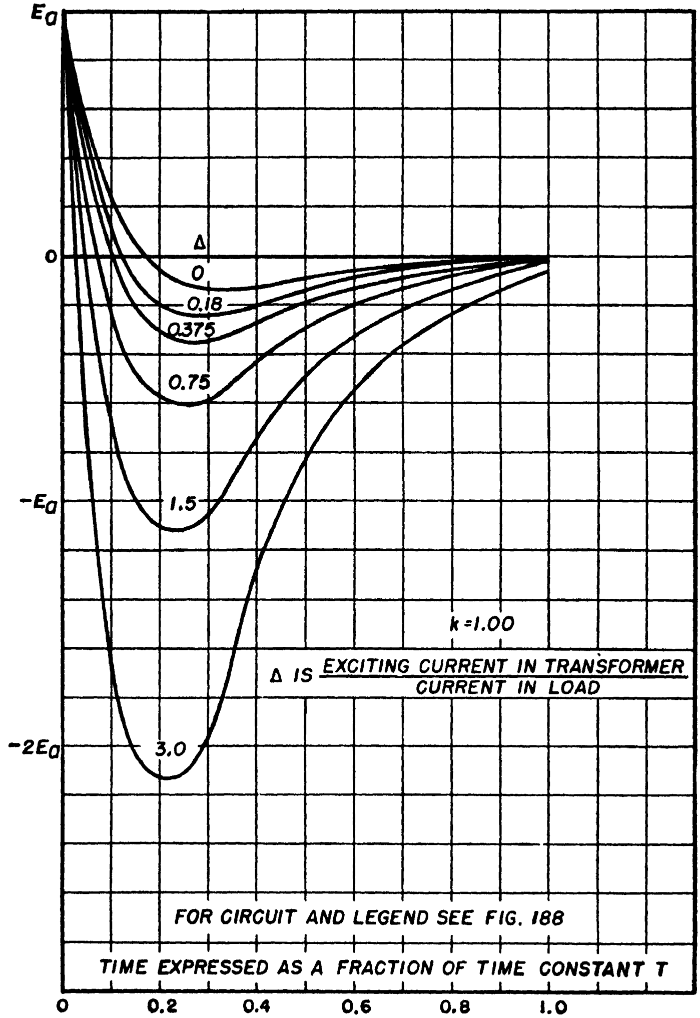


FIG. 189. Trailing edge of pulse, borderline case.

for the damped circuits corresponding to values of $k \geqq 1.0$. The same is also true for lower values of k, but with diminishing emphasis, so that in Fig. 188 exciting current has less influence on the oscillatory

backswings. These afford poor reproduction of the original pulse shape, but occasionally large backswing amplitudes are useful, as mentioned in Section 112.

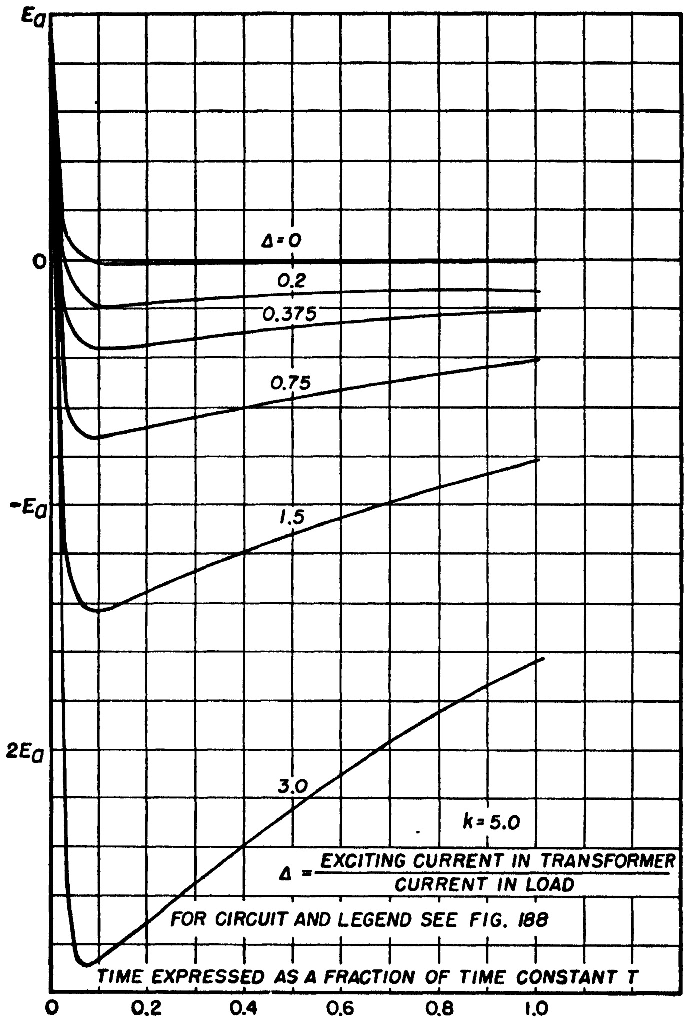


FIG. 190. Trailing edge of pulse, highly damped transformer.

In transformers with oscillatory constants the backswing becomes positive again on the first oscillation. In some applications this would appear as a false and undesirable indication of another pulse. The

conditions for no oscillations are all included in the real values of the equivalent circuit angular frequency, or by the inequality

$$\frac{1}{4R_2{}^2C_2{}^2} > \frac{1}{LC_2}$$

or

$$\sqrt{\frac{L}{C_2}} > 2R_2$$

Terms are defined in Fig. 188, with $R = R_2$.

The quantity $\sqrt{L/C_2}$ may be regarded as the open-circuit impedance of the transformer. Its value must be more than twice the load resistance (on a 1:1 ratio basis) to prevent oscillations after the trailing edge. This requires low distributed capacitance.

Likewise the negative backswing may prove objectionable in certain apparatus. Certain conditions for avoiding all backswing are those represented in Fig. 190 by $k = 5$ and $\Delta = 0$; these require good core material, low exciting current, low distributed capacitance, and a loaded transformer.

102. Total Response.[2] By means of the curves we can now construct the pulse shape delivered to load R_2. Suppose that a transformer with the following properties is required to deliver a flat top pulse of 15 microseconds' duration.

Primary leakage inductance (secondary short-circuited)	$= 1.89 \times 10^{-4}$ henry
Primary open-circuit inductance	$= 0.1$ henry
Primary/secondary turns ratio N_p/N_s	$= 1:3$
Source resistance R_1	$= 800$ ohms
Load resistance (primary equivalent) R_2	$= 5000$ ohms
Primary effective capacitance C_2	$= 448\ \mu\mu f$

From the expressions given in Fig. 182,

$$\begin{aligned} m &= 2.34 \times 10^6 \\ T &= 1.8 \text{ microsecond} \\ k &= 0.68 \end{aligned}$$

The front of the wave follows a curve between those marked $k = 0.4$ and $k = 0.8$ in Fig. 182. Value E_a is reached in $0.5T$ or 0.85 microsecond, and an overshoot of about 10 per cent occurs in 1.2 microseconds.

[2] This section is based on the author's "Iron-Core Components in Pulse Amplifiers," *Electronics*, Aug. 1943, p. 115.

The top of the wave slopes down to a voltage determined by the product $tR_1/L = 0.12$, and by a curve between those for $R_2 = \infty$ and $R_2 = 2R_1$ in Fig. 186. Voltage E' at b is evidently $0.9E_a$.

The trailing edge is approximated in Fig. 189. Here

$$\begin{aligned} m &= 0.223 \times 10^6 \\ T &= 42.2 \times 10^{-6} \\ k &= 1.5 \\ \Delta &= \tfrac{5800}{800} \times 0.09 = 0.65 \end{aligned}$$

Load voltage reaches zero in $0.05T$ or 2.11 microseconds. The negative loop has maximum amplitude of about 50 per cent E' at

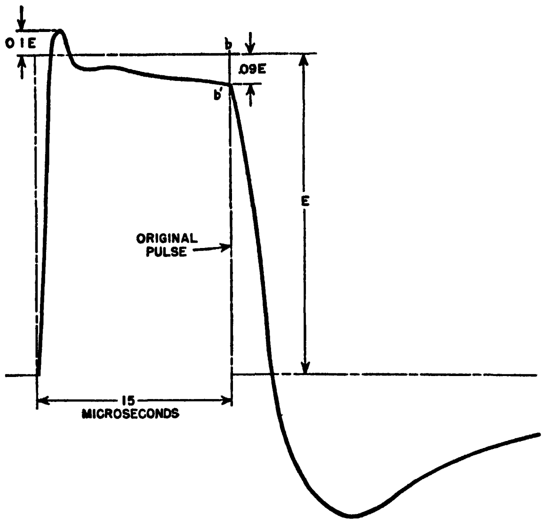


FIG. 191. Output voltage of pulse transformer.

$0.2T$ or 8.44 microseconds beyond the pulse edge b. The pulse delivered to load R_2 is shown in Fig. 191, in terms of E instead of E_a.

So far we have assumed that the pulse source is disconnected at the end of the pulse. In some applications the source remains connected. This would result if switch S (Fig. 185) were left closed, and battery E were short-circuited. Under these conditions the leakage inductance remains in the circuit, and an additional transient occurs which depends upon the same constants as in Fig. 182. The transient has the shape of one of the curves of Fig. 182, but is inverted and superposed on the backswing voltage due to OCL. In the example just given, this superposed oscillation has an amplitude of 10 per cent of E, with a total result similar to the oscillogram of Fig. 192. Because of the

distributed nature of leakage inductance and capacitance, higher frequency superposed oscillations may sometimes occur even when the load is disconnected at the end of a pulse. By their very nature, the conditions for these oscillations are difficult to state with certainty,

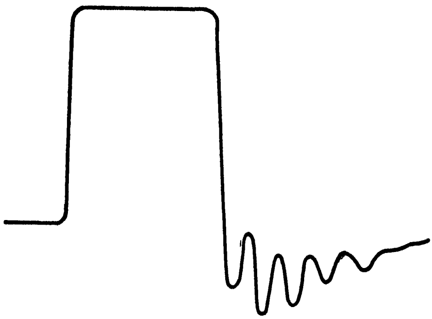


FIG. 192. Oscillogram of voltage pulse.

but if oscillations occur on the front edge they are likely to appear on the trailing edge, superposed on the voltage backswing.

103. Step-Down Transformers. The circuits of Figs. 181 and 185 for step-up pulse transformers are essentially the same as those of Figs. 89 (*e*) and 89 (*c*), respectively, for audio transformers. Low frequency response corresponds to the top of the pulse and high frequency response to the front edge. In step-down pulse transformers the top is unchanged, but the front edge corresponds to Fig. 95. Step-down transformers are analyzed in Appendix B for the oscillatory case. This analysis shows that the form of equation 113 (Appendix B) is similar to that for step-up transformers, except that the

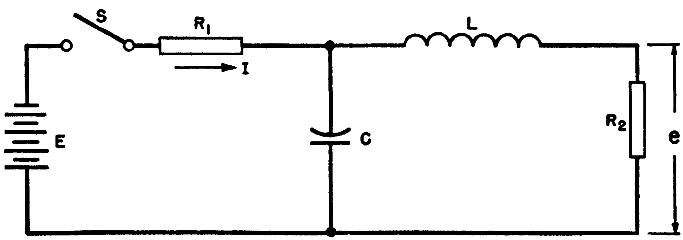


FIG. 193. Step-down transformer equivalent circuit.

damping factor for the sine term is greater by the quantity $R_2/L\beta$. Also, the decrement, although still composed of two terms, has the resistances R_1 and R_2 in these two terms reversed with respect to the corresponding terms for the step-up transformer. Except for this, the

front edge curves are little different in shape from those of Fig. 182, and for most purposes can be regarded as being virtually the same.

Pentode amplifiers, with their constant current characteristics, can be represented by the circuit of Fig. 193. Here I is the current entering the primary winding from the tube, and is constant over most of

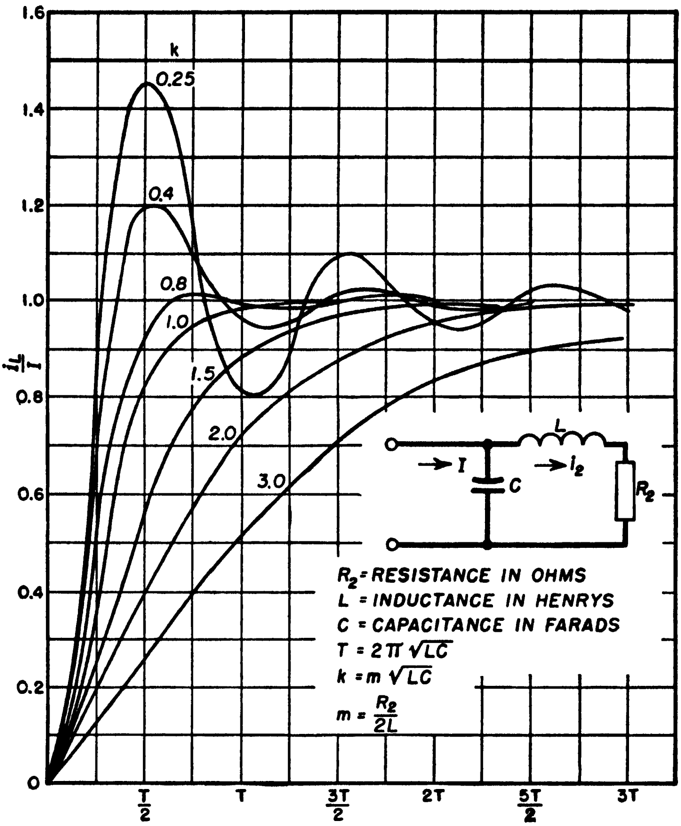


FIG. 194. Pentode amplifier front edge response.

the voltage range. The transformer is usually step-down for the reasons of impedance mentioned in Section 63 (Chapter V). It is shown in Appendix B that the front edge response of these transformers is the same as in Fig. 182 if the rise in load current is expressed as a fraction of final current I, and that the decrement is changed to $R_2/2L$. It is reproduced in Fig. 194 with this change in constants. Flatness of top is approximately that of the curve $R_2 = 0.1R_1$ in Fig. 186. Trailing edge is the same as in Figs. 188, 189, and 190.

It is evident that many practical cases are represented by these figures. If transformer constants are outside the curve values, the pertinent equation should be plotted to obtain the response. If none of the equations appearing in the Appendix has been derived for a particular circuit, another derivation is necessary, but the same general methods of attack can be used.

104. Frequency Response and Wave Shape. Because of the prevalent thinking of engineers in terms of frequency response rather than wave shape, it is sometimes necessary to correlate the two concepts. The matter of phase shift enters, for the reason that the relative phase of the different frequency components affects wave shape. It is sometimes convenient to know whether a transformer, whose frequency response is known, can deliver a given wave shape. Starting with the low frequency response, assume equal source and load resistances; the upper curve of Fig. 90 (p. 120) applies. This curve shows 90 per cent of maximum response at the frequency for which $X_N/R_1 = 1$. How does this frequency compare with the reciprocal of the pulse width at the end of which there is 10 per cent droop in the top of the pulse? X_N/R_1 can be written

$$\frac{2\pi fL}{R_1} = 1 \quad \text{or} \quad f = \frac{R_1}{2\pi L} \tag{98}$$

Likewise, from the proper curve of Fig. 186, for 10 per cent droop,

$$\frac{tR_1}{L} = 0.2 \tag{99}$$

Combining equations 98 and 99 gives $f = 0.0318\ (1/t)$, or the transformer should be not over 1 db down at a frequency about $1/30$ of the reciprocal of the pulse width. For example, if a maximum of 10 per cent droop is desired at 2 microseconds the response should be not more than -1 db at $0.0318 \times 0.5 \times 10^6 = 16$ kc. Maximum phase shift is 27 degrees (from Fig. 111, p. 148), but this is taken into account in Fig. 186.

Similarly, steepness of the front edge can be related to transformer high frequency response, which for the case of $R_1 = R_2$ is found in Fig. 91. The corresponding front edge curves are found in Fig. 182. Parameter k of these curves is related to B in Fig. 91 as follows.

$$k = m\sqrt{LC} = \frac{R_1\sqrt{LC}}{2L} + \frac{\sqrt{LC}}{2R_1C} \quad (\text{for } R_1 = R_2)$$

$$B = \frac{X_C}{R_1} = \frac{X_L}{R_1} \quad \text{at frequency } f_r$$

$$= \frac{2\pi f_r L}{R_1} = \frac{L}{R_1\sqrt{LC}}$$

$$k = \frac{1}{2B} + \frac{B}{2} \tag{100}$$

From equation 100 we can prepare Table XV.

TABLE XV. PARAMETERS FOR FREQUENCY RESPONSE AND WAVE SHAPE

B	k
1.0	1.0
0.8, 1.25	1.025
0.67, 1.5	1.08
0.5, 2	1.25
0.25, 4	2.125

If a transformer has frequency response according to the curve for $B = \frac{1}{2}, 2$ in Fig. 91, its front edge will rise between curves for $k = 1$ and $k = 1.5$ in Fig. 182.

Transformer OCL, leakage inductance, and effective capacitance must be known to make this comparison, but these quantities are already known if it is established that the frequency response is given by Figs. 90 and 91, or the wave shape by Figs. 182 and 186. If conditions other than $R_1 = R_2$ prevail, another set of response curves can be used, and corresponding relations can be found in the manner here outlined.

Pulse transformer windings are similar to those in the high frequency transformers described in Section 82 (Chapter VII). Resonance frequency f_r is determined largely by leakage inductance and winding-to-winding capacitance. With pulse operation, partial resonances of sections of a coil, and even turn-to-turn resonance, may appear because of the steep front edge of voltage impressed on the transformer. If these resonances cause pronounced oscillations in the output wave form, larger coil or turn spacings or fewer turns may be necessary to reduce them.

105. Core Material. In Chapter VII it was shown that core permeability decreases with frequency, especially at frequencies higher than audio. This decrease also occurs with short pulse widths. When a pulse is first applied to the transformer, there is initially very little penetration of flux into the core laminations because of eddy currents. Hence initially only a fraction of the total core is effective, and the apparent permeability is less than later in the pulse, or after the flux density becomes uniform throughout the laminations.

A typical *B-H* curve for pulse transformers is shown in Fig. 195. Flux density builds up in the core in the direction shown by the arrows.

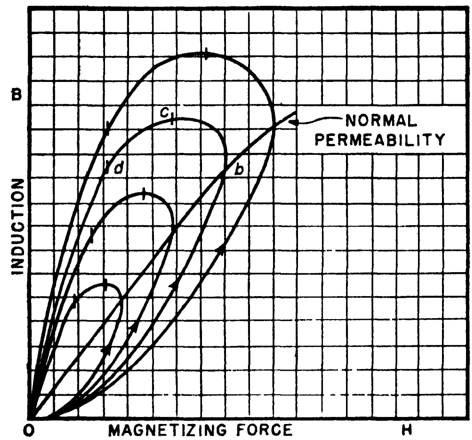


FIG. 195. Pulse *B-H* loops.

For a typical loop such as *obcd*, the slope of the loop (and hence the permeability) rises gradually to the end of the pulse *b* which corresponds to point *b′* in Fig. 191. Since magnetizing current starts decreasing at this point, *H* also starts decreasing. Current in the windings does not decay to zero immediately, but persists because of winding capacitance, and sufficient time elapses for permeability to increase. Therefore, flux density *B* may also increase during a short interval after point *b*. The trailing edge of the pulse voltage soon reaches zero, and this corresponds to point *c* on the loop. At some interval later, the maximum backswing amplitude is reached, which corresponds to point *d* on the loop. At this point the slope or permeability is several times as great as at point *b*.

For any number of pulses of varying amplitudes but of the same width, there are corresponding loops having respective amplitudes *c*. A curve drawn through point *b* of each loop is called the normal per-

meability curve, and this is ordinarily given as the permeability curve for the material. The permeability μ for a short pulse width is less than the 60-cycle or d-c permeability for the same material. Values of pulse permeability for 2-mil grain-oriented steel are given in Fig. 196. The permeability values include the irreducible small gap which exists in type C cores; the cores on which the measurements were made had a ratio of gap to core length $l_g/l_c \approx 0.0003$, but the data are not critically dependent on this ratio. The effect of penetration time is clear.

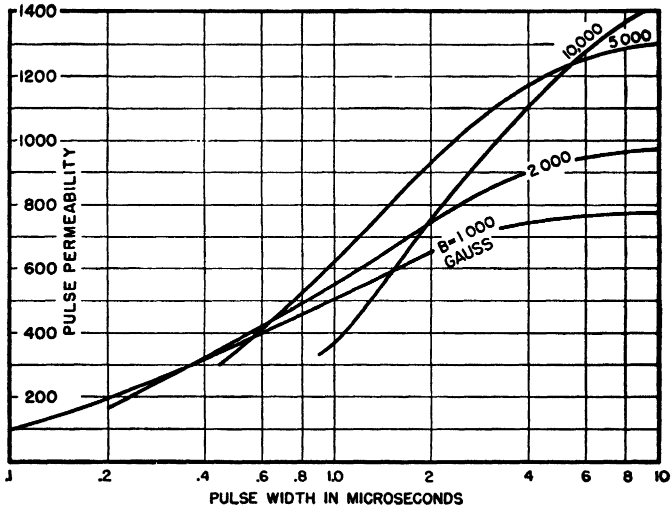


FIG. 196. Effective permeability versus pulse width.

Flux densities attained in pulse transformers may be low for small units where very little source power is available, or they may be high (several thousand gauss) in high power units. This is true whether the pulse width is a few microseconds or 1000 microseconds.

The nickel-iron alloys in general have lower saturation densities, but higher permeabilities below saturation than either grain-oriented or ordinary silicon steel. Depending on the flux density chosen, the increase of permeability with the use of a nickel-iron alloy may vary from zero to 300 per cent. This increase holds also for long-time pulses, during which permeability may approach the 60-cycle value.

In order to overcome the net d-c pulse magnetization which is in the same direction throughout each pulse, an air gap may be inserted in the core to prevent it from returning only to the remanent magnetism value B_r at the end of each pulse, and thereby limiting its useful pulse flux density range ΔB to the difference between maximum flux density

B_m and remanence B_r (see Fig. 197). This gap increases the effective length of the magnetic path and reduces *OCL* from the value it has with symmetrical magnetization. The reduction is less with core materials of low permeability. To maintain the advantage of high permeability in nickel-iron alloys, the core is "reset." This is done by arranging the circuit so that, during the period of backswing, sufficient negative current flows through the windings to overcome coercive force

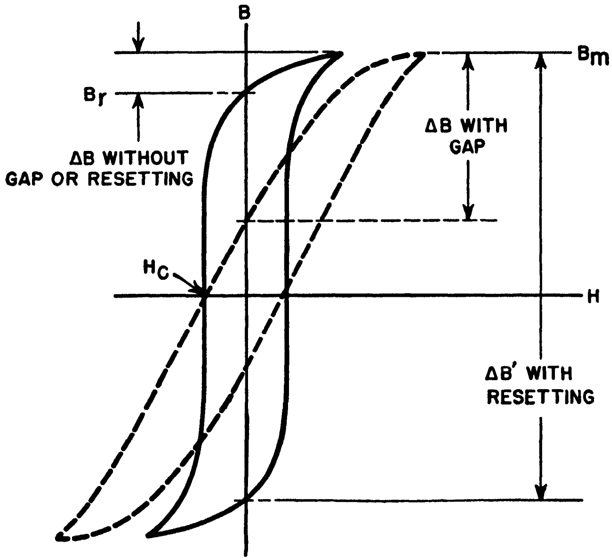


FIG. 197. Flux density range in pulse transformer cores.

H_c and drop the flux density to the negative value of remanent magnetism. Then nearly twice the previous maximum flux density ($\Delta B'$ in Fig. 197) is available for the pulse. Where resetting is possible, it is advantageous to use nickel-iron alloys; where resetting is not practicable, grain-oriented silicon steel is preferable.

106. Windings and Insulation. Pulse transformers generally have single-layer concentric windings with solid insulation between sections. For high load impedance, a single section each for primary and secondary as in Fig. 148 is favorable, as the effective capacitance is lowest. For low load impedance, more interleaving is used to reduce leakage inductance. To reduce capacitance to a minimum, pie-section coaxial windings may be used. In these, coil capacitance is kept low by the use of universal windings, and intersection capacitance between windings is low because the dielectric is air. Such coils are more diffi-

cult to wind, require more space, and therefore are used only when necessary.

Coil sections can be wound with the same polarity as in Fig. 148 (p. 188) or with one winding reversed. Effective capacitance between P and S is given below for three turns ratios. Capacitance is based on 100 $\mu\mu$f measurable capacitance.

Turns Ratio	Effective Capacitance Referred to Primary	
N_1/N_2	Same Polarity	Reversed Polarity
1:5	533	1200
1:1	0	133
5:1	21	48

From this it can be seen that the polarity shown in Fig. 148 is preferable for reducing effective capacitance, but that the percentage difference is greatest for turns ratios near unity and less as the ratio increases.

Attention to the insulation so far has centered around capacitance. The insulation also must withstand the voltage stress to which it is subjected. It can be graded to reduce the space required. Low frequency practice is adequate for both insulation thickness and end-turn clearances.

Small size is achieved by the use of solventless varnish. Small size with consequent low capacitance and low loss results in higher practicable impedance values and shorter pulses.

In order to utilize space as much as possible, or to reduce space for a given rating, core-type construction is often used. Low capacitance between high voltage coils is possible in such designs. It is advantageous in reducing space to split the secondary winding into non-symmetrical sections. Although the leakage inductance is higher with non-symmetrical windings, there is less distributed capacitance when the high voltage winding has the smaller length. Lower capacitance obtains with two coils than with a shell-type transformer of the same interleaving. In core-type transformers high voltage windings are the outer sections. It is preferable to locate terminals or leads in the coil directly over the windings, in order to maintain margins. Insulating barriers may be located at the ends of the windings to increase creepage paths.

Autotransformers, when they can be used, afford opportunity for space saving, because there are fewer total turns and less winding space is needed. Less leakage inductance results, but not necessarily less capacitance; this always depends on the voltage gradients.

Initial distribution of voltage at the front edge of a pulse is not uniform because of turn-to-turn and winding-to-winding capacitance. In a single layer coil the total turn-to-turn capacitance is small compared to the winding-to-ground capacitance, because the turn capacitances add in series but the ground or core capacitances add in shunt. Therefore a steep wave of voltage impressed across the winding sends current to ground from the first few turns, leaving less voltage and less current for the remaining turns. Initially, most of the pulse voltage appears across the first few turns.

After a short interval of time, some of the current flows into the remaining turns inductively. Before long the capacitive voltage distribution disappears, all the current flows through all the turns, and the voltage per turn becomes uniform. This condition applies to most of the top of a pulse. Between initial and final current distribution, oscillations due to leakage inductance and winding capacitance may appear which extend the initially high voltage per turn from the first few turns into some of the remaining turns.

Winding capacitance to ground is evenly distributed along the winding of a single coil, and so is the turn-to-turn capacitance. If a rectangular pulse E is applied to one end of such a winding, and the other end is grounded, the maximum initial voltage gradient is [3]

$$\frac{\alpha E}{N} \coth \alpha$$

where N = number of turns in winding
$\alpha = \sqrt{C_g/C_w}$
C_g = capacitance of winding to ground
C_w = capacitance across winding
= turn-to-turn capacitance/N.

Practical values of α are large, and $\coth \alpha$ approaches unity. Then

$$\text{Maximum gradient} \approx \alpha E/N \tag{101}$$

or the maximum initial voltage per turn is approximately α times the final or average voltage per turn.

If the other end of the winding is open instead of grounded, equation 101 still governs. This means that maximum gradient is independent of load. If there is a winding N_1 between the pulsed winding N_2 and

[3] For the development of this expression see "Surge Phenomena," British Electrical and Allied Industries Research Association, 1941, pp. 223–226.

ground, α depends on C_{1-2} and C_1 in series. The initial voltage in winding N_1 is [4]

$$E_1 = \frac{EC_{1-2}}{C_{1-2} + C_1} \tag{102}$$

where E_1 = initial voltage in winding N_1
E = pulse voltage applied across N_2
C_{1-2} = capacitance between N_1 and N_2
C_1 = capacitance between N_1 and core.

Thus the initial voltage in winding N_1 is independent of the transformer turns ratio. It is higher than the voltage which would appear in N_2 if N_1 were pulsed, because then current would flow from N_1 to ground without any intervening winding. If winding N_1 is the low voltage winding (usually true), applying pulses to it stresses turn insulation less than if N_2 is pulsed.

Reinforcing the end-turns of a pulsed winding to withstand better the pulse voltages is of doubtful value, because the additional insulation increases α and the initial gradient in the end-turns. Increasing insulation throughout the winding is more beneficial, for although α is increased the remaining turns can withstand the oscillations better as inductance becomes effective. Decreasing winding-to-core capacitance is better yet, for then α decreases and initial voltage gradient is more uniform.

107. Efficiency. Circuit efficiency should be distinguished from transformer efficiency. Magnetization current represents a loss in efficiency, but it may be returned to the circuit after the pulse. Circuit efficiency may be estimated by comparing the area of the actual wave shape across the load to that impressed upon the transformer; it includes the loss in source resistor R_1 (Fig. 185). Except for this loss, the circuit and transformer efficiency are the same when the source is cut off at the end of the pulse. It is important in testing for losses to use the proper circuit.

Core loss can be expressed in watt-seconds per pound per pulse. A convenient way to measure core loss is to use a calorimeter. The transformer is located in the calorimeter, and the necessary connections are made by through-type insulators. Dielectric loss is included in such a measurement. It is appreciable only in high voltage transformers, and may be separated from the iron loss by first measuring the loss of the complete transformer and then repeating the test with the high voltage winding removed. At 6000 gauss and 2 microseconds pulse width,

[4] See "Surge Phenomena," pp. 227–281.

the loss for 2-mil grain-oriented steel is approximately 6000 watts per pound, or 0.012 watt-second per pound per pulse. For square pulses, core loss varies (*a*) as B^2 or E^2 for constant pulse width and (*b*) as pulse width, for constant voltage and duty tf, where t is the pulse width and f is the repetition rate. Dielectric loss is independent of pulse width and varies (*a*) as the repetition rate, for constant voltage, and (*b*) as E^2 for constant repetition rate.

Copper loss is usually negligible because of the comparatively few turns required for a given rating, if a wire size somewhere near normal for the rms current is used. If the windings are used to carry other current, such as magnetron filament current, the copper loss may be appreciable but this is a circuit loss.

Efficiencies of over 90 per cent are common in pulse transformers, and with high permeability materials over 95 per cent may be obtained. These figures are for pulse power of 100 kw or more. Maximum efficiency occurs when the iron and dielectric losses are equal.

108. Non-Linear Loads. The role played by leakage inductance and distributed capacitance in determining pulse shape has been mentioned in Sections 99 and 101. It has been shown that the first effect is a more or less gradual slope on the front edge of the pulse, and that the second effect consists of oscillations superposed upon the voltage backswing following the cessation of the pulse. Consider the additional influence of non-linear loads upon the first effect, that is, upon the pulse front edge.

Figure 182 is based on the following assumptions:

(*a*) Load and source impedances are linear.
(*b*) Leakage inductance can be regarded as "lumped."
(*c*) Winding capacitance can be regarded as "lumped."

Assumptions (*b*) and (*c*) are approximately justified. Pulses effectively cause the coils to operate beyond natural resonance, like the higher frequency operation of r-f coils in Section 88 (Chapter VII). The distributed nature of capacitance and leakage inductance, as well as partial coil resonance, may cause superposed oscillations which require correction. But the general outline of output pulse shape is determined by low frequency leakage inductance and capacitance.

Assumption (*a*) may be a serious source of error, for load impedances are often non-linear. Examples are triodes, magnetrons, or grid circuits driven by pulse transformers. In a non-linear load with current flowing into the load at a comparatively constant voltage, the problem is chiefly that of current pulse shape. First assume that no current flows into this load for such time as it takes to reach steady voltage E.

During this first interval, the transformer is unloaded except for its own losses, and is oscillatory. After voltage E is reached, the current rises rapidly at first and then more slowly, as determined by the new load R_2. The sudden application of load at voltage E damps out the oscillations which would exist without this load, and furnishes two conditions for finding the initial current. A rigorous solution of the problem involves overlapping transients and is complicated.

The problem can be simplified by assuming that the voltage pulse has a flat top E. When the pulse voltage reaches E, capacitance C_2 ceases to draw current. At the instant t_1 (Fig. 198) when voltage E is first reached, the current in L which was drawn by capacitance C_2 flows immediately into the load. Also since the voltage was rising rapidly at instant t_1, the energy which would have resulted in the first positive voltage loop (shown shaded in Fig. 198) must be dissipated in the load. The remaining oscillations also are damped. Prior to the time t_1, all the current through L flowed into C_2. The value of this current is $C_2 de/dt$. Therefore we may find the slope of the appropriate front edge voltage curve and multiply by the transformer capacitance to obtain the initial current. Unloaded transformer front edge voltage is shown in Fig. 182, and the front edge slope at voltage E is given in Fig. 199, the ordinates of which are $(Tde/E)/dt$, with E corresponding to the E_a of Fig. 182. Ordinates of Fig. 199 are multiplied by C_2E/T to find the initial load current.

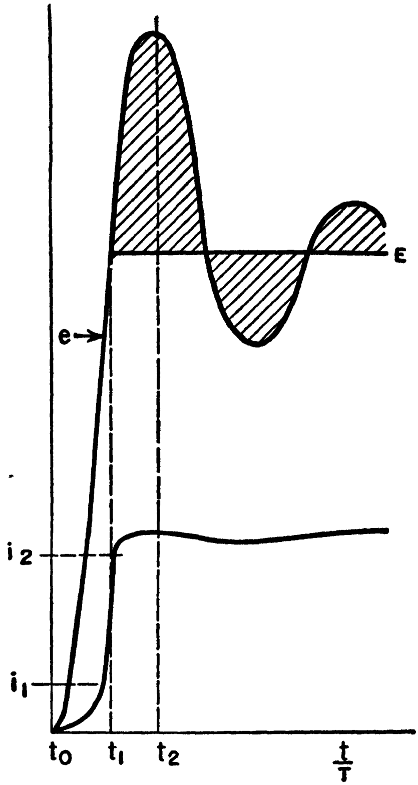


FIG. 198. Non-linear load voltage and current pulse shapes.

Few non-linear loads have absolutely zero current up to the time that voltage E is reached, and the foregoing assumptions are thus approximate. In spite of this, the following procedure gives fair accuracy.

(*a*) Find the initial capacitance current as just outlined.

(*b*) Estimate the current at which the load e-i curve departs from a straight line (i_1, in Fig. 198).

(*c*) Add currents (*a*) and (*b*). This gives i_2 (Fig. 198), as the total initial current.

Pulse current continues to rise beyond the value i_2 if the initial current value just found is less than the final operating current correspond-

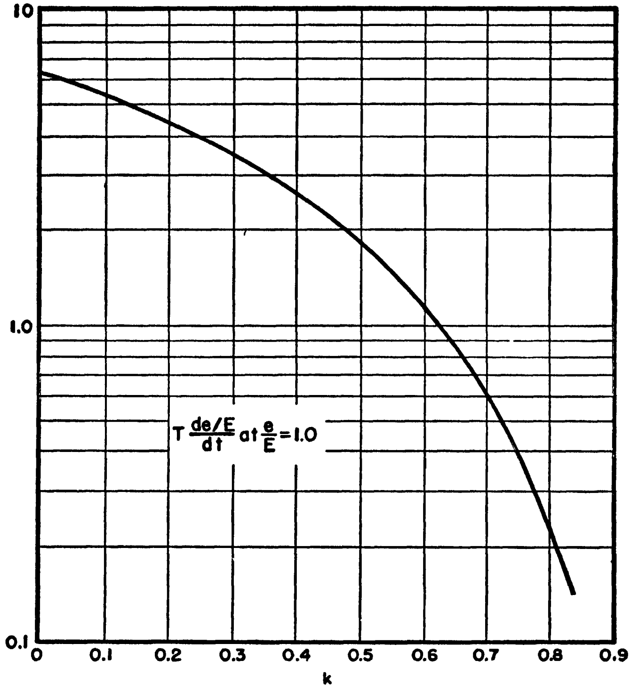


FIG. 199. Front edge slope of pulse transformer.

ing to the voltage E; it will droop if the initial current is higher than the load current at voltage E. To obtain constant current over the greater part of the pulse width, i_2 should equal the load current at voltage E. When this equality does not exist, the rate of rise or droop is determined by transformer leakage inductance, source impedance, and load resistance. Where the mode of operation depends upon the rate of voltage rise, as it does in some magnetrons, the initial current may drop off to nearly zero before the main current pulse starts. When there is negligible initial current i_1, the condition for a good current pulse is $E/i_2 \approx \sqrt{L/C_2}$, where L is the leakage inductance.

At the end of the pulse, when the source voltage is reduced to zero

(point b, Fig. 178), the circuit reverts to that of Fig. 187, but the transformer immediately loses all the load except its own losses. Since by this time it has drawn exciting current, the higher values of Δ in the backswing curves apply. Large backswing amplitudes are common with non-linear loads. This loss load also governs the superposed oscillations due to leakage L and C_2. A typical voltage wave shape is that of Fig. 192.

109. Design of Pulse Transformers. (*A*) *Requirements.* The performance of a pulse transformer is usually specified by the following

(*a*) Pulse voltage
(*b*) Voltage ratio
(*c*) Pulse duration
(*d*) Repetition rate
(*e*) Power or impedance level
(*f*) Slope of front
(*g*) Droop on top
(*h*) Amount of backswing permissible
(*i*) Type of load

Design data for insuring that these requirements are met are provided in the foregoing sections, in several sets of curves. Below are outlined the steps followed in utilizing these curves for design purposes.

(*B*) *Start of Design.* The first step in beginning a design is to choose a core. It is helpful if some previous design exists which is close in rating to the transformer about to be designed.

After choosing the core to be used, the designer must next figure the number of turns. In pulse transformers intended for high voltages, the limiting factor is usually flux density. If so, the number of turns may be derived as follows.

$$e = \frac{N\,d\phi}{dt} \times 10^{-8} = NA_c\frac{dB}{dt} \times 10^{-8}$$

$$\int e\,dt = NA_c\int dB \times 10^{-8} \tag{103}$$

For a square wave, $e = E$ and

$$Et = NA_cB \times 10^{-8}$$

or

$$N = \frac{Et \times 10^8}{6.45BA_c} \tag{104}$$

where E = pulse voltage
t = pulse duration in seconds
B = allowable flux density in gauss
A_c = core section in square inches
N = number of turns.

In many designs, the amount of droop or the backswing which can be tolerated at the end of the pulse determines the number of turns, because of their relation to the *OCL* of the transformer.

After the turns are determined, appropriate winding interleaving should be estimated and the leakage inductance and capacitance calculated.

With the leakage inductance and winding capacitance estimated, the front end performance for linear loads can be found from Fig. 182. Likewise, from *OCL* and winding capacitance, the shapes of the top and trailing edge are found in Figs. 186, 188, 189, and 190. If performance from these curves is satisfactory and the coil fits the core, the design is completed.

(*C*) *Final Calculations.* Preliminary calculations may show too much slope on the front edge of the pulse (as often happens with new designs. Two damping factors $R_1/2L$ and $1/2R_2C$ contribute to the front edge slope, and the preliminary calculations show which one is preponderant. Sometimes it is possible to increase leakage inductance or capacitance without increasing time constant T greatly, and this may be utilized in decreasing the slope.

If the front edge slope is still too much after these adjustments, the core chosen is probably inadequate. Small core dimensions are desirable for low leakage inductance and winding capacitance. Small core area A_c may require too many turns to fit the core. These two considerations work against each other, so that the right choice of core is a problem in any design.

If the calculated front edge slope is nearly good enough it may be improved by one of the following means:

(*a*) Change number of turns
(*b*) Reduce core size
(*c*) Change interleaving
(*d*) Increase insulation thickness
(*e*) Reduce insulation dielectric constant

High capacitance is a common cause of poor performance and items (*b*) to (*e*) may often be changed to decrease the capacitance. It is sometimes possible to rearrange the circuit to better advantage and thereby make a deficient transformer acceptable. One illustration of this is the termination of a transmission line. Line termination resistance may be placed either on the primary or secondary side. If it is placed on the primary side there is usually a much improved front edge. Figure 182 does not show this improvement inasmuch as it was plotted for Fig. 181. For resistance on the primary side, the damping

factor reduces to the single term

$$\alpha = \frac{R_1 R_2}{2(R_1 + R_2)L} \tag{105}$$

Improvement of the trailing edge performance usually accompanies improvement of the front edge.

Core permeability is important because it requires fewer turns to obtain the necessary OCL with high permeability core material. Permeability at the beginning of the trailing edge (point b', Fig. 191) is most important, for two reasons: the droop at this point depends on the OCL, so that for a given amount of droop the turns on the core are fixed; also, the normal permeability data apply to such points as b'. Flux density is chosen with two aims: it should be as high as possible for small size, but not so high as to result in excessive magnetizing current and backswing voltage.

(*D*) *Example.*

Assume that the performance requirements are

Pulse voltage ratio 2000:10,000 volts
Pulse duration 2 microseconds
Pulse repetition rate 1000 per second
Impedance ratio 50:1250 ohms (linear)
To rise to 90 per cent of final voltage in ¼ microsecond or less
Droop not to exceed 10 per cent in 2 microseconds
Backswing amplitude not to exceed 60 per cent of pulse voltage
50-ohm source

The final design has the following:

Primary turns = 20
Secondary turns = 100
Core: 2-mil silicon steel with 1-mil gap per leg
Core area = 0.55 sq in.
Core length = 6.3 in. ($l_g/l_c = 0.0003$)
Core weight 0.75 lb, window ⅝ in. × 1⁹⁄₁₆ in.
Primary leakage inductance = 2 microhenrys
Effective primary capacitance = 1800 $\mu\mu$f
No load loss equivalent to 400 ohms (referred to primary)

$$\text{Flux density} = \frac{10{,}000 \times 2 \times 10^2}{6.45 \times 100 \times 0.55} = 5600 \text{ gauss}$$

At 2 microseconds and $B = 5600$ $\mu \approx 600$

$$\text{Primary } OCL = \frac{3.2 \times 400 \times 0.55 \times 10^{-8}}{0.002 + \dfrac{6.3}{600}} = 550\ \mu\text{h}$$

Front edge performance is figured as follows.

$$m = \frac{R_1}{2L} + \frac{1}{2R_2C} = \frac{50 \times 10^6}{4} + \frac{10^6}{0.1 \times 1.8} = 18 \times 10^6$$

$$T = 6.28 \times 10^{-6}\sqrt{2 \times 0.0018} = 0.375 \text{ microsecond}$$

$$k = 1.08$$

According to Fig. 182, this value of k gives 90 per cent of E_a in $0.625T$ or 0.235 microsecond.

The top is figured at

$$\frac{tR_1}{L} = \frac{2 \times 50 \times 10^{-6}}{550 \times 10^{-6}} = 0.18$$

and from the curve $R_1 = R_2$ in Fig. 186, the top droops 9 per cent.
The magnetizing current is

$$\frac{R_1 + R_2}{R_1} \times 9 \text{ per cent or 18 per cent of the load current}$$

For the backswing

$$m = \frac{10^6}{0.1 \times 1.8} = 5.5 \times 10^6$$

$$T = 6.28 \times 10^{-6}\sqrt{550 \times 0.0018} = 6 \text{ microseconds}$$

$$k = 5.2$$

From Fig. 190, the backswing is 20 per cent of E_a. If the load resistance is connected to the transformer when the pulse voltage is removed, the backswing superposed oscillation has the same k (1.08) as the front edge; that is, there is no oscillation and the total backswing voltage is 20 per cent of E_a.

Suppose the load were non-linear; the voltage would rise up to E within $\frac{1}{4}T$ or 0.094 microsecond. The front edge

$$m = \frac{50 \times 10^6}{4} + \frac{10^6}{0.8 \times 1.8} = 13.2 \times 10^6$$

and

$$k = 0.8$$

From Fig. 199,

$$\frac{T}{E}\frac{de}{dt} = 0.22$$

The secondary effective capacitance is $1800/25 = 72\ \mu\mu$f and the initial load current is

$$C\frac{de}{dt} = \frac{72 \times 0.22 \times 10{,}000}{0.375 \times 10^6} = 0.42 \text{ amp}$$

Final load current is 10,000/1250 = 8 amp, so that current is non-uniform during the pulse. The backswing constants are

$$m = \frac{10^6}{0.8 \times 1.8} = 0.7 \times 10^6$$

$$T = 6 \text{ microseconds}$$

$$k = 0.665$$

Δ is 9 times the linear load current, or 81 per cent

From Figs. 188 and 189, long term backswing is about 60 per cent. Added to this is a slight oscillation overshoot corresponding to $k = 0.8$ in Fig. 183, but it decays to zero before maximum backswing. This transformer just meets the requirements. It could be improved by using 0.002-in. grain-oriented material having $\mu = 1000$ at 5600 gauss. *OCL* then becomes 1000 μh, Δ reduces to 45 per cent, $k = 0.94$, and the long term backswing becomes 40 per cent of E.

Secondary current is 8 amp. The rms value of this current is, from Table I (p. 10),

$$I_{\text{rms}} = 8\sqrt{2 \times 10^{-6} \times 1000} = 0.36 \text{ amp}$$

and the primary current is $5 \times 0.36 = 1.8$ amp. The wire insulation must withstand $10{,}000 \div 100 = 100$ volts per turn, and with single-layer windings

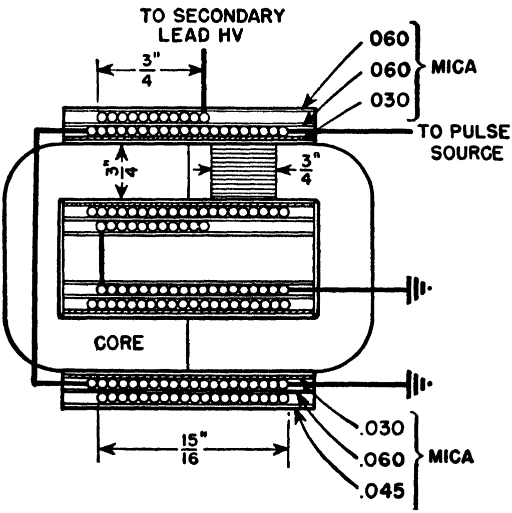


FIG. 200. Section of pulse transformer.

this normally requires at least 0.0014 in. of covering insulation. Heavy enamel wire, No. 28, has a margin of insulation over this value. This is further modified by the initial non-uniform voltage distribution as figured below. A sectional view of a two-coil design is shown in Fig. 200, with No. 28 heavy enamel

wire in the secondary and No. 22, wound two in parallel to occupy the form fully, in the primary. The core section is ¾ in. by ¾ in. The primary turn length is 3.75 in. and that of the secondary is 4.13 in. Primary and secondary d-c resistances are 0.05 and 2.3 ohms, and the respective copper losses are

$$(1.8)^2 \times 0.05 = 0.162$$

$$(0.36)^2 \times 2.3 = 0.3$$

$$\text{Total} = 0.462$$

The no-load loss is $[(2000)^2/400] \times 1000 \times 2 \times 10^{-6} = 20$ watts. Copper loss is therefore of little significance.

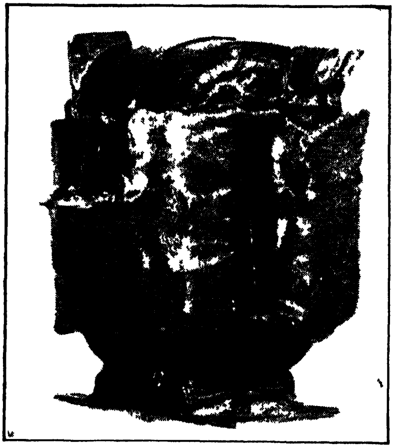


FIG. 201. Pulse transformer with coils of Fig. 200.

From the coil dimensions and insulation thicknesses we can figure the capacitances. The total winding traverse for both coils is 1.875 in. The primary-to-core capacitance is

$$\frac{0.225 \times 3.75 \times 1.875 \times 5}{0.030} = 264\ \mu\mu\text{f}$$

and the secondary-to-primary capacitance is

$$\frac{0.225 \times 4.13 \times 1.875 \times 5}{0.060} = 145\ \mu\mu\text{f}$$

so that these two capacitances in series are 94 $\mu\mu$f. Secondary turn-to-turn capacitance is, approximately,

$$\frac{0.225 \times 4.13 \times 0.0126 \times 3}{0.0019} = 18.4$$

or

$$C_w = 0.184$$

α is therefore $\sqrt{94/0.184} = 22.5$, and the wire enamel initially must withstand 2250 volts per turn.

Figure 201 is a photograph of the transformer with Fosterite-treated coils.

110. Sawtooth Transformers.[5] Probably the most common application of sawtooth amplifier transformers is to provide a linear sweep to horizontal plates of a cathode ray oscilloscope. The spot is moved at a uniform rate across the screen, and quickly returned to repeat the trace. In such a circuit, the load on the transformer can be regarded as negligible. Assume a linearly increasing voltage as shown in Fig. 202 to be applied to the circuit of Fig. 203. Analysis shows (see Appendix C) that the voltage e across the inductance L has the same slope as

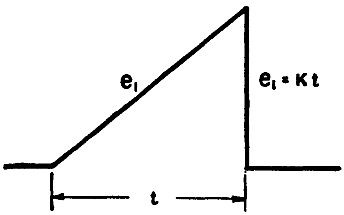


FIG. 202. Sawtooth wave.

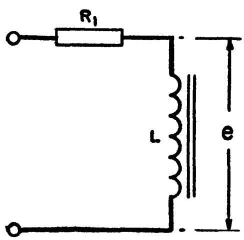


Fig. 203. Sawtooth transformer circuit.

the applied voltage times an exponential term which is determined by the resistance R_1 of the amplifier, the OCL of the transformer, and the time between the beginning and the end of the linear sweep. Under the conditions assumed, the value of the exponential for any interval of time can be taken from the curve marked $R_2 = \infty$ in Fig. 186. For example, suppose that the sweep lasts for 500 microseconds, that the sweep amplifier tube plate resistance is 800 ohms, and that the transformer inductance is 10 henrys. The abscissa of Fig. 186 is 0.04 and, since the slope of this exponential curve equals its ordinate, the slope of the voltage applied to the plates of the oscilloscope will be, at the end of the sawtooth interval, 96 per cent of the slope which it had at the beginning of the interval.

Assume that at the end of the time interval t (Fig. 202) the amplifier tube is cut off. Then the sweep circuit transformer reverts to that of Fig. 187, in which C_2 has the same meaning as before, but R_2 includes only the losses of the transformer, which were neglected in the analysis for linearity of sweep. That is to say, the voltage does not immediately

[5] Sections 110 and 111 are based on the author's "Iron-Core Components in Pulse Amplifiers," *Electronics*, Aug. 1943, p. 115.

disappear, but follows the curves of Figs. 188, 189, and 190 very closely, just as the trailing edge of the square wave pulse does. Backswing voltage may be kept from affecting the screen by suitable spacing of the applied wave forms or biasing the cathode ray tube grid.

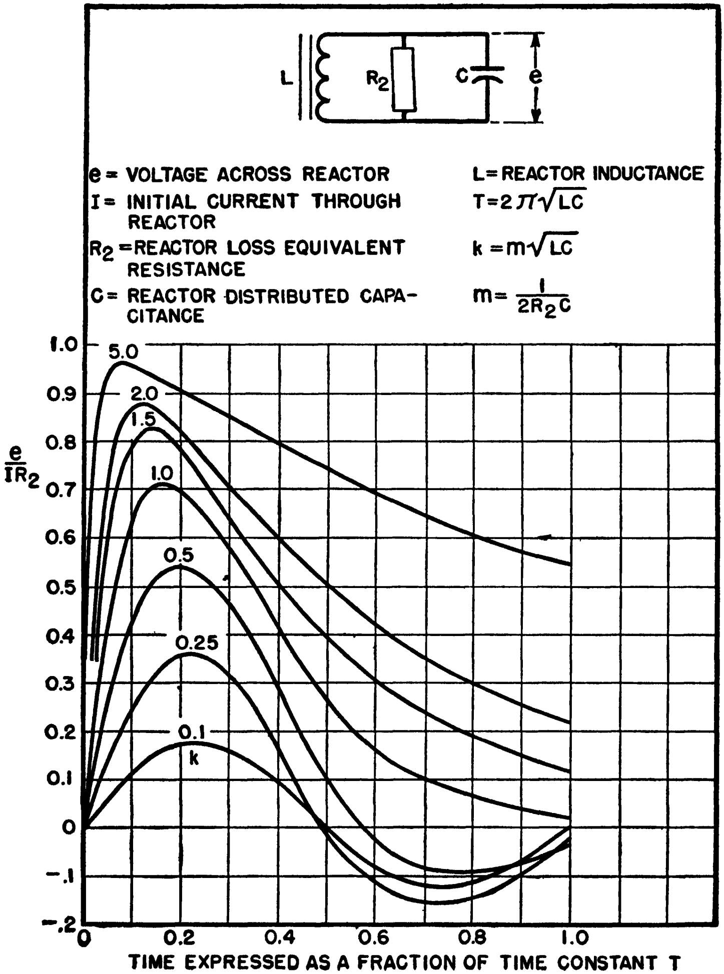


FIG. 204. Reactor voltage rise.

111. Voltage Rise in Pulse-Forming Reactors. Numerous circuits have been devised for the development of relatively high voltages by interrupting the current through a reactor. The interruption of current through the choke forces it to discharge into its own capacitance and loss resistance R_2 as in Fig. 187.

Voltage rise in this choke is shown in Fig. 204. The abscissas are defined by the time constant of the choke in terms of its *OCL* and distributed capacitance. The voltage curves depend on the factor k, which has the same significance as in the trailing edge of a pulse transformer. The ordinates on the curve are the voltage rise divided by the product of the initial choke current and loss resistance R_2. For high values of k the voltage rise is high and steep. This requires low capacitance. The choke voltage rises up to its peak value in about the same length of time that the voltage on the trailing edge of a pulse transformer decreases to its minimum value. For small values of k the voltage amplitude is small. Usually in the operation of such chokes, only the relatively linear portion of the voltage rise curve is used. Figure 204 shows that with higher values of k it may be possible to utilize more of such a voltage, and require less subsequent amplification. Figure 204 is plotted from equations 116 and 117, Appendix D.

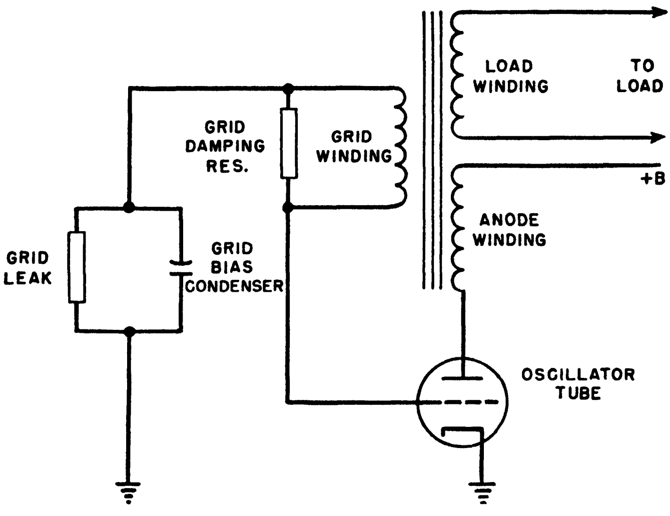


Fig. 205. Blocking oscillator.

112. Blocking Oscillators. Blocking oscillators are used to obtain pulses at certain repetition rates. The pulse may be used to drive a pulse amplifier, or it may be used to modulate a *UHF* oscillator.

A typical blocking oscillator circuit is shown in Fig. 205. The grid is driven hard, and grid current usually is comparable in magnitude to anode current. Grid and anode winding turns are approximately equal. The oscillator operates as follows.

If the grid is only slightly negative, the tube draws plate current and because of the large number of grid turns the transformer drives

the grid positive, increases the plate current, and starts a regenerative action. During this period, the grid draws current, charging the bias capacitor to a voltage depending on the grid current flowing into the bias resistor-capacitor circuit. The negative plate voltage swing is determined by grid saturation, so that large positive swings of grid voltage result in virtually constant plate voltage. This continues for a length of time determined by the constants of the transformer, after which the regenerative action is reversed. Because of lowered plate voltage swing, the plate circuit can no longer drive the low impedance

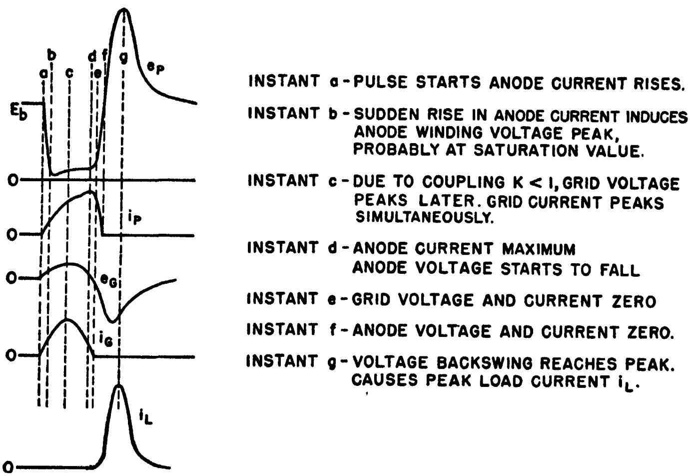


FIG. 206. Blocking oscillator voltages and currents.

reflected from the grid, and the charge accumulated on the bias capacitor becomes great enough to decrease plate current rapidly in a degenerative action. Plate current soon cuts off, and then the plate voltage overshoots to a high positive value and the grid voltage to a high negative value. Grid voltage decays slowly because of the discharge of the bias capacitor into the grid leak. The next pulse occurs when the negative grid voltage decreases sufficiently so that regenerative action starts again. Hence the repetition rate depends on the grid bias R and C.

Either the negative or the positive pulse voltage may be utilized. Instantaneous voltages and currents are shown in Fig. 206, for a load which operates only on the positive pulse. The general shapes of these currents and voltages approximate those in a practical oscillator, except for superposed ripples and oscillations which often occur.

The negative pulse has a much squarer wave shape than the positive pulse, and consequently it is used where good wave shape is required.

No matter how hard the grid is driven, plate resistance cannot be lowered below a certain value; so a limit to the negative amplitude is formed. There is no such limit to the positive pulse, and this characteristic may be used for a voltage multiplier.

If the transformer has low *OCL*, the leakage inductance may be high enough to perform like an air-core transformer. That is, there are optimum values of coupling for maximum power transfer, grid drive, and negative pulse shape, but they are not critical. Comparison of peak voltages in Fig. 206 shows that the usual 180 degrees phase relationship between grid and plate swings do not hold for such a blocking oscillator, if the term "phase" has any meaning in this case.

The front edge slope of the negative pulse is determined by leakage inductance and capacitance as in Fig. 182, with two exceptions: The pulse is negative and the load is non-linear; hence there are no oscillations on the inverted top. The slope of this portion can be computed from Fig. 186, provided tube and load resistances are accurately known. The positive pulse can be found from Figs. 188, 189, and 190, if these curves are inverted.

Pulse width, shape, and amplitude also are affected by the ratio of grid turns to plate turns in the transformer. Voltage rise is steeper as this ratio is greater, with the qualification that grid capacitance increases as the square of the grid turns; the ratio is seldom greater than unity. The exact ratio for close control depends on tube data which may not be available and must be determined experimentally. The situation parallels that of the class C low *Q* oscillators mentioned in Chapter VI.

113. Testing Technique. Tests for open circuits, short circuits, turns ratio, and d-c resistance are made on pulse transformers in the same manner as in other transformers. The instruments used must be suitable for the low values of inductance encountered, but otherwise no special precautions are necessary. Usually the d-c resistance is somewhat lower than the winding resistance during most of the pulse, but even the latter value is so low that it causes no significant part of the transformer loss. Losses are measured as described in Section 105.

Various methods have been used to check effective pulse *OCL*. These may involve substitution of known inductances, or current build-up, or decay, depending on the time constant of the transformer inductance and an external known resistance. When such measurements are attempted under pulse conditions, there is usually a certain amount of error due to reflections, incidental capacitance, and the like. A method involving the measurement of pulse permeability and calculation by the *OCL* formula is given here.

If the air gap and pulse permeability are known, the OCL for a given core area and number of turns can be calculated. If the gap used is purposely made large to reduce saturation, proper allowance for it can be made in equation 26 (p. 75). If the gap is the minimum obtainable, it is necessarily included in the permeability measurement, but this is often done in taking pulse permeability data, as it was in the data of Fig. 196. With this definition of permeability equation 26 reduces to

$$OCL = \frac{3.2\mu N^2 A \times 10^{-8}}{l_c} \tag{106}$$

Equation 106 is valid only when $l_c/\mu >> l_g$.

B-H data for a pulse transformer are taken by means of a circuit similar to that of Fig. 207. Primary current flowing through small

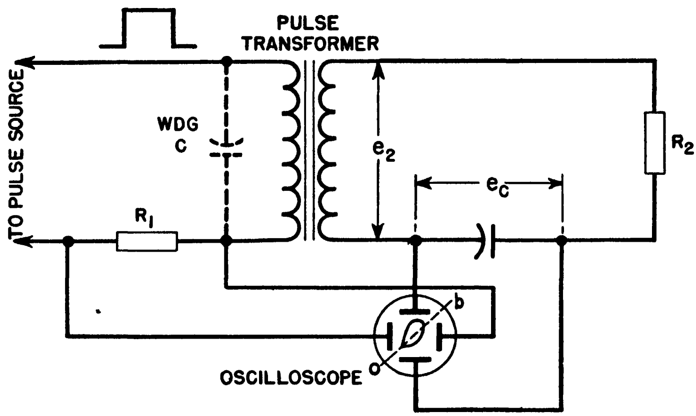


FIG. 207. B-H test for pulse transformers.

resistor R_1 (10 to 50 ohms) gives a horizontal deflection on the oscilloscope proportional to I and therefore H for a given core. If the voltage drop across a high resistance load R_2, ($\approx$ 50 times normal pulse load) is almost the entire secondary voltage e_2, then the voltage e_c applied to the vertical plates is the time integral of e_2 and is therefore proportional to flux density at any instant. (See equation 103.)

Short leads and the reduction of incidental capacitance are essential to obtain accurate measurements. Distributed capacitance of the winding, shown dotted, should be minimized, as it introduces extraneous current into the measurement of H. One way to minimize this capacitance is to omit the high voltage winding, and make all measurements from the low voltage, low capacitance coil only.

The pulse source should be the kind for which the transformer is to be used. If it must be loaded to obtain proper pulse shape, a diode may be used to prevent backswing discharge through this load and therefore a reset core, unless reset core data are desired. Difficulty may be experienced in seeing the *B-H* loops of pulses having a low ratio of time on to time off because of the poor spot brilliance, unless an intensifier is used to brighten the trace.

With a calibrated oscilloscope it is possible to determine the slope of the dotted line *ob* in Fig. 207, drawn between the origin and the end of the pulse, and representing effective permeability μ at instant *b*, Fig. 178. This value of μ can be inserted in equation 106 to find *OCL*. Cores failing to meet the *OCL* should first be examined for air gap.

Effective values of leakage inductance and capacitance are difficult to measure. The calculations of capacitance and leakage inductance are based on the assumption of "lumped" values, the validity of which can be checked by observing the oscillations in an unloaded transformer when pulse voltage is applied. The frequency and amplitude of these oscillations should agree with those calculated from the leakage inductance and effective capacitance. The pulse source should be chosen for the squareness of its output pulse. Because of the light load, the transformer usually will be oscillatory, and produce a secondary pulse shape of the kind shown in Fig. 208. In this figure, the dot-dash line is that of the impressed pulse and the solid curve is the resulting transformer output voltage. This curve is observed by connecting the vertical plates of a synchroscope (oscilloscope with synchronized sweep) across the transformer output winding.

The first check of leakage L and C is made by finding the time constant T from

$$T = 2\pi\sqrt{LC}$$

This time constant can be related to the time interval $t_0 - t_1$ in Fig. 208 by consulting Fig. 182. Formulas in this figure can be used for finding values of parameter k using L, C, source resistance R_1, and load resistance R_2. This load resistance will be that corresponding to transformer losses only. With this value of k, the increase or overshoot of the first voltage oscillation over the flat top value E may be found from Fig. 183, and may be compared with that observed in Fig. 208. When the load is resistive, or when the voltage pulse is the criterion of pulse shape, these are the only checks that need to be made on leakage inductance and distributed capacitance.

When the load is a magnetron, triode, blocking oscillator, grid circuit, or other non-linear load, the shape of the current pulse is important.

Ordinarily the current will not build up appreciably before time t_1 in Fig. 208. The shape of this current pulse and sometimes the operation of the load are determined to a large extent by slope AB of the no load voltage at time t_1. This time is the instant when the first oscillation crosses the horizontal line E in Fig. 208. As shown in Fig. 199, there

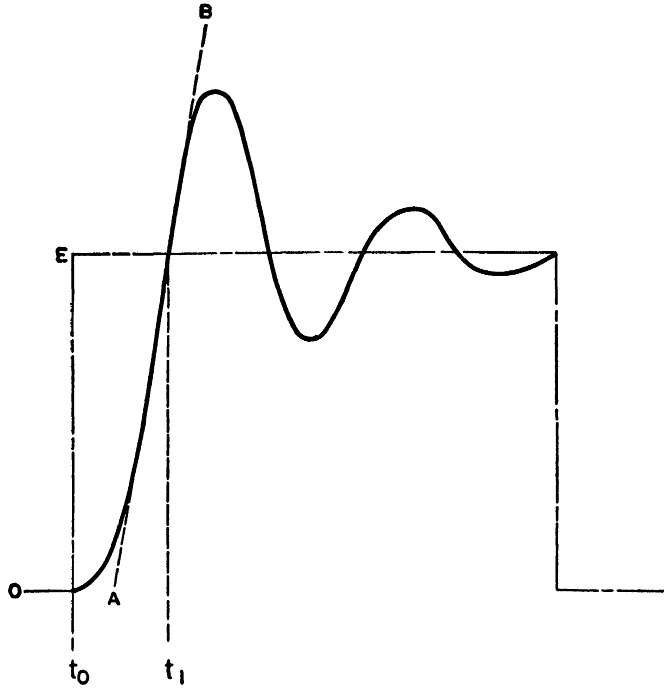


FIG. 208. Transformer constants may be found from pulse shape.

is a relationship between this slope and the parameter k. If the slope AB is confirmed, correct current pulse shape is also assured.

Insulation can be tested in one of two ways, depending on whether the insulation and margins are the same throughout the winding or whether the insulation is graded to suit the voltage. In the former case an equivalent 60-cycle peak voltage, applied from winding to ground at the regular 60-cycle insulation level, is sufficient. But if the winding is graded, this cannot be done because the voltage must be applied across the winding and there is not sufficient OCL to support low frequency induced voltage; hence a pulse voltage of greater than normal magnitude must be applied across the winding. Adequate margins support a voltage of the order of twice normal without insulation failure.

Such pulse testing also stresses the windings as in regular operation, including the non-uniform distribution of voltage gradient throughout the winding. The higher voltage test ought to be done at a shorter pulse width so that saturation does not set in. In cases of saturation, the voltage backswing is likely to exceed the pulse voltage of normal polarity, and thus subject the insulation to an excessive test. This backswing may be purposely used to obtain higher voltage than the equipment can provide, but it must be carefully controlled. Corona

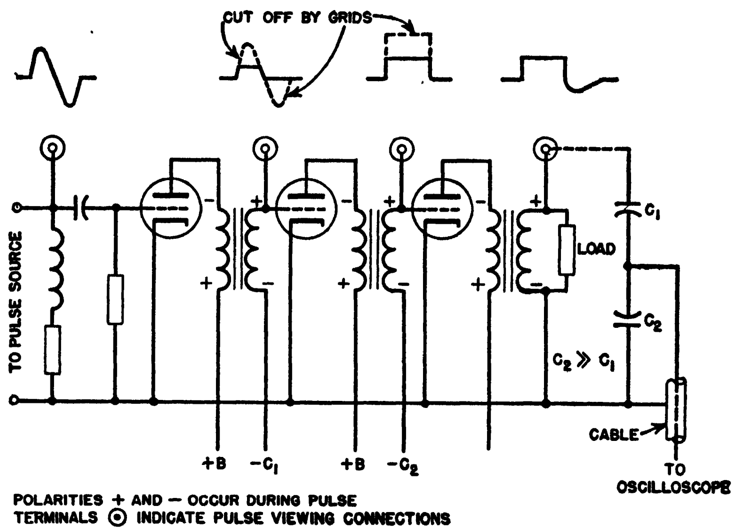


FIG. 209. Pulse amplifier with oscilloscope connections.

tests are sometimes used in place of insulation tests, and this can be done, where the insulation is not graded, by using a 60-cycle voltage and a sensitive receiver to pick up the corona noise. With graded insulation a high frequency must be used. The method becomes too difficult to use because the receiver may pick up part of the high frequency power emitted from the transmitter, or the transmitter parts may generate a certain amount of corona which is more troublesome than at 60 cycles.

In pulse amplifiers, the mode of operation of the tubes and circuit elements is important. A round irregular pulse may be changed by grid saturation, or by non-linear loading of some other sort, into a practically square wave pulse. It may take several stages of amplification to do this in certain instances, and a transformer may be used at each stage. Often the function of the transformer is to invert the pulse for each stage; that is, the transformer changes it from a negative pulse

at the plate of one stage to a positive pulse on the grid of the next. Polarity is therefore important, and should be checked during the turns ratio test. If the transformer fails to deliver the proper shape of pulse, it may be deficient in one of the properties for which tests are mentioned above. Figure 209 shows a pulse amplifier with normal pulse shapes for each stage. Checking each stage at the points indicated, without spoiling the pulse shape by the measuring apparatus, requires attention to circuit impedance, stray capacitance, cable termination, and lead length.

PROBLEMS

1. Derive the expression 97*a* (p. 226) for current at the end of a pulse.

2. In the example given in Section 109*D* for a non-linear load, the current pulse slopes considerably. Find how much greater the initial current can be made by increasing the primary leakage inductance to 6 microhenrys, leaving core, turns, and capacitance the same. Are the other requirements then met?

Ans. After 0.05 microsecond additional delay, initial load current is 1.88 amp.

3. In Prob. 2 assume a core-type design, with a primary and secondary coil on each leg. Each coil has 1.5-in. total length. What is the optimum division of turns between the secondary coils from an insulation standpoint? *Ans.* 36 and 64.

4. In Prob. 2, what is the transformer efficiency? *Ans.* 89 per cent.

5. A transformer has a frequency response of -1 db at 100 cycles and at 100,000 cycles when source and load impedances are equal. $B = 1$ at 100,000 cycles. For what range of pulse widths does its output voltage rise to 90 per cent of flat top in 25 per cent of minimum pulse width and droop 10 per cent at the end of maximum pulse width? *Ans.* 25 to 318 microseconds.

6. What primary inductance is required in a sawtooth transformer to maintain linearity of sweep within 1 per cent of a straight line if the source resistance is 2000 ohms and the sweep lasts 0.01 sec? *Ans.* 2000 henrys.

7. In a pulse transformer the winding configuration is similar to that in Fig. 102, but pulses are applied at D; E and C are grounded, and pulse output appears at A. Prepare a table similar to that in Section 106 for the same turns ratio. Are the conclusions of Section 106 supported?

8. Design a pulse transformer with the following rating and using the same core as in Section 109*D*:

Voltage ratio 4000/8000, step-up
Pulse width 2 microseconds
Voltage must rise to 90 per cent of final value in ¼ microsecond
Voltage must not droop more than 10 per cent in 2 microseconds
Pulse repetition rate 500 per second
Source impedance 50 ohms, load impedance 200 ohms

Calculate the trailing edge and efficiency.

9. With $i_1 = 0$ in Fig. 198, derive a relation between E, i_2, leakage inductance L, and capacitance C_2 for optimum current pulse shape. Assume negligible resistance in the source and no loss in the core. *Ans.* $E/i_2 = \sqrt{L/C_2}$.

APPENDIX

A. ANALYSIS OF STEP-UP TRANSFORMERS FOR SQUARE WAVES

Front Edge. Refer to Fig. 181 (p. 222). The operational impedance of the circuit is

$$Z(p) = R_1 + Lp + \frac{R_2}{1 + R_2C_2p}$$

and

$$\frac{e}{E} = \frac{R_2(1)}{R_2LC_2p^2 + (R_1R_2C_2 + L)p + R_1 + R_2}$$

The roots of this equation are

$$\alpha \pm j\beta = \frac{-R_1}{2L} - \frac{1}{2R_2C_2} \pm \sqrt{\left(\frac{R_1}{2L} + \frac{1}{2R_2C_2}\right)^2 - \frac{R_1 + R_2}{R_2LC_2}}$$

so that

$$\beta = \sqrt{\frac{R_1 + R_2}{R_2LC_2} - \alpha^2}$$

For the oscillatory case,

$$e = \frac{ER_2}{R_1 + R_2} + \epsilon^{\alpha t}[M \sin \beta t + N \cos \beta t]$$

When $t = 0$, $e = 0$, whence

$$N = \frac{-ER_2}{R_1 + R_2}$$

When $t = 0$, $i = 0 = (e/R_2)(1 + R_2C_2p)$ whence

$$M = -\frac{\alpha}{\beta}N$$

The complete solution is

$$\frac{e}{E} = \frac{R_2}{R_1 + R_2}\left(1 + \epsilon^{\alpha t}\frac{\alpha}{\beta}\sin \beta t - \epsilon^{\alpha t}\cos \beta t\right) \tag{107}$$

For the non-oscillatory case,

$$e = \frac{ER_2}{R_1 + R_2} + M\epsilon^{m_1t} + N\epsilon^{m_2t}$$

When $t = 0$, $e = 0$, and

$$N = -\frac{ER_2}{R_1 + R_2} - M$$

When $t = 0$, $i = 0$, and

$$M = \frac{ER_2 m_2}{(R_1 + R_2)(m_1 - m_2)}$$

The complete solution is

$$\frac{e}{E} = \frac{R_2}{R_1 + R_2}\left[1 + \frac{m_2 \epsilon^{m_1 t}}{m_1 - m_2} - \left(1 + \frac{m_2}{m_1 - m_2}\right)\epsilon^{m_2 t}\right] \tag{108}$$

Figure 182 consists of plots of equations 107 and 108, with

$$T = \frac{1}{\beta}, \quad m = -\alpha, \quad m_1 = -m\left(1 + \sqrt{1 - \frac{1}{k^2}}\right),$$

$$m_2 = -m\left(1 - \sqrt{1 - \frac{1}{k^2}}\right), \quad k = m\sqrt{LC_2}, \quad E_a = \frac{ER_2}{R_1 + R_2}$$

Top of Pulse. The operational impedance of the circuit of Fig. 185 is

$$Z(p) = R_1 + \frac{R_2 Lp}{R_2 + Lp}$$

and

$$\frac{e}{E} = \frac{R_2 Lp}{R_1 R_2 + (R_1 + R_2)Lp}$$

The voltage $e = A\epsilon^{-mt}$, where

$$m = \frac{R_1 R_2}{(R_1 + R_2)L}$$

When $t = 0$, $e = R_2 E/(R_1 + R_2)$, whence

$$A = \frac{ER_2}{R_1 + R_2} = E_a$$

so that

$$\frac{e}{E_a} = \epsilon^{-\frac{R_1 R_2 t}{(R_1 + R_2)L}} \tag{109}$$

Figure 186 consists of equation 109 with the following parameters:

For $R_2 = 0.1R_1$,

$$\frac{e}{E_a} = \epsilon^{-\frac{0.09 R_1 t}{L}}$$

For $R_2 = R_1$,

$$\frac{e}{E_a} = \epsilon^{-\frac{0.5 R_1 t}{L}}$$

For $R_2 = 2R_1$,

$$\frac{e}{E_a} = \epsilon^{-\frac{0.67 R_1 t}{L}}$$

For $R_2 = \infty$,

$$\frac{e}{E_a} = \epsilon^{-\frac{R_1 t}{L}}$$

Trailing Edge. Refer to Fig. 187 and assume zero initial current in L, and current at any time in $R_2 = i$. Then all the current initially comes by discharge of C_2, or

$$-C_2pe = \frac{e}{R_2} + \frac{e}{Lp}$$

From this follows

$$LC_2p^2 + \frac{L}{R_2}p + 1 = 0$$

$$m_1, m_2 = -\frac{1}{2R_2C_2} \pm \sqrt{\frac{1}{4R_2{}^2C_2{}^2} - \frac{1}{LC_2}}$$

The voltage $e = A\epsilon^{m_1t} + B\epsilon^{m_2t}$.

When $t = 0$,

$$e = E \quad \text{and} \quad A = E - B$$

When $t = 0$,

$$i = -C_2pe = \frac{E}{R}, \quad \text{and} \quad A = \frac{Em_1}{m_1 - m_2}; \quad B = \frac{-Em_2}{m_1 - m_2}$$

Thus for the non-oscillatory case,

$$\frac{e}{E} = \frac{m_1\epsilon^{m_1t} - m_2\epsilon^{m_2t}}{m_1 - m_2} \tag{110}$$

For the oscillatory case let $m_1 = \alpha + j\beta$, $m_2 = \alpha - j\beta$. Then

$$\begin{aligned} \frac{e}{E} &= \frac{\epsilon^{\alpha t}}{2j\beta}[(\alpha + j\beta)\epsilon^{j\beta t} - (\alpha - j\beta)\epsilon^{-j\beta t}] \\ &= \frac{\epsilon^{\alpha t}}{\beta}[\alpha \sin \beta t + \beta \cos \beta t] \end{aligned} \tag{110a}$$

If there is appreciable magnetizing current, the initial conditions alter somewhat. Let the voltage at point b' (Fig. 191), be E'. Then the load current

$$i_R = \frac{E'}{R_2}$$

Let the magnetizing current be expressed as a fraction Δ of i_R; that is, $i_L = \Delta i_R$. Then C initially maintains both i_L and i_R. Hence

$$-Cpe = (1 + \Delta)\frac{E'}{R_2}$$

This changes only the coefficients of the transient terms, not the exponents; consequently equation 110 becomes

$$e = \frac{E'}{m_1 - m_2}[(m_1 + 2\Delta m)\epsilon^{-m_1t} - (m_2 + 2\Delta m)\epsilon^{-m_2t}] \tag{111}$$

For the oscillatory case, equation 110a becomes

$$\frac{e}{E'} = \frac{\epsilon^{\alpha t}}{\beta} [(\alpha + 2\Delta\alpha) \sin \beta t + \beta \cos \beta t] \tag{112}$$

Equations 111 and 112 are plotted in Figs. 188, 189, and 190 after the substitutions for m and k are made.

B. STEP-DOWN TRANSFORMERS

In the circuit of Fig. 193 (p. 233) first let i_1 = variable input current; i_c = current in C; e_c = voltage across C; and i_2 = current in R_2. Then the following conditions prevail:

$$e = R_2 i_2$$

$$i_1 = i_c + i_2$$

$$i_c = Cpe_c$$

$$e_c = Lpi_2 + R_2 i_2$$

$$E = R_1 i_1 + e_c$$

Combining these we obtain

$$\frac{e}{E} = \frac{R_2(1)}{R_1LCp^2 + (R_1R_2C + L)p + R_1 + R_2}$$

and

$$\alpha + j\beta = -\frac{R_2}{2L} - \frac{1}{2R_1C} \pm \sqrt{\left(\frac{R_2}{2L} + \frac{1}{2R_1C}\right)^2 - \frac{R_1 + R_2}{R_1LC}}$$

When $t = 0$, $e = 0$, and

$$N = -\frac{ER_2}{R_1 + R_2}$$

When $t = 0$, $i_2 = 0$, and

$$e_c = 0 = \left(\frac{Lp}{R_2} + 1\right) e$$

whence

$$\frac{L}{R_2}(M\beta + N\alpha) + N = 0$$

$$M = \frac{ER_2}{R_1 + R_2}\left(\frac{\alpha}{\beta} + \frac{R_2}{L\beta}\right)$$

and

$$\frac{e}{E} = \frac{R_2}{R_1 + R_2}\left[1 + \left(\frac{\alpha}{\beta} + \frac{R_2}{L\beta}\right) \epsilon^{\alpha t} \sin \beta t - \epsilon^{\alpha t} \cos \beta t\right] \tag{113}$$

Note the similarity between equations 113 and 107, and the transposition of the resistance terms in α.

If the entering current in Fig. 193 has a constant value I after switch S is closed, such as would be the case of a pentode with constant positive voltage suddenly applied to the grid, then

$$\frac{i_2}{I} = \frac{Z_T}{R_2 + Lp}$$

where Z_T is the impedance to the right of the resistor R_1, or

$$Z_T = \frac{R_2 + Lp}{1 + R_2Cp + LCp^2}$$

If R is used in place of R_2

$$\frac{i_2}{I} = \frac{1}{1 + RCp + LCp^2}$$

and

$$\alpha + j\beta = -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$$

For the oscillatory case,

$$\frac{i_2}{I} = 1 + A\epsilon^{\alpha t} \sin \beta t + B\epsilon^{\alpha t} \cos \beta t$$

$$= 0 \quad \text{when } t = 0$$

Whence $B = -1$.

When $t = 0$, $L\dfrac{di}{dt} = 0$. Differentiating gives

$$A = -\frac{B\alpha}{\beta} = \frac{\alpha}{\beta}$$

and

$$\frac{i_2}{I} = 1 + \epsilon^{\alpha t}\left(\frac{\alpha}{\beta} \sin \beta t - \cos \beta t\right) \tag{114}$$

which is the same as equation 107 except for the coefficient and α. Equation 114 is plotted in Fig. 194.

C. LINEARITY OF SAWTOOTH TRANSFORMERS

Let the voltage e_1 of Fig. 202 (p. 251) be impressed upon the left-hand terminals of Fig. 203. Then

$$e_1 = Kt$$

$$Lpe_1 = LK$$

$$\frac{e}{e_1} = \frac{Lp}{Lp + R_1}$$

$$e = \frac{LK}{Lp + R_1}$$

$$= \frac{LK}{R_1} + A\epsilon^{\alpha t}$$

where $\alpha = -R_1/L$.

When $t = 0$, $e = 0$, and $A = -LK/R_1$,

$$e = \frac{LK}{R_1}(1 - \epsilon^{\alpha t})$$

$$\text{Slope of } e = \frac{de}{dt} = K\epsilon^{\alpha t} \tag{115}$$

Exponential $\epsilon^{\alpha t}$ is the same as that of the curve $R_2 = \infty$ in Fig. 186.

D. ANALYSIS OF VOLTAGE RISE IN A REACTOR

Let the circuit of Fig. 187 (p. 226) represent a reactor with initial voltage across it equal to zero, and initial current through it equal to I. How fast and to what value does the voltage rise after the source is disconnected?

The reactor now charges its capacitance C_2 and sends current into its loss resistance R_2.

$$e = -Lpi = 0 \quad \text{initially}$$

Reactor $i = A\epsilon^{m_1 t} + B\epsilon^{m_2 t}$. m_1 and m_2 are defined as in equation 110.

When $t = 0, i = I, A = I - B$. When $t = 0, e = 0, A = -Im_2/(m_1 - m_2)$, $B = Im_1/(m_1 - m_2)$. Whence

$$e = -\frac{LIm_1m_2}{m_1 - m_2}(\epsilon^{m_2 t} - \epsilon^{m_1 t})$$

$$\frac{e}{IR_2} = \frac{k}{\sqrt{k^2 - 1}}(\epsilon^{m_2 t} - \epsilon^{m_1 t}) \tag{116}$$

If the circuit is oscillatory, equation 116 becomes

$$\frac{e}{IR_2} = \frac{m}{\beta}\epsilon^{\alpha t}\sin\beta t$$

$$= \frac{k}{\sqrt{1 - k^2}}\epsilon^{\alpha t}\sin\beta t \tag{117}$$

Equations 116 and 117 are plotted, and m, k, and T are defined in Fig. 204.

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